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ALHACEN ON IMAGE-FORMATION AND DISTORTION IN MIRRORS

A Critical Edition, with English Translation
and Commentary, of Book 6
of Alhacen's *De aspectibus*

VOLUME ONE
Introduction and Latin Text

VOLUME TWO
English Translation

A. Mark Smith

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VOLUME ONE

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To Lois, still and always my sine qua non

CONTENTS

VOLUME I

Preface xi
Introduction xv
1. Alhacen's Analysis of Image-Distortion in Mirrors: An Overview xv
Plane Mirrors xv
Convex Spherical Mirrors xvi
Convex Cylindrical Mirrors xxii
Convex Conical Mirrors xxiii
Concave Spherical Mirrors xxv
Concave Cylindrical Mirrors xxxi
Concave Conical Mirrors xxxiii
2. The Sources for Alhacen's Analysis and Its Reception in the Latin West xxxiv
3. Conclusion xxxix
Notes xli
Manuscripts and Editing xlv
Textual Issues in the Manuscripts xlv
The Critical Text xlviii
Diagrams xlix
The Critical Apparatus xlix
The Translation and Commentary xlix
Notes li

LATIN TEXT
Chapter 1 3
Chapter 2 4
Chapter 3 5
Analysis of Plane Mirrors: Proposition 1 5
Chapter 4 7
Analysis of Convex Spherical Mirrors: Propositions 2-15 7
Chapter 5 39
Analysis of Convex Cylindrical Mirrors: Propositions 16-19 39

Chapter 6 48
Analysis of Convex Conical Mirrors:
Propositions 20-22 48
Chapter 7 58
Analysis of Concave Spherical Mirrors:
Propositions 23-32 60
Chapter 8 81
Analysis of Concave Cylindrical Mirrors:
Propositions 33-36 81
Chapter 9 92
Analysis of Concave Conical Mirrors:
Propositions 37 and 38 93
Figures for Translation and Commentary 97

VOLUME II

ENGLISH TRANSLATION
Topical Synopsis 155
Chapter 1 161
Chapter 2 161
Chapter 3 162
Analysis of Plane Mirrors:
Proposition 1 163
Chapter 4 164
Analysis of Convex Spherical Mirrors:
Propositions 2-15 165
Chapter 5 188
Analysis of Convex Cylindrical Mirrors:
Propositions 16-19 189
Chapter 6 196
Analysis of Convex Conical Mirrors:
Propositions 20-22 196
Chapter 7 204
Analysis of Concave Spherical Mirrors:
Propositions 23-32 205
Chapter 8 221
Analysis of Concave Cylindrical Mirrors:
Propositions 33-36 221

Chapter 9 230
Analysis of Concave Conical Mirrors:
Propositions 37 and 38 230
Notes 233
Figures for Introduction and Latin Text
Introduction 261
Latin Text 304

APPENDIX 330
Latin-English Index 339
English-Latin Glossary 359
Bibliography 371
General Index 387

PREFACE

As I remarked in the preface to the previous edition of books 4-5, it is easy to be misled by Alhacen's analytic focus in those books into supposing that his primary aim in studying reflection was to explain how light interacts physically with reflecting surfaces. This supposition is not unreasonable in light of Alhacen's painstaking efforts in book 4 to validate the equal-angles law of reflection experimentally and in book 5 to determine with mathematical precision the point or points on various convex and concave mirrors from which a ray of light emanating from a given spot will reflect to a given center of sight. But these efforts were part of a more comprehensive program to put the cathetus-rule of image-location on the most secure footing possible. That, for instance, is why Alhacen undertook at the beginning of book 5 to demonstrate empirically that the image of any spot on an object facing any mirror will be seen along the normal, or cathetus, dropped from it to the mirror's surface or to a plane tangent to the mirror's surface. Whereas this principle has nothing to do with the physics of light, it has everything to do with the psychology of sight. Without it Alhacen had no meaningful way to define image-location and therefore no meaningful way to explain how and why things appear to us as they do in mirrors.

Viewed from this perspective, Alhacen's study of image-distortion in book 6 takes on dual significance as an end to his reflection-analysis, not simply because it concludes that analysis but because it represents the ultimate goal for it. Accordingly, Alhacen's purpose in this, the sixth book, is to apply the cathetus-rule to an analysis of the various misperceptions that arise in the seven types of mirrors chosen for study in the previous two books. Some of these misperceptions, he informs us, are common to all mirrors, an example being image-displacement. Under no circumstances does an object actually lie where it is perceived to lie in a mirror, no matter where it appears in the reflecting surface. Certain other misperceptions, however, are specific to the type of mirror in which the image appears. These are the ones to which Alhacen devotes most of his attention in book 6. Limited to size, shape, spatial disposition, and number, such misperceptions lead us to see things as diminished or magnified, or as more curved than they are, or as reversed or inverted in orientation. Out of those misperceptions others can arise, a diminution in apparent size leading to an increase in apparent distance, and so forth.

In order to explain such misperceptions, Alhacen extends the analysis of object-points and image-points developed in book 5 to object-lines and image-lines, which he treats according to their constituent points. For the most part, in fact, he restricts his analysis to endpoints and midpoints, leaving it to us to extrapolate from them to the remainder of the line. He also leaves it to us to extrapolate from single lines to the visible surfaces containing them as cross-sections. As a result, the overall analysis in book 6 seems somewhat sketchier than that in book 5. It is also less mathematically demanding. But whatever it may lack in mathematical complexity it makes up for at the level of spatial conceptualization because so many of the constructions and proofs in it involve three rather than two dimensions. It is in this regard, in his ability to think in space, that Alhacen's peculiar analytic genius and imagination shine forth in flashes over the course of book 6. It is in this regard, as well, that this book represents not just an end, but a fitting end to Alhacen's reflection-analysis

As with the previous two editions, so with this one, I have amassed a variety of debts that I am pleased to acknowledge. First and foremost, I wish to thank the NSF for its generous support during 2005-2007 (SES 0521372). The NSF's continuing support for this project from the beginning has lightened my load considerably, and I appreciate it deeply. I also appreciate the advice and encouragement of the two program officers, Michael Sokal and Ronald Rainger, with whom I have dealt from the outset. Thanks are also due to the MU Research Council and the UM Research Board, both of which have provided generous supplementary funding over the course of this project. Thanks, too, to the American Philosophical Society for the various Franklin Grants it has awarded me during the past years. I am deeply indebted as well to Danielle Jacquart, Jean-Marc Mandosio, and various students at the École Pratique des Hautes Études, Paris, for giving me the opportunity to organize and hone my thoughts on Alhacen in a course of lectures I gave there in May and June of 2005. It was an intellectually stimulating experience that I will always cherish.

Warm thanks are also due the librarians and archivists in charge of the manuscript collections I consulted at the following libraries: Biblothèque Nationale, Paris; Biblioteca Nazionale Centrale, Florence; Bibliothèque de l'agglomération, St-Omer; Royal College of Physicians, London; Corpus Christi College, Oxford; Trinity College, Cambridge; and the Crawford Library of the Royal Observatory, Edinburgh. In one way or another each of these librarians has gone beyond the call of duty not only to facilitate my research but also to make me feel welcome at his or her institution.

As to more personal debts, they are typically legion. Thanks, first, to various colleagues in the history department here at the University of Missouri for serving as sounding boards during the course of this edition. They know

who they are; and they know how much I appreciate their patience. Thanks, too, to my family, especially my wife, Lois, for bearing with my somewhat obsessive pursuit of this editing project, which has at times conflicted with family obligations. Thanks, finally, to my editorial assistant, M. Leo Shaw, of the Anthropology Department here at the University of Missouri, who has been of invaluable help throughout the publication of this edition from the very beginning. The care and intelligence with which he has scrutinized the entire edition for mistakes and infelicities have contributed immeasurably to the final result, as have his punctilious efforts at indexing. I honestly don't know what I would have done without him.

INTRODUCTION

1. *Alhacen's Analysis of Image-Distortion in Mirrors: An Overview*

Unlike book 5, book 6 is organized in an absolutely straightforward manner that reflects its purpose to explain image-distortion in the seven types of mirrors analyzed in books 4 and 5. Thus, after two introductory chapters, Alhacen devotes the next seven, in order, to: plane mirrors (chapter 3), convex spherical mirrors (chapter 4), convex cylindrical mirrors (chapter 5), convex conical mirrors (chapter 6), concave spherical mirrors (chapter 7), concave cylindrical mirrors (chapter 8), and concave conical mirrors (chapter 9). As in book 5, so in book 6, the two types of spherical mirrors receive the most extensive treatment because their analysis forms the basis for that of the corresponding cylindrical and conical mirrors.

Plane Mirrors: Before examining misperceptions specific to image-formation in each of the seven types of mirrors—i.e., plane; convex spherical, cylindrical, and conical; and concave spherical, cylindrical, and conical—Alhacen discusses those that are common to all mirrors. First and most obvious is the spatial discrepancy between the object and its image, which invariably appears "in" the mirror, whatever its shape, and is perceived as if it were an object directly before the eyes. As such, it is subject to the same perceptual scrutiny and judgment that applies to objects seen in direct vision. Accordingly, we perceive a given image as if it were an object of a certain kind, having a certain shape, size, and color, lying at a certain distance, disposed in a certain way, and so forth.

But, as Alhacen has shown in book 3, these perceptions can be skewed if the threshold conditions for veridical vision are not properly met. Adequate illumination is one of those threshold conditions, and it is affected by the natural weakening of light due to reflection. As a result, the image may be dimmed to the point that certain of its defining features cannot be seen, or its apparent distance cannot be properly judged, or its shape cannot be properly perceived. In addition, the inherent color of the reflecting surface mingles with the color of the object's form, further dimming the image and causing a misperception of its hue. All told, then, these two factors can conspire to cause a host of misperceptions that are independent of, yet added to, the misperceptions arising from the particular shape of the reflecting surface.

Having made these points in the second chapter, Alhacen turns in the third to the simplest case of image-distortion, that due to reflection from plane mirrors. The actual analysis occupies only one theorem (proposition 1), in which Alhacen establishes that the image is the same size as its generating object and lies the same distance below the mirror as the object does above it. Granted, then, that the mirror is virtually colorless and highly reflective, the image will look almost precisely like its object in terms of color, brightness, shape, size, distance, etc. The only distortions that occur are image-displacement and image-reversal, according to which the left-hand side of the object corresponds to the left-hand side of the image, as seen from a viewpoint above the mirror on the side of the object. Thus, if the image were taken as an actual object facing the object that generates it, and if a viewer were posed exactly halfway between it and the object itself so that it could view the two directly, the left-hand side of the one would correspond to the right-hand side of the other, and vice-versa. Otherwise, the two would be perfect replicas of one another. The only distortions specific to plane mirrors are therefore image-displacement and image-reversal, and from these arises a misperception of location and spatial disposition or orientation (*situs*).

Convex Spherical Mirrors: Alhacen opens his analysis of convex spherical mirrors by pointing out that all the distortions or misperceptions arising from reflection in plane mirrors occur in these mirrors as well. This of course includes image-reversal. In addition, spherical mirrors distort both the size and shape of the image. Alhacen addresses size-distortion in the first two theorems of the chapter (propositions 2 and 3), showing in the first of them that images are almost always diminished in spherical convex mirrors. The qualification "almost always" takes us to proposition 3, which is by far the longest, most complex, and most original of the theorems in book 6. Alhacen's purpose in that theorem is to demonstrate that images seen at the very outer edge of a spherical convex mirror can actually appear magnified rather than diminished. The construction and demonstration can be summarized as follows.

Let A in figure 1, p. 261, be the center of a convex spherical mirror whose diameter is AD, and let a plane be passed through the mirror along AD to form a great circle on which arc DB lies. Extend AD to point Z such that ZD is signifcantly smaller than AD. Bisect ZD at H, produce a circle of diameter AH centered on A, and let QH be an arc on that circle. From point H draw chord QH equal to half HD. Find point T on AH such that AH:HD = HD:HT, and connect QT. Find point C such that HC = 3HT, and then find point I such that IA is the mean proportional between CA and HA, which is to say that CA:IA = IA:HA. Produce a circle of diameter IA through I, and extend line QA to intersect that circle at point N. Draw chord NI. A key consequence of the construction to this point is that NI = QT.

Now find point M on line IH in figure 1a, p. 261, such that IM:MT = AI:AT, which translates to the expression AI:IM = AM:MT, and find point L on that same line such that IL:LH = AI:AH, which translates to AI:IL = AH:LH. From points M and L in figure 2, p. 262, drop tangents MG and LB to the surface of the mirror within plane ADGB. According to proposition 7 of book 5 (in Smith, *Alhacen on the Principles*, 404), if AI:IM = AM:MT, then it follows that M is the endpoint of tangency for reflection-point G, I is the object-point, and T is the image-point. By the same token, if AI:IL = AH:LH, then it follows that L is the endpoint of tangency for reflection-point B, I is the object-point, and H is the image-point. Let us therefore drop normal AGZ_1 to reflection-point G, and let us draw line of incidence IG and line of reflection TG, extending this latter line to point T'. Consequently, angle of incidence IGZ_1 = angle of reflection $T'GZ_1$, and T will be the image of I for any center of sight placed on line T'G. Likewise, if we drop normal ABZ_2 to reflection-point B and draw lines IB and HB, extending this latter to point H', angle of incidence IBZ_2 = angle of reflection $H'BZ_2$, and H will be the image of I for any center of sight placed on line H'B.

Let us recapitulate figure 2 in figure 3, p. 263, excising tangents MG and LB as well as line of incidence IB and line of reflection HBH'. Let us then mark off arc MB on arc DB equal to arc FD. From the construction we know that points N and Q on line AN correspond to points I and H on line AI, and M has a corresponding position on line AM to that of point B on line AB. Therefore, if N is taken as an object-point and M as a point of reflection, Q will be the image, and it will be seen along line of reflection MQ, which extends to point Q'.

At the beginning of the construction it was specified that ZD be significantly smaller than AD. In figures 1-3, for instance, ZD is somewhat less than half AD, and in that case arc GB is considerably smaller than arc MG. ZD can, however, be as long as we please, and as it augments, so do IH and NQ in concert with DH and QF. Meantime, arcs FD and MB get progressively larger, while arc GB increases in size relative to arc MB until the point is reached at which arc GB = arc MG, as represented in the inset to figure 4, p. 264 (= figure 6.4.3c, p. 102). If, therefore, we continue augmenting ZD, arc GB will become larger than arc MG by tiny increments, as represented in the inset to figure 5, p. 265 (= figure 6.4.3d, p. 103).[1]

Let arc GB (i.e., RC) = arc MG (i.e., RU), as illustrated in figure 4. In that case, Alhacen goes on to demonstrate, lines of reflection QM and TG, when extended to the right, will intersect at point Z on the outer arc NZ. Accordingly, if the center of sight is placed at Z, it will see the entire line QT as the image of line NI. Furthermore, since NI = QT according to the construction, the image will be the same size as its object. Now take the case illustrated in figure 5. In this instance, with arc GB > arc MG—which it actually is, although by a minuscule amount in the figure—the two lines of reflection QM and TG, when extended to the right, will intersect at point L between points G and Z. Thus, QT will be the image of line NI for a center of sight at L. Finally, let arc GB < arc MG. In that case, as illustrated in figure 6, p. 266 (= figure 6.4.3e, p. 104), it can happen that the intersection of the two lines of reflection will occur to the left of both points of reflection, i.e., at point X between M and NI, so QT will not be visible to a center of sight posed at that point. The same in fact holds for the case in which arc GB is greater than arc MG; where the intersection occurs depends on the relative size of arcs GB and MG. Accordingly, there are innumerable instances in which the lines of reflection will intersect to the left of M and/or G, leaving QT as a whole invisible to a center of sight placed at that point. It follows more or less self-evidently, therefore, that QT will be visible only when the lines of reflection intersect to the right of G, and when it is visible, it will be the same size as its object.

So far the analysis is deficient in two respects. First, it applies to a restricted number of cases: i.e., those in which the lines of reflection from Q and T intersect to the right of G. Second, although it does show that the image can be the same size as its generating object, it does not show that it can be larger. In response to these two issues, the analysis takes a remarkable turn. Suppose that GB is smaller enough than MG that the lines of reflection QM and TG intersect at point X to the left of M, as illustrated in figure 6, p. 266. Extend TG to O and QM to Z, and drop normals AMU and AGR to the points of reflection. Hence, angles of incidence NMU and IGR = angles of reflection ZMU and OGR, respectively.

Now imagine triangle IAO containing normal AG and line of reflection TGO, all highlighted in thick lines in figure 7, p. 267, to be a rigid flap hinged along line IA so that it can be pivoted upward out of the plane of the page toward some point S above line AI. Imagine the same for triangle NAZ containing normal AM and line of reflection QMZ and hinged along line AQN. Bring both flaps together at point S, as illustrated in figure 8, p. 268 (= figure 6.4.3g, p. 106). Thus, as illustrated in figure 9, p. 269 (= figure 6.4.3h, p. 107), line AO will have migrated to AS, normal AG to AY, line of incidence IG to IY, and line of reflection TO to TS. Moreover, point Y will lie on the sphere of the mirror, which is cut along arc O'YD by the plane of reflection IAS. The entire figure SAI with all its constituent points and lines thus corresponds perfectly to the entire figure IAO with all its constituent points and lines, which means that angle of incidence IYP = angle of reflection SYP. T is therefore the image of I for the center of sight at S. By the same token, the plane of flap ASN cuts an arc on the mirror, and there is a point of reflection on that arc that corresponds to point M on arc DG. From point S, therefore, the image of point N in figure 8, p. 268, will appear at point Q, so from point S image QT of line NI will be seen in its entirety, and it will be the same size as NI.

All that remains is to show that the image can actually be larger than the object. This can be easily demonstrated by recourse to figure 10, p. 270 (= figure 6.4.3k, p. 108). For a start, we know that angle INQA is acute, so if we drop perpendicular IP from I to AQN, it will fall between Q and N and will be shorter than IN. We also know from book 5, proposition 17 (in Smith, *Alhacen on the Principles*, 414-415), that, as the object-point approaches the mirror's surface, its image recedes from the mirror's center—which is to say that it too approaches the mirror's surface from the opposite direction. Thus, since P lies nearer the mirror's surface than N, its image Q′ will lie nearer N than N's image Q. But IP < IN, and Q′T > QT, so *a fortiori* image Q′T > object IP. Moreover, any line between IN and IP will be shorter than IN, whereas the image of any such line will be longer than QT, so there are innumerable cases in which the image can be larger than its generating object.

With the issue of size-distortion out of the way in proposition 3, Alhacen broaches the subject of shape-distortion, opening his account with a series of four fairly rudimentary lemmas. In the first, which occupies proposition 4, he demonstrates that, if T and D in figure 11, p. 271 (= figure 6.4.4, p. 109), represent object-points whose forms are reflected to center of sight E from spherical convex mirror AQB, and if they are both equidistant from the mirror's center G, then the image L of point T, which lies farther from center of sight E than does D, will lie farther from centerpoint G than the image F of point D, which lies closer to E. In short, LG > FG. Furthermore, endpoint of tangency N for T, the point farther from E, will lie farther from centerpoint G than endpoint of tangency S for the nearer point D. In short, GN > GS. Then follow three interrelated lemmas, starting with lemma 2 (proposition 5), where it is demonstrated that, if line AB in figure 12, p. 271 (= figure 6.4.5, p. 109), is divided so that AB:BD = AG:GD, if three lines are produced from points B, D, and G to intersect at E, and if some line AT is drawn from A to intersect BE at T, that second line will be divided according to the same ratio: i.e., AT:TH = AZ:ZH. In lemma 3 (proposition 6), which is the obverse of lemma 2, Alhacen demonstrates that, if line AB in figure 13, p. 272 (= figure 6.4.6, p. 110), is divided according to the ratio AB:BD = AG:GD, and if line AT dropped at a slant from A is divided according to the ratio AT:TH = AL:LH (L being a surrogate for Z from the previous proposition), then, when they are extended, the lines passing from B through T, D through H, and G through L will intersect at some point E. The fifth and final lemma (proposition 7) is so simple as to be virtually self-evident: if line AB in figure 14, p. 272 (= figure 6.4.7, p. 110), is divided according to the ratio AB:BD = AG:GD, if lines GZ, DH, and BT are parallel, and if line AT is dropped to BT, that line will be cut according to the same ratio—i.e., AT:TH = AZ:ZH.

In the next four propositions, Alhacen deals with the situation in which various object-lines face the center of sight more or less frontally. The first point he wants to establish is that in convex spherical mirrors images are curved with respect to the mirror's center of curvature, not its surface. Were the latter to be the case—that is, were the image to take on the shape of the mirror's surface—it would follow that the image is somehow impressed on that surface, a point that Alhacen was at some pains to refute in book 4.[2] Alhacen addresses this issue in proposition 8, where he starts by supposing that arc AEB in figure 15, p. 272 (= figure 6.4.8, p. 110), is concentric with the mirror, which is centered on G. Arc Z'Y' is the great circle on the mirror formed by the plane passing through AEB and G. From center of sight D drop DG perpendicular to this plane. In that case, Alhacen concludes, image QML of arc AEB will also be concentric with the mirror. However, since it lies nearer than the mirror's surface to centerpoint G, its curvature will be sharper than that of the mirror's surface.

On the other hand, if DG is not perpendicular to the plane of the arc, as in figures 16 and 17, p. 273 (= figure 6.4.8a and 6.4.8b, p. 111), then the curvature of image TQF of arc ECB in figure 17 will clearly not conform to that of the mirror's surface. In order to define the curvature of image TQF, Alhacen argues as follows according to that figure. N is the endpoint of tangency for object-point E, and F is its image.[3] Likewise, M is the endpoint of tangency for point C, and its image is Q, whereas L is the endpoint of tangency for point B, and T is its image. Therefore, according to book 5, proposition 7, GE:EN = GF:FN, GC:CM = GQ:QM, and BG:GL = GT:TL. By proposition 5, lemma 2, then, lines BC, ML, and QT, when extended, will intersect at point O, and when line OQ is extended toward EG, it will intersect it at point K. According to the same lemma, lines EC, NM, and FQ, when extended, will intersect at point P. Since image-point F of E lies below point K, the whole image TQF must be curved. As we will see, Alhacen applies this same analytic technique in several subsequent theorems.

In propositions 9 and 10 Alhacen undertakes to show that the images of curved lines facing the center of sight with their concavity toward the mirror's surface and their endpoints equidistant from the mirror's center (such as ADB and AEB in figure 19, p. 275 [= figure 6.4.9, p. 112]) will appear curved. Likewise, the images of straight lines facing the center of sight with their endpoints equidistant from the mirror's center (such as ACB in figure 20, p. 275 [= figure 6.4.10, p. 112]) will appear curved. Also, given the two curved lines ADB and AEB in figure 19, the one that is more sharply curved (i.e., AEB) will yield a less sharply curved image than the other. The same holds for arc AEB in figure 20; its image ZTH is less sharply curved than image ZMH of straight line ACB. Alhacen then concludes by showing in proposition 11 that straight line AB in figure 21, p. 276 (= figure 6.4.11, p. 113), whose endpoints A and B are not equidistant from the mirror's center, will yield a curved image, i.e., ZNT, which is more sharply curved than image MZ of curved line AE. In all these cases, moreover, neither the visible line-segments nor their extensions ever touch the mirror's surface.

The next two theorems, propositions 12 and 13, are devoted to showing that, when a given straight line-segment does touch the mirror's surface, that line-segment will generally yield a curved image. For instance, in proposition 12 Alhacen describes the following situation illustrated in figure 22, p. 277 (= figure 6.4.12, p. 114). Let AB be a visible line-segment whose extension BE is tangent to the mirror at E, and let D be the center of sight. Let AB be posed with respect to the center of sight such that plane of reflection DGA cuts arc PZ on the mirror, while plane of reflection DGB cuts arc PH on the mirror. To demonstrate that the image of AB will be curved, Alhacen has recourse to the analytic technique he employed in proposition 8. Accordingly, N on line AG represents the endpoint of tangency for point A, whose image is I, whereas M on line BG represents the endpoint of tangency for B, whose image is O. On that basis, it follows from book 5, proposition 7, that GA:AN = GI:IN, and GB:BM = GO:OM. Since both lines are divided equiproportionally, then, by proposition 5, lemma 2, the lines passing through image-points I and O, endpoints of tangency N and M, and object-points A and B will intersect at some point Q. Find image-point U of Q. Since this point lies below line IOQ, the overall image IOU of line ABQ must be curved, as must be the image of its segment AB. The case in which a rectilinear object-line or its extension actually cuts the mirror's surface is then taken up in proposition 13, whose primary purpose is to show that in every such case but one the image will be curved. The exception occurs when the line or its extension intersects the mirror's center, in which case the image will be rectilinear, a point Alhacen has already established empirically early in the second chapter of book 5.[4]

In the following theorem, proposition 14, Alhacen discusses a range of situations in which object-lines posed in various ways with respect to the mirror and the center of sight may or may not yield a visible image. He then concludes his analysis of shape-distortion with proposition 15, where he demonstrates that, when a rectilinear object-line and the center of sight lie in the same plane, and when the line does not touch the mirror, its image will be curved. Overall, Alhacen's analysis of shape distortion in convex spherical mirrors is organized according to the orientation of the object-line with respect to the center of sight. First, the orientation may be frontal, as illustrated in the top diagram of figure 23, p. 278, where E is the center of sight and AB the object-line, as seen from above. This is the orientation assumed for propositions 8-11. On the other hand, the orientation may be oblique, as illustrated in the middle diagram of figure 23. This is the orientation assumed for proposition 12 and 13. Or, finally, the object-line and the center of sight may lie in the same plane, as illustrated in the bottom diagram of figure 23. This, of course, is the orientation for proposition 15. Whatever its orientation, the object-line may be disposed in various ways with respect to the mirror's surface according to whether its endpoints are equidistant from the center of curvature, and according to that disposition, it or its extension may or may not touch the mirror's surface.

Convex Cylindrical Mirrors: In chapter 5 Alhacen passes on to convex cylindrical mirrors, pointing out in the introductory paragraph that images in these mirrors are subject to the same distortions of size and shape that arise in convex spherical mirrors, although those distortions are more pronounced in the former. Before addressing actual cases, however, Alhacen offers a prefatory lemma in proposition 16. In this, the fifth and penultimate lemma of the book, Alhacen proposes the following. Let a plane be passed through a cylindrical mirror to form an elliptical section on its surface. Let that section be RBEOA in figure 24, p. 279 (= figure 6.5.16, p. 124), let point B on it be a point of reflection, and choose some other point E on it. Drop normal BD from the point of reflection so as to intersect the cylinder's axis at point D. This normal will thus form the diameter of circle BTO passing through point B, and that diameter will be the minor axis of the elliptical section. Then drop normal EU from point E within the plane of the ellipse. When extended, Alhacen concludes, this normal will intersect the extension of normal BD at some point U, and it will bypass the axis. The purpose of this lemma is to establish that, if the elliptical section lies in the plane of reflection, and if EU is the cathetus of incidence dropped from an object-point, such as N, it will lie farther from the cylinder's axis than the normal EK dropped through point E within circle ES because that normal necessarily intersects the axis at the circle's center.

Alhacen begins his actual analysis of image-distortion in proposition 17 with two cases, in the first of which no distortion in fact occurs. Illustrated in figure 25, p. 280 (= figure 6.5.17, p. 126), this is the case in which straight object-line TH and center of sight E lie in the same plane, which cuts the mirror's surface along line of longitude AG and passes through axis ZK. Image T'H' of line TH will therefore be deformed in neither size nor shape because it is produced in precisely the same way it would have been produced in a plane mirror.[5] In the second case, however, object-line TH, which is still posed upright and parallel to axis ZK, does not lie in the same plane as the center of sight E. Thus, as illustrated in figure 25a, p. 280 (= figure 6.5.17a, p. 126), E is displaced to the side of TH. That point granted, Alhacen goes on to prove that the entire form of line TH will reflect to center of sight E from some line of longitude GA on the mirror's surface. On that basis he shows in proposition 18 that image ICS of line TH in figure 26, p. 281 (= figure 6.5.18, p. 127), will be curved, although its curvature will be virtually indiscernible to center of sight E because its convexity faces it directly.

To demonstrate this point, Alhacen assumes that object-point Q, reflection-point B, and center of sight E lie in a plane of reflection that forms circle BF on the mirror, whereas planes of reflection TGE and HAE form elliptical sections on the mirror, T and H being equidistant from Q. According to proposition 16, lemma 5, then, catheti TU and HU pass behind the axis, while cathetus QL intersects it, so TU and HU lie farther from E than does QL. From this it follows that image-points I and S also lie farther from E than image-point C—hence the curvature of the image.

Finally, in proposition 19 Alhacen demonstrates that, if line TQH faces the mirror horizontally rather than vertically, as in figure 27, p. 282 (adapted from figure 6.5.19, p. 128), and if the center of sight E lies above it, TQH's image will be convex with respect to E. In this case, Q is assumed to be the midpoint of TH and to lie in the plane formed by line EX, which passes through the center of the top base of the cylinder, and axis DX. Thus, the form of H will reflect to E from point B, and the form of T will reflect to E from point G, and the image of H will lie at R, where line of reflection EB intersects cathetus HU. Likewise, the image of T will lie at point Y on cathetus TU, as represented in the bottom diagram of figure 27, which gives a bird's-eye view of the situation in the top diagram. The form of Q, however, will reflect from point K on line of longitude AZ, which lies in the plane formed by EZ and XD, so its image will lie at point P on cathetus QAC, where it is intersected by line of reflection EK. As is clear from the lower diagram, point P lies significantly closer to the reflecting surface than line RY, and it lies in a lower plane. The resulting image RPY is therefore convex and manifestly curved with respect to center of sight E. It is also smaller than its object. From all this it is therefore clear that the more line TH approaches the vertical—which is to say the closer it is to being parallel to the mirror's axis—the less curved its image will appear. It is also clear that the more the line approaches the horizontal, the more sharply curved its image will appear and, in addition, the more it will shrink in size along the horizontal.

Convex Conical Mirrors: As Alhacen observes at the beginning of chapter 6, convex conical mirrors produce the same sorts of image-distortions as do convex cylindrical mirrors, and they do so according to the same conditions.

Thus, the closer to vertical the object-line, the less curved its image will appear, whereas the closer to horizontal it is, the more curved and shrunken its image will appear. Unlike those in convex cylindrical mirrors, of course, the distortions in convex conical mirrors are affected by the conical shape of the mirror, the distortion being most pronounced at the mirror's vertex and least pronounced at its base.

In the first theorem of this chapter, proposition 20, Alhacen provides the sixth and final lemma, which is to the analysis of convex conical mirrors precisely what proposition 16, lemma 5 is to the analysis of convex cylindrical mirrors. Let E in figure 28, p. 283 (= figure 6.6.20, p. 129), be a point of reflection on convex conical mirror AGZR, and let conic section BFEZ pass through it. That section will thus lie in the plane of reflection. Pick some point Z on the conic section. Then, within the plane of the conic section drop normal ED to point E and normal ZX to point X. Those two normals will intersect at point X, and ZX, which will be the cathetus of incidence for object-point H, will bypass axis AK of the cone to its right.

In the next two theorems Alhacen follows essentially the same line of analysis he followed in propositions 17 and 18 pertaining to convex cylindrical mirrors, albeit with some modifications. Thus, in proposition 21, he demonstrates that, if straight line-segment ON in figure 29, p. 283 (= figure 6.6.21, p. 129), faces the mirror such that its continuation OA intersects the mirror's vertex, and if C is a center of sight facing the mirror to the side of ON, the entire form of ON will reflect to C from some line of longitude AZE on the mirror. Presumably, Alhacen ignores the case in which ON and C lie in the same plane, which passes through the mirror's axis, because it is obvious that in such a case the image of ON will be produced as if the reflection had occurred from a plane mirror. Furthermore, in this case, unlike those in propositions 17 and 18, ON is not parallel to a line of longitude on the mirror's surface because, if it were, its form would not reflect to C from a straight line on that surface.

Having therefore demonstrated both that and how the form of ON will reflect to the center of sight from line of longitude AE, Alhacen shows in proposition 22 that, under these conditions, the image of the entire line AON, as seen from center of sight R in figure 30, p. 284 (= figure 6.6.22a, p. 131), will be slightly curved, that image being represented by line APY. As was the case in proposition 18, the image's curvature will be essentially indiscernible to the eye at R both because it is slight and because its convexity faces the eye directly. However, unlike the image in proposition 18, this one will be slightly shorter than the line itself insofar as straight cross-section AY of APY < AN.

The transition from proposition 21 to 22 marks a shift in translators that is indicated in several ways. First, and perhaps most obvious, the initial half of proposition 22 is simply a recapitulation of proposition 21 in somewhat different format. Second, from the beginning of proposition 22 on, there are significant changes of both vocabulary and style. And finally, in two of the seven manuscripts collated for the critical text an alternative version of the beginning of book 6 to the end of proposition 20 is interpolated between propositions 21 and 22, this latter theorem flowing naturally from proposition 20 of the interpolated text.[6]

Having finished the demonstration in proposition 22, Alhacen brings chapter 6 to a close with a general description of how and why line-segments appear distorted in size and shape according to their position and orientation with respect to the surface of a convex conical mirror. The nearer to the mirror's vertex a line-segment's image appears, for instance, the more curved and truncated it will be. Likewise, the more horizontal the line-segment is with respect the mirror's surface, the more curved and truncated its image will be.

Concave Spherical Mirrors: Aside from the misperceptions due to the weakening of light and color, Alhacen informs us in the rather extensive introduction to chapter 7, that concave spherical mirrors produce a host of misperceptions that do not occur in plane and convex mirrors. For one thing, size-distortion is far more variable and complex in spherical concave mirrors than in convex mirrors. For another thing, concave spherical mirrors can yield as many as four images of a single object-point, whereas plane and convex mirrors can never yield more than one image. Much of book 5 is devoted to demonstrating these points. For yet another thing, images are always located behind the reflecting surface of plane and convex mirrors, whereas in concave spherical mirrors they can lie behind, on, or in front of the reflecting surface.[7] Furthermore, while reflection from plane and convex mirrors causes image-reversal, reflection from concave spherical mirrors can yield upright and reversed images as well as inverted ones. And, finally, reflection from concave spherical mirrors can cause shape-distortion in the resulting images.

Having laid out the basic order of analysis for chapter 7 in this preliminary section, which occupies the first seven paragraphs of the chapter, Alhacen addresses three of the issues raised there in propositions 23-28: size-distortion, variation in image-location, and variation in image-orientation. Accordingly, he begins in proposition 23 by locating the center of sight T in figure 31, p. 285 (= figure 6.7.23, p. 132), on radius OU of the mirror BUG such that it lies between midpoint O of that radius and the reflecting surface. In that case, he concludes, if line MTN is taken very roughly to represent a cross-section of the surface of an eye facing the mirror, then, as seen by center of sight T, image FQ of that cross-section will appear behind the reflecting surface and will be larger than its object.[8] With that established, Alhacen goes on in proposition 24 to erect KT at point T perpendicular to plane BUG of the mirror. Let K in figure 32, p. 285 (= figure 6.7.24, p. 132), be a center of sight on that line, and let ABK be a plane of reflection within which the form of some point M reflects to K from point B, and let its image be F. Let AGK be a plane of reflection within which the form of point N reflects to K from point G, and let its image be Q. Therefore, image FQ of line MN will appear behind the mirror, and it will be larger than its object.

In the next three propositions, i.e., 25-27, Alhacen shows that, under the right circumstances, the image seen in a concave spherical mirror can be the same size as, larger than, or smaller than its object. It can also appear reversed or upright, and it can be seen in front of or behind the reflecting surface. In proposition 25, for instance, Alhacen has ZAB in figure 33, p. 286 (= figure 6.7.25, p. 133), be a great circle on a concave spherical mirror centered on E, with GZE a randomly drawn line in the plane of that circle. Line DG is erected perpendicular to line GE, and from point D lines DA, DB, and DE are drawn to the plane of circle ZAB. DE is extended below that plane toward point O. Then, from some point K on DA line KE is passed through point E in the plane of circle ZAB and extended to point L so that KE = EL. Point T is located on line DB such that TE = KE, and TE is extended to point H such that TE = EH. Then lines AE and BE are drawn in the plane of the circle. Consequently, angles DKAE, LAE, DTBE and HBE will all be equal, and lines KT and HL will also be equal.

Let D be a center of sight and LH an object-line inside the mirror. The form of L will thus reflect to D from A, the form of H will reflect to D from B, and the image of line HL will be KT, where lines of reflection DA and DB intersect the extensions of catheti LE and HE. Since KT = LH, therefore, the image will be the same size as its object, and it will lie between the reflecting surface and the center of sight. Moreover, since the right-hand side H of the object will lie on the left-hand side T of the image with respect to center of sight D, and conversely, since the left-hand side L of the object will lie on the right-hand side K of the image, the image will be reversed and, in fact, inverted if LH is oriented parallel to DG.

Now connect HB and AL, and let them intersect at O, through which the extension of DE passes. Let O be a center of sight and KT an object-line behind the center of sight. From O's point of view, then, KT's image will be LH, and it will be the same size as its object. Furthermore, it will lie between the center of sight and the reflecting surface. It will not be reversed or inverted, however, because the right-hand side T of the object as viewed face-on from O is also the right-hand side H of its image as viewed face-on from O, while the left-hand side K of the object is also the left-hand side L of the image from the same point of view.

Using the same approach in proposition 26, Alhacen extends lines BH and AL in figure 34, p. 286 (= figure 6.7.26, p. 133), to points R and M respectively so that RB = AM. He then draws RE and ME, continuing them past point E until they intersect DA and DB at points U and N. Since RE > EN, and ME > EU, it follows that RM > UN. If, therefore, MR is an object-line and D a center of sight, MR's image will be NU, and it will be smaller than its object.[9] It will also be inverted. On the other hand, if O is the center of sight and NU the object-line, NU's image RM will be larger than it and will appear upright. In the same vein, finally, Alhacen shows in proposition 27 that, if CI in figure 35, p. 287 (= figure 6.7.27, p. 134), is the object-line and QF its image as seen from center of sight O, the image will be smaller than the object and will appear upright. On the other hand, if FQ is the object-line and D the center of sight, image CI will be longer than its object and will appear inverted.

In the preceding five propositions Alhacen has dealt with two basic cases. In the first case, which occupies propositions 23 and 24, the center of sight and the object-line lie in a plane between the mirror's center of curvature and the reflecting surface. In the second case, which occupies propositions 25-27, the center of sight and the object-line are located either between the center of curvature and the reflecting surface or on either side of the center of curvature. Alhacen addresses the third and final case in proposition 28, where both the center of sight and the object-line lie beyond the center of curvature. Hence, as illustrated in figure 36, p. 288 (= figure 6.7.28, p. 135), G is the mirror's center of curvature, BDA an arc on the reflecting surface, HZ the object-line, which represents the surface of the eye, and E the center of sight. Under these conditions, Alhacen concludes, image LK of HZ will be smaller than its object and will lie between the center of sight and the reflecting surface BDA. It will also be inverted.

The remainder of chapter 7, which consists of propositions 29-32, is devoted to showing not only how the shape and orientation of images can be distorted in concave spherical mirrors, but also how such distortion can vary according to the number of images formed. In proposition 29, for instance, Alhacen shows that, if we choose two diameters OEA and DEB in concave spherical mirror BDO represented in figure 37, p. 289 (= figure 6.7.29, p. 136), then, when Z is taken as a center of sight and GR as a rectilinear object-line, the image LK will be smaller than its object. It will, however, be rectilinear, like its object, and it will have the same left-to-right and top-to-bottom orientation with respect to the mirror: i.e., point G, which lies to object-line GR's right from the mirror's perspective, will appear at point K, which lies to image-line LK's right from the perspective of E, whereas point R, which lies to GR's left from the mirror's perspective, will appear at LK's left from the perspective of E.

The same holds by extension when the object-line GR is convex or concave. Thus, as Alhacen shows in proposition 30, if the object-line is GNR in figure 38, p. 289 (= figure 6.7.30, p. 138), and if it is convex with respect to reflecting surface ODB, its image LIK will be convex with respect to center of sight E It will also be smaller than its object and will have the same orientation. On the other hand, if the object-line is GQR, which is concave with respect to the reflecting surface, its image LCK will be concave with respect to center of sight E. It too will be smaller than its object and will have the same orientation.

In the last two theorems of chapter 7, Alhacen takes up the problem of multiple images and the various ways in which they can be distorted. He starts in proposition 31 by assuming that E in figure 39, p. 290, is a center of sight and Z an object-point, both of them facing reflecting surface ABDX and both equidistant from centerpoint G of the mirror. It follows, then, that the form of Z can reflect to E from three points on the arc directly facing E: i.e., D, B, and A. Accordingly, Z will have three images, of which two are of special interest here: the one lying at M, where the extension of line of reflection BE intersects the extension of cathetus GZ, the other lying at L, where the extension of line of reflection DE intersects that same cathetus. Granted these conditions, finally, choose some point N on that cathetus such that its form will reflect to center of sight E from some point K′ on the mirror. Its image will lie at Q, where the extension of line of reflection K′E intersects the extension of cathetus GZ. Altogether, then, object-points Z and N will yield three images along cathetus GZ: L and M for Z and Q for N.

Now let us drop a line CZR through point Z perpendicular to plane ABDX, as illustrated in figure 39a, p. 290, and taking NG as radius, let us form circular segment CNR intersecting perpendicular CZR at points C and R such that CZ = ZR. The planes formed by line EG, which lies in the plane of circle ABX, and lines CG and RG oblique to the plane of circle ABX will intersect the sphere of the mirror to form arcs on it equivalent to arc ABDX in figure 39, and within those arcs there will be points perfectly equivalent to K′ from which the forms of C and R will reflect to E, so, from center of sight E's perspective, S will be the image of C within plane of reflection EGC and O the image of R within plane of reflection EGR. Accordingly, since Q is the image of N, the whole of arc SQO will be the image of arc CNR, and it will appear behind the reflecting surface. Moreover, since arc CNR is convex with respect to the reflecting surface, its image will be concave with respect to that same surface as well as to the center of sight, and the image will be larger than its object. On the other hand, if straight line CZR is taken as the object-line, its image will be SLO, which lies behind the mirror and is concave with respect to the center of sight. It too will be larger than its object.

It was established earlier that for center of sight E point Z has at least two images within arc ABD of the mirror, one at L, the other at M. However, from book 5, proposition 37, case 3 (in Smith, *Alhacen on the Principles*, 456-458), we know that, given the conditions of the analysis, according to which center of sight E and object-point Z are equidistant from the center of the mirror's curvature G, object-point Z will produce four images for E. Consequently, as illustrated in figure 39b, p. 291, the two images L and M are produced by the reflection of Z's form from points D and B, respectively, and they lie where lines of reflection DE and BE intersect cathetus ZG. The form of Z also reflects from point A to yield an image at point F, where line of reflection AE intersects cathetus ZG. And the fourth reflection is from point I directly opposite B on diameter BG, its image being T′, where line of reflection IE intersects cathetus ZG. It has also been established that from E's perspective points C and R in figure 39c, p. 291, produce images at S and O, respectively. Accordingly, as we noted before, concave line SLO is an image of straight line CZR.

Imagine that center of sight E is able to scan the entire arc on the mirror just ahead of point B and just beyond line AEF, as represented by the thick lines in both figures 39b and 39c. Imagine, as well, that it can see all four images of Z at L, M, T′ and F, as well as images S and O of points C and R. Accordingly, straight line CZR will yield four separate images according to the four images of midpoint Z, all four images having S and O as their endpoints and all four being concave with respect to the reflecting surface. As we already noted, one of these images will be curved line SLO in figure 39c. Another will be the curved line passing from S through M to O. Yet another will be the curved line passing from S through T′ to O. And the final one will be the curved line passing from S through F to O. Furthermore, according to their placement with respect to center of sight E, points C and R can have as many as four images each within the facing portion of the mirror, so the number of images for straight line CZR will be multiplied commensurately.

Having established in proposition 31 that both straight and convex lines can yield multiple concave images, Alhacen concludes the theorematic portion of chapter 7 by showing in proposition 32 that straight lines can yield convex images, whereas concave lines can yield straight or convex images. Let us start by assuming that arc AG in figure 40, p. 292, lies on the surface of a spherical concave mirror centered on point D. Let H be a center of sight lying on radius DA of the mirror, and on line DGQ let O and U be object-points whose forms are reflected to H from points B and F on the mirror. Point Q, where line of reflection BH intersects cathetus DO, will thus be the image of O as seen from H, and point N will be the image of U as seen from H.

Rotate line DHA 90° counterclockwise out of the plane of circle GBA on axis DQ so that center of sight H is carried to point H′ in figure 40a, p. 293, which represents a three-quarter view of the situation from above. Using DO as a radius, form arc EOZ, and pass line EUZ through U perpendicular to line DO so as to intersect arc EOZ at points E and Z. According to previous analysis, the form of O reflects to center of sight H from point B to yield an image at Q. So too, the form of U reflects to H from point F to form an image at N. Since DH′ is perpendicular to DH, and since the two lines are equal, it follows that the plane formed by DH′ and DNQ will cut an arc on the mirror equivalent to arc AG. This arc is A′G in figure 40b, p. 293, and within it there will be two points B′ and F′ equivalent to points B and F in arc AG. Thus, the form of O will reflect to H′ from point B′ to yield an image at Q, and the form of U will reflect to H′ from point F′ to yield an image at N.

Now extend line DE toward K and DZ toward T in figure 40a so that DK and DT are both equal to DQ. Since points E and Z lie precisely the same distance from the reflecting surface as point O, then within their respective planes of reflection H′DK and H′DT their forms will reflect to H′ from points equivalent to point B′ on arc A′G in figure 40b. K and T will therefore be their respective images, and they will lie the same distance from the center of sight as does Q. The same holds for every point on arc EOZ, so its image will be arc KQT of radius DQ. Hence, line EOZ, which is convex with respect to the reflecting surface, will yield an image KQT that is concave with respect to center of sight H'. Furthermore, since the image of point U is at N, the overall image of straight line EUZ will have its endpoints at K and T and its midpoint at N, which means that the image will be convex with respect to the center of sight. In both cases the image will also be larger than its object.

In order to demonstrate, finally, that a concave line can yield a convex image, Alhacen has us select some random point M on line UZ in figure 41, p. 294 (= figure 6.7.32d, p. 143), and form a circle with radius UM such that it will intersect arc EOZ at points R and F to create concave arc RUF. We are then to draw lines DR and DF, extending them to points C and I on arc KQT. Since points R and F lie on arc EOZ, and since we have shown that every point on that arc yields an image on arc KQT, C and I will be the images of R and F from H′s point of view. On the other hand, point U on concave arc RUF has its image at N according to that same point of view. Therefore, the image of concave arc RUF will pass from C through N to I and will be convex with respect to the reflecting surface.

The reason for this rather elaborate analysis of multiple images and their distortion in concave spherical mirrors comes clear in the closing paragraphs of chapter 7, where Alhacen points out that the surfaces of visible objects consist of various lines and that the way those lines are perceived will determine the way those surfaces will be perceived. Thus, if a given straight line on a flat surface yields a multitude of images of various curvatures and locations, then the surface will appear deformed according to those curvatures and locations. Suffice it to say, when the surface as a whole is analyzed according to several of its constituent lines, the resulting composite image will be confused to the point of incoherence. That, of course, is why objects posed close to the surfaces of concave spherical mirrors so often yield chaotic, blurry images spread out over the reflecting surface.

Concave Cylindrical Mirrors: Occupying chapter 8 of book 6, Alhacen's analysis of image-distortion in these sorts of mirrors unfolds in propositions 33-36. In the first theorem of this series, Alhacen reverts to the construction for proposition 18, which deals with image-formation in convex cylindrical mirrors. That construction is illustrated in figure 26, p. 281. First, let the cylinder represent a convex mirror faced by object-line TQH and center of sight E. According to the conditions set by Alhacen in this context, the form of straight line TQH will reflect to E from line of longitude GBA on the mirror's surface, and the resulting image will be curved line ICS. As part of the construction, moreover, Alhacen has continued lines of incidence TG, QB, and HA to converge at point O and then demonstrated that EG = GO, EB = BO, and EA = AO.

Now let us reverse the situation by designating O as a center of sight and ICS as an object-line, both of them facing the concave surface of the cylinder. By extension from the previous account of reflection from the convex surface of the cylinder, the form of I will reflect from G to O, the form of C will reflect from B to O, and the form of S will reflect from A to O, and the resulting image will be TQH. Thus, the image will lie behind the mirror, it will be larger than its object, and it will be rectilinear, unlike its object, which is convex with respect to the reflecting surface. The amount and type of curvature can vary, however, if point C is moved to and fro along line of reflection EB. Accordingly, the image of C can appear beyond point Q, leaving the whole image TQH concave with respect to the center of sight at O. Or it can appear between points Q and B, leaving the whole image TQH convex with respect to O. Furthermore, depending on its location with respect to center of sight O and the axis of the cylinder, C can have as many as four images, so there can be as many as four images of line ICS with endpoints I and S fixed in the position illustrated in the figure. Meantime, of course, I and S can each have as many as four images, so the number of possible images of ICS will be multiplied commensurately.

As in proposition 33, so in proposition 34 Alhacen reverts to an earlier construction, this one pertaining to proposition 19, dealing with convex cylindrical mirrors. In that theorem it was shown that, when straight line TQH and center of sight E in figure 27, p. 282, face the convex surface of the cylinder, with E lying above TQH, the resulting image RPY behind the reflecting surface will be noticeably convex and shorter than its object. One step in the construction for that theorem involved extending lines of incidence HB and TG until they intersect at point L, through which line ED is drawn.

Now, if we take L as a center of sight facing the concave surface of the mirror, and if we take straight line RY as an object line, then it follows from symmetry that, just as the form of H reflects to E from B, the form of R will reflect from B to L, and the same holds for point Y, which will reflect from G to L. Accordingly, the image of R will lie at H, where the extension of line of reflection BL intersects cathetus HU, and the image of Y will lie at T, where the extension of line of reflection GL intersects cathetus TU. Let M in figure 42, p. 295 (= figure 6.8.34b, p. 149), be the point at which line RY is bisected by the plane formed by EX' and axis X'XD. The form of M will therefore reflect from point F on line of longitude AZ within that plane, as represented in the lower diagram of the figure. Its image will lie at S, where the extension of line of reflection FL intersects cathetus SQM, which lies on the diameter of the circle passing through Q with its center at axis X'XD. Furthermore, as is clear from the diagram, S lies beyond line TH and higher than it. It follows, then, that image HST of straight line RMY will be larger than its object, and it will be manifestly concave with respect to center of sight L.

Proposition 35 addresses the case in which the plane of reflection is elliptical, as represented in figure 43, p. 296 (= figure 6.8.35, p. 150), where BG represents an arc on an ellipse that also passes through point A on the opposite side of the cylinder. Since G and A lie at the intersection of the ellipse and the circle passing through G and A, diameter GA of that circle will be the minor axis of the ellipse. Let BK be normal to the ellipse at point B. Thus, BK and GA will intersect at some point E beyond the axis of the cylinder, and line BK will lie behind that axis. Let LMK represent an object-line, let G and A represent points of reflection at the respective ends of minor axis GA, and let D be a center of sight lying on DG, which is constructed parallel to BK. According to D's particular placement, then, the form of K will reflect to D from O, and its image will be N. Meantime, the form of M will reflect to D from A, and its image will be T, and the form of L will reflect to D from G, yielding an image at G on the very surface of the mirror.[10] All told, therefore, the resulting image TGN of straight line LMK will be concave with respect to D.

In proposition 36, finally, Alhacen reverts back to the analysis of size-distortion and image-reversal in concave spherical mirrors provided in propositions 26 and 27. Thus, in figure 44, p. 297 (= figure 6.8.36, p. 151), D and O can be taken as respective centers of sight, and IC, KT, and UN can be taken as cross-sections of either objects or images tied, respectively, to QF, HL, or RM as the corresponding images or objects. Thus, if IC is taken as an object-line and D as the center of sight, the image QF will be smaller than its object, and it will appear reversed (but not inverted) in a concave cylindrical mirror. With IC as an object-line and O as a center of sight, on the other hand, the image QF will still be smaller than its object, but it will not be reversed. By the same token, if QF is taken as the object-line and IC its image, then the image will be larger than the object for both D and O, but for D the image will be reversed, whereas for O it will not. And the same sort of analysis can be applied to the other two object-image pairs KT and LH and UN and RM.

Concave Conical Mirrors: Consisting of only two theorems, i.e., propositions 37 and 38, chapter 9 on concave conical mirrors is by far the shortest of book 6. In the first of these theorems, Alhacen reverts back to proposition 22, which deals with the formation of convex image APY of straight line AON in the top diagram of figure 30, p. 284. According to his analysis there, the form of the entire line AON reflects to center of sight R from line of longitude AZE. Accordingly, lower endpoint N reflects to R from E to yield an image at Y, upper endpoint A is identical with its image at the cone's vertex, and midpoint O reflects to R from Z to yield an image at P. In proposition 37 Alhacen merely reverses the analysis, as illustrated in figure 45, p. 298 (= figure 6.9.37, p. 152), where APY is the object-line and F the center of sight, both facing the concave surface of the cone. Hence, the form of Y reflects to F from E to yield an image at N, the image of A lies at point A itself, and the form of P reflects to F from Z to yield an image at O. In this case, then, line APY, which is convex with respect to the reflecting surface, will yield a rectilinear image. If, however, midpoint P is moved to and fro in line with RZP, then the resulting image AON can take on a convex or concave curvature with respect to the center of sight. Furthermore, in this case, with A and N the endpoints of whatever image is produced, the image will be larger than its object.

The thirty-eighth and final proposition of book 6, is essentially a recapitulation of proposition 36, where Alhacen provides a general analysis of size-distortion and image-reversal in concave cylindrical mirrors. The same analysis applies to concave conical mirrors and is based on figure 44, p. 297, where CI and QF, KT and HL, and NU and RM can be taken as object-image couples according to whether the center of sight is posed at D or O. On that basis it can be shown that a given image can be equal to, smaller than, or larger than its object, depending on the placement of the object and the center of sight with respect to the mirror's center of curvature at E. On that same basis, moreover, it can be shown that the image will sometimes be reversed and sometimes unreversed—again depending on the relative placement of the object and the center of sight with respect to the mirror's center of curvature.

2. *The Sources for Alhacen's Analysis and Its Reception in the Latin West*

Not surprisingly, the mathematical and optical sources for Alhacen's analysis of reflective image-distortion in book 6 do not go beyond those used in book 5.[11] In fact, unlike his analysis in book 5, that of book 6 makes no explicit use of Apollonius' *Conics*, so the proofs in book 6 are based entirely on Euclidean reasoning. As regards optical sources, there is no need to look past Ptolemy's account of image-distortion in plane and convex mirrors in books 3 and 4 of his *Optics*.[12] Aside from elaborating on, and re-organizing that account to include cylindrical and conical mirrors, Alhacen's only real innovation—and he admits as much—lies in his demonstration in proposition 3 that images seen at the very edge of convex spherical mirrors can be magnified. Otherwise, his analysis is essentially Ptolemaic in style, apart, of course, from his reliance on light-rays rather than visual rays and his more rigorous approach to mathematical analysis.

Like his general account of vision, Alhacen's analysis of image-distortion in mirrors followed the path along which *De aspectibus* as a whole was disseminated either directly, through manuscript copying, or indirectly, through derivative works, such as Roger Bacon's *Perpsectiva*, Witelo's *Perspectiva*, and John Pecham's *Perspectiva communis*.[13] As was the case with his study of reflection in book 5, the complex mathematical details of Alhacen's analysis of image-distortion in book 6 seem to have excited little or no interest among his Latin followers—apart, that is, from Witelo.[14] Thus, although both Bacon and Pecham were cognizant of Alhacen's demonstration in proposition 3 that images can sometimes be larger than their objects when seen in convex spherical mirrors, neither of them made any effort to explain that conclusion geometrically, despite the fact that it is the only truly original contribution Alhacen made in book 6.[15] The same applies to their analyses of image-distortion in general; although they drew upon Alhacen for certain salient points, they ignored most of the analytic details of his account. As a result, their accounts of both image-formation and image-distortion in mirrors are both sketchy and relatively unsophisticated. Neither author, in fact, took the topic of image-distortion much further than Ptolemy.

The question of precisely how Alhacen's account of image-distortion in mirrors might have influenced certain practical and theoretical developments in catoptrics during the fifteenth and sixteenth centuries has come to the fore recently in two arenas. On the one hand, the artist David Hockney, with the collaboration of Charles Falco, an optical researcher at the University of Arizona, has suggested that, from the early fifteenth century, Renaissance artists routinely used spherical concave mirrors or spherical convex lenses to project real images of the scenes or portraits they wanted to paint directly onto their boards or canvasses and then used those images to establish landmarks for the actual painting.[16] This thesis has aroused considerable controversy among art historians and historians of technology, the latter arguing that suitable concave mirrors were not technologically feasible at the time.[17] Perhaps most problematic is the lack of textual evidence, or at least convincing textual evidence, to support Hockney's and Falco's claim. In response to this issue, the two have appealed to the fact that a full theoretical justification of image-projection from concave spherical mirrors and convex lenses was well within the range of Perspectivist optics. Hence, the ability of concave spherical mirrors to project real images could easily have been discovered and explained through the application of Perspectivist ray-analysis.

Meantime, several scholars have been looking closely at technological and theoretical developments leading to the invention and improvement of optical devices, telescopes in particular, during the sixteenth century. Among such developments, according to Sven Dupré, was the recognition in that century of a correlation between the focal point of concave spherical mirrors and the "inversion point" (*punctum inversionis*) at which images seen in such mirrors flip upside-down.[18] Since the focal point determines where real, inverted images will begin to be physically projected by concave spherical mirrors, establishing the correlation between that projection and the inversion of images *seen* beyond that point linked the formation of both real (physical) and virtual (psychological) images in concave mirrors. The same applies to convex lenses, where the focal point also constitutes the inversion-point of images seen in them.

The very concept of a focal point (from Latin *focus* = hearth, fire place) harks back to antiquity and the effort to perfect burning mirrors. Within that context, the focal properties of concave spherical mirrors had been accurately determined by at least Archimedes' time according to the ability of such mirrors to ignite combustible material at or around a particular point on its diameter. Such is clear from the treatise of Diocles on burning mirrors. Most likely written in the early second century B.C., this work provides a rigorous mathematical analysis of the focal properties not only of concave spherical mirrors but also of paraboloidal mirrors.[19]

Whether Alhacen was familiar with Diocles' account of burning mirrors is an open question, although there are grounds to suppose that he was.[20] Beyond doubt, however, is that Alhacen was fully aware of the focal properties of concave spherical mirrors discussed and demonstrated in that account and, moreover, that he was also well aware of the focal properties of concave paraboloidal mirrors.[21] Clearly, then, Alhacen had at his disposal the analytic tools needed to explain the projection of real images by concave spherical mirrors, although the context within which he applied those tools was specific to the physical capacity of light-rays to create heat rather than images. The question thus arises whether Alhacen recognized the correlation between the two capacities, and the answer lies in how he might or might not have brought that correlation to bear in his analysis in book 6, chapter 7, of image-formation and image-distortion in concave spherical mirrors.

That such a correlation is at least implicit in his analysis of image-formation in concave spherical mirrors can be seen from the account he provides in book 5, proposition 32 (in Smith, *Alhacen on the Principles*, 446-448). The point of this proposition is to show that image-locations for object-points vary according to how and whether the line of reflection and the cathetus of incidence dropped from the object-point intersect. To illustrate, let the circle in figure 47, p. 300 (adapted from figure 5.2.32 [in Smith, *Alhacen on the Principles*, 250] upon which the proposition is based), be a concave spherical mirror centered on D, and let A be a center of sight posed on diameter IB. Let M be an object-point on diameter HDO, and let its form reflect to A from N. As seen from A, therefore, its image will lie at L behind the reflecting surface. Now let T be an object-point on the same diameter, and let its form be reflected to A from point E such that reflected ray EA is parallel to cathetus of incidence TD. Since the two lines will never intersect, it follows that there will be no definite image; instead, a confused image will appear at reflection-point E.[22] Then let Q be an object-point, and let its form reflect to A from G. Its image O will therefore lie behind the center of sight. If, on the other hand, Z on diameter IAB is the object-point, and if its form reflects to A from E, its image will lie at the center of sight itself. And, finally, if K is the object-point, and if its form reflects to A from C, its image S will lie between the center of sight and the mirror.

These same points emerge from an analysis of figure 47a, p. 300, which is adapted from the diagram accompanying Ptolemy's account of the same phenomena in book 4 of the *Optics*.[23] Let the arc represent a portion of a concave spherical mirror centered on C, and let E represent the center of sight, R a point of reflection defining reflected ray ER, and O_6R a line of incidence upon which object-point O is posed at various locations from O_1 to O_6. Accordingly, if O_1 is an object-point whose form reflects to E from R, its image, as seen from E, will lie at I_1 behind the mirror. On the other hand, if O_2 is taken as the object-point, and if its cathetus of incidence O_2C is parallel to reflected ray RE, its image will appear at R on the mirror's surface, according to Alhacen's account in book 5, proposition 32. When the object is O_3, however, and when the center of sight is shifted to I_3, the image will lie at the center of sight itself. In the same vein, finally, if the center of sight is at point I_4 with the object-point remaining at O_3, the resulting image I_3 will lie behind the center of sight, whereas if the center of sight remains at I_4 with the object-point at O_5 or O_6, the resulting image I_5 or I_6 will lie between the center of sight and the reflecting surface.

Let us modify figure 47a somewhat as follows. First, let us pass radius CF through line of incidence O_6R so that it is bisected at point F. F is therefore a focal point of the mirror. Then let us find point R′ on the other side of radius CF such that FR = FR′, as illustrated in figure 47b, p. 301. Likewise, let us locate point O_1' on line FR′ such that it lies the same distance from F as O_1, and let center of sight E lie well beyond center of curvature C, where the extension of CF intersects reflected rays RE and R'E. As we just determined in the previous analysis, the form of O_1 will reflect to E from R to yield an image at I_1 behind the mirror. Symmetry dictates that the form of O_1' reflect from R′ to E to yield an image at I_1' behind the mirror. Hence, if we take O_1O_1' to represent an object-line posed between the mirror's surface and focal point F, its image I_1I_1' will appear behind the mirror from E's perspective, and it will be larger than its object. If we assume that O_1 is at the top of the object and O_1' at the bottom, the image will appear upright, insofar as top-most point O_1 of the object will appear at top-most point I_1 of the image.

Now, following the same analytic model, let O_2O_2' in figure 47c, p. 301, represent an object-line whose top and bottom endpoints O_2 and O_2' reflect from R and R′ such that the two reflected rays RE and R′E are parallel, respectively, to the two catheti O_2C and $O_2'C$. In that case, there will be no definite image, only a confusion at R and R′. Consequently, when the object is placed just above the focal point, it will yield a blurry image anchored to the mirror's surface at those two points.

What happens when the object-line falls between center of curvature C and focal point F is illustrated in figures 47d and 47e, p. 302, where the two object-lines O_3O_3' and O_4O_4', with points O_3 and O_4 at the top, lie between the focal point and the center of curvature. Their respective images I_3I_3' and I_4I_4' will be larger than their objects and will appear inverted, insofar as the top-most points O_3 and O_4 of the objects will appear at the bottom-most points I_3 and I_4 of the images. When the object-line reaches the center of curvature, as in figure 47f, p. 303, and when O_5 is at the top and O_5' at the bottom, its image I_5I_5' will coincide with it. That image, however, will have diminished in size so as to be smaller than its object, although it will remain inverted. Finally, in figure 47g, p. 303, when the object O_6O_6' passes beyond the center of curvature (i.e., between the center of curvature and the center of sight), its image I_6I_6' will have gotten even smaller relative to the object itself, but it will remain inverted. As the object moves progressively farther from the center of curvature toward the center of sight, its image will get progressively smaller relative to the object, but it will continue to be inverted.

Several features of this analysis merit attention. First, it is consistent with experience. If a viewer faces a concave spherical mirror from a position well beyond the center of curvature, and if an object of moderate size is placed close to the reflecting surface, the viewer will see its image behind the mirror, upright and magnified. As the object is drawn away from the reflecting surface toward the viewpoint, its image becomes progressively larger as it recedes farther and farther behind the surface until a point is reached at which the image becomes blurry and incoherent. Then, as the object is drawn just a bit farther from the reflecting surface toward the viewpoint, its image resolves back into a clear representation that is inverted and larger than its object. From that point on, as the object is drawn ever farther from the reflecting surface, its image gets progressively smaller while remaining inverted. A second feature of the analysis that bears remarking is that it ties the inversion of images seen in concave spherical mirrors clearly and explicitly to the focal point of those mirrors. The final and perhaps most significant feature to consider is that the analysis itself follows directly and logically from Ptolemy's account of image-location and its variation in book 4 of the *Optics*. That account in turn forms the basis for Alhacen's own account of the phenomenon in book 5, proposition 32. Within this context, therefore, the connection among heat-production, the projection of real images, and the inversion of images seen in concave spherical mirrors seems every bit as evident as its grounding in the focal point of such mirrors.

Did Alhacen in fact recognize this seemingly obvious connection? As we have seen from our examination of propositions 23-28, where Alhacen deals specifically with image-inversion and magnification in concave spherical mirrors, there is no clear indication that he did. True, the construction in proposition 23 calls for locating the object-line, as well as the center of sight, between the mirror's focus and the reflecting surface, as illustrated in figure 31, p. 285, where O lies at the midpoint of radius AU, with MTN representing the object-line and T the center of sight. But the fact that O is the focal point for the mirror is incidental to the resulting demonstration. Other than specifying the diameter of circle ZOE to which tangents TE and TZ are dropped, it has no explicit function in that demonstration. The same, of course, applies to proposition 24, which is merely a variant of proposition 23. In the remaining four propositions devoted to image-inversion and magnification, moreover, the cardinal reference-point for analysis is the mirror's center of curvature, not the focal point. No more is needed to confirm this point than a brief look at figures 33-36, pp. 286-288, and their construction in propositions 25-28. All told, propositions 23-28 are designed to show *that* image-inversion, along with magnification or diminution, can occur in concave spherical mirrors under particular circumstances, not to explain precisely how and why they occur according to general conditions, as defined by the focal properties of such mirrors.

Does this mean that Alhacen failed to see the connection between the inversion of images seen in concave spherical mirrors and the focal properties of such mirrors implied by his analysis in book 5, proposition 32? Perhaps, but to pose the question in terms of failure is misleading because the two phenomena lie in entirely different domains from an Alhacenian perspective. The gathering of light to a burning point in concave spherical mirrors is a physical phenomenon, as is the projection of real images beyond that point, both phenomena being tied to the mirror's focal point. On the other hand, the inversion of images seen in such mirrors is a psychological phenomenon involving not real, physical images but imaginary, or virtual, ones. Granted, the two phenomena are "optical" in that they involve light, but they are absolutely different in terms of their respective effects.[24] So Alhacen was bound by the framework of his analysis in the *De aspectibus*—where the focus is upon the psychological (or "subjective") effects of light rather than its physical (or "objective") effects—to treat image-inversion as a visual rather than a physical phenomenon. There is simply no place for real images within such an analytic framework.[25] Nor, for that matter, is there any clear functional place for the focal point within it. Consequently, the analysis of reflection in general and image-inversion in particular according to Alhacen and his Perspectivist disciples did not lend itself to the discovery, much less a coherent explanation of the image-projecting capacity of spherical concave mirrors.

3. *Conclusion*

Our examination of Alhacen's analysis of image-formation and distortion in the seven types of mirrors, concave spherical mirrors in particular, reinforces the conclusion drawn in the previous edition of books 4 and 5 that, from a post-Keplerian perspective, Alhacen's treatment of reflection represents little more than wasted effort as far as the development of modern physical optics, catoptrics in particular, is concerned.[26] The reason is not far to seek. Modern, post-Keplerian catoptrical analysis is based primarily on focal points and real images, whereas Alhacenian catoptrics is based on viewpoints and virtual images. Hence, as far as the evolution of modern physical optics is concerned, Alhacen's account in the *De aspectibus* of image-formation and distortion in mirrors represented a dead end because it had no practical ramifications for the improvement of optical devices, such as the *camera obscura* or the reflecting telescope, which was of utmost concern to sixteenth- and seventeenth-century opticians.

Proposition 3 of book 6 exemplifies this lack of concern for practical application. On the one hand, the fact that images can be magnified in convex spherical mirrors opens the possibility of using such mirrors as telescopic devices. On the other hand, the circumstances under which such magnification occurs are so contrived and narrowly constrained that such an application would be fruitless. Why, then, bother to analyze the phenomenon at such excruciating length? The answer lies in the analysis itself. For one thing, it is brilliant in conception, fully commensurate with Alhacen's extraordinary imagination and skills as a mathematician. For another, it was possible to do and therefore had to be done for the sake of completeness. Alhacen thus approached reflection as a mathematician concerned with explaining visual anomalies as rigorously and as comprehensively as possible, not as a physicist or technical practitioner bent on exploiting those anomalies. It is both unfair and irrelevant, then, to judge him and his contribution to catoptrics from this latter perspective. To do so—and to find him deficient according to it—is to judge him not for what he actually accomplished but for what we, as post-Keplerians, would prefer him to have accomplished. What he did accomplish was to take the science of catoptrics, as conceived by his Greco-Arabic predecessors, Ptolemy foremost among them, to the highest level of sophistication and mathematical elegance it had ever attained and would ever attain. That, in its own right, is a remarkable achievement.

NOTES

[1]See note 24 to the translation, pp. 236-237, for an explanation of these points.

[2]Book 4, chapter 4, in A. Mark Smith, *Alhacen on the Principles of Reflection*, Transactions of the American Philosophical Society, 96.2 (Philadelphia: APS, 2006), 324-325.

[3]This point is illustrated in figure 18, p. 274, where the sphere of the mirror is delineated by the gray arc ZY passing through line EG. EGD is the plane within which the form of E reflects to D, and it forms the great circle Z'Y' on the mirror's surface. Accordingly, the form of E reflects to D from point R on that arc, and N is the endpoint of tangency insofar as the line drawn from it to arc Z'Y' is tangent to that arc at reflection-point R.

[4]See book 5, chapter 2, paragraphs 2.6-2.9, in Smith, *Alhacen on the Principles*, 387-388.

[5]It is of course true that, if the mirror faces the center of sight with its axis horizontal rather than vertical, the image of line TH will suffer left-right reversal.

[6]For a more detailed account of the change in translators and the interpolated text, see the section on manuscripts and editing, pp. xlv-xlviii.

[7]See book 5, proposition 32, in Smith, *Alhacen on the Principles*, 446-448.

[8]It should be noted that, when comparing image-size to object-size, Alhacen is comparing rectilinear cross-sections rather than actual object-lines or image-lines. Hence, in this proposition line MTN represents a cross-section of the surface of the eye, not that surface itself, and likewise line FQ represents a cross-section of MN's image, not the image itself. In fact, as is clear from figure 31, the image of point X, as seen from T, will lie far beyond the reflecting surface where line of reflection RT and cathetus AX intersect. Later on in book 6, when Alhacen deals with actual object-lines and their resulting image-lines, he defines them according not only to their two endpoints but also to their midpoints.

[9]Suffice to say, if MR is to serve as an object-line, it must be short enough to fit inside the mirror. Hence, ER and MR can be no longer than the radius of the mirror; otherwise, it will extend outside the mirror. On the other hand, if MR serves as an image, it can be as long as we please, since it can extend outside the mirror, which is to say that it will appear behind the reflecting surface.

[10]According to Alhacen's account in book 5, paragraph 2.312 (in Smith, *Alhacen on the Principles*, 448), when the line of reflection and the cathetus are parallel, the image is seen by the eye at the point of reflection. In this case, of course, BK, being normal to the ellipse at point B, is the cathetus, whereas GD is the line of reflection, which has been constructed parallel to BK. Hence, the image of point L will appear at point G.

[11]For an account of these sources, see Smith, *Alhacen on the Principles*, lxvi-lxxvii.

[12]See A. Mark Smith, *Ptolemy's Theory of Visual Perception*, Transactions of the American Philosophical Society, 86.2 (Philadelphia: APS Press, 1996), theorems

III.13-III.16, pp. 166-171, and theorems IV.30-IV.38, pp. 205-222. Ptolemy's analysis of convex mirrors is restricted solely to spherical mirrors, and his analysis of concave mirrors is slanted mostly toward spherical mirrors.

[13]For a brief account of this line of dissemination, see A. Mark Smith, *Alhacen's Theory of Visual Perception*, Transactions of the American Philosophical Society, 91.4 (Philadelphia: APS, 2001), lxxxii-lxxxiv.

[14]For more on this point, see Smith, *Alhacen on the Principles*, lxxxii-lxxxvi.

[15]Bacon's acknowledgement of this conclusion has already been discussed in Smith, *Alhacen on the Principle*s, cii, n. 82; for Pecham, see David C. Lindberg, ed. and trans., *John Pecham and the Science of Optics* (Madison, WI: University of Wisconsin Press, 1970), proposition 33, 187-189. Although both Bacon and Pecham rely heavily upon geometrical diagrams, their analyses are generally descriptive rather than rigorously demonstrative.

[16]See David Hockney, *Secret Knowledge: Rediscovering the Lost Techniques of the Old Masters* (New York: Viking, 2001). The primary optical device used by Renaissance artists, according to Hockney, was the *camera obscura*, within which a spherical concave mirror could be placed facing the small opening that allows light into the chamber so as to project a real image of the scene beyond that opening onto a screen. Much the same thing can be accomplished by placing a spherical convex lens at the opening.

How a concave spherical mirror projects real images can be easily understood by recourse to figure 46, p. 299. Let EAK be an arc on a concave spherical mirror centered on C. Let F lie at the midpoint of radius AC of that mirror, let the parallel lines O_1E, O_2D, etc., striking the mirror's surface at E, D, B, A, G, H, and K, represent light-rays reaching the mirror from a distant luminous object, and let AE = AK, AD = AH, and AB = AG. According to the law of equal angles, each of the light-rays reaching the mirror's surface will reflect at equal angles from the point at which it strikes the mirror, so it follows that ray O_1E will reflect along EX, which intersects diameter AC at point X, such that angle of incidence O_1EC = angle of reflection CEX, and so forth. Moreover, since AE = AK by construction, it follows that ray O_7K will reflect at equal angles from point K to form reflected ray KX, which also intersects diameter AC at point X. The same holds by extension for ray AC (which reflects back onto itself), as well as for ray-couple O_2D and O_6H and ray-couple O_3B and O_5G. Furthermore, it is clear from the diagram that, as the incident ray approaches diameter AC, the reflected ray intersects diameter AC at a point approaching focal point F. Thus, as far as the focusing of rays is concerned, the effective portion of the mirror is quite small, limited, say, to arc BG. Moreover, focal point F represents the limit toward which the reflected rays all tend, so there is a small area between points X and F where enough rays congregate to provide the heat of ignition.

One other point comes clear from the diagram: all the reflected ray-couples, such as EX and KX, continue past their respective points of intersection, so if a screen were to be placed beyond point F, as represented by line K'E', the continuing rays would reach it at points corresponding to the points of reflection from which they originate. Accordingly, the ray reflected from E would strike the screen at E', and so forth on down the line to K, which would reach the screen at K′. Imagine a viewer facing the screen and, therefore, also facing the object. From his perspective the right-hand side O7 of the object would appear at the left-hand side K′ of the image, whereas the left-hand side O1 of the object would appear at the right-hand side E′ of the image. By the same token, if O1 were taken to represent the top of the object, that point would appear at E′, which lies at the bottom of the image. Hence, the image would appear inverted. In modern parlance such images are called "real," whereas images seen behind a reflecting surface are called "virtual."

[17]A representative sample of reactions to Hockney's and Falco's thesis among both academics and non-academics from various disciplines can be found in *Early Science and Medicine,* 10.2 (2005), a special issue containing papers given at a European Science Foundation workshop entitled "Optics, Optical Instruments and Painting: The Hockney-Falco Thesis Revisited" that was held at Ghent University in 2003. For a critique of Hockney's thesis on technological grounds, see Sara J. Schechner, "Mirrors and their Imperfections in the Renaissance," ibid., 137-162. While Schechner's argument that the technology of mirrors was inadequate to produce suitable concave mirrors for image-projection in the early fifteenth century has some merit, Vincent Ilardi's long-awaited study of early eyeglasses, *Renaissance Vision from Spectacles to Telescopes* (Philadelphia: American Philosophical Society, 2007), makes it abundantly clear that adequate convex lenses for such purposes were readily available by that time. Over the past several years Charles Falco has vigorously defended the Hockney-Falco thesis on the basis primarily of computer analyses of various paintings from the fifteenth and sixteenth centuries. See, e.g., "The Art of the Science of Painting," Proceedings of the Symposium on Effective Presentation & Interpretation in Museums, The National Gallery of Ireland, Dublin, 2003, available in pdf format at www.optics.arizona.edu/SSD/NatlGallery.pdf.

[18]See, e.g., Sven Dupré, "Ausonio's Mirrors and Galileo's Lenses: The Telescope and Sixteenth-Century Practical Optical Knowledge," *Galileana*, 2 (2005): 145-180.

[19]See G. J. Toomer, *Diocles on Burning Mirrors* (Berlin/Heidelberg/New York: Springer-Verlag, 1976), esp. proposition 3, 56-62. Toomer's edition and translation of Diocles' work is based on an Arabic version, which exists in a single manuscript completed in AH 867 (1462/3 A.D.) according to the testimony of the scribe. In all likelihood, the original Arabic translation predates this particular copy, but by how much is subject to speculation. Toomer opines that it may have been done by Quṣṭā ben Lūqā, which would take it back to around 900 A.D. (see ibid., 21).

[20]See Toomer, *Diocles*, 18-23. Cf., however, A. I. Sabra, *The Optics of Ibn al-Haytham: Books I-III on Direct Vision* (London: Warburg, 1989), p. 52, where he emphasizes that Diocles' proof of the focusing properties of paraboloidal mirrors "*may* have been known to" Alhacen. Presumably this would apply to Diocles' proof for the focusing properties of concave spherical mirrors as well.

[21]See H. J. J. Winter and W. 'Arafat, "A Discourse on the Concave Spherical Mirror by Ibn al-Haitham," *Journal of the Royal Asiatic Society of Bengal*, 16 (1950): 1-16, and "Ibn al-Haitham on the Paraboloidal Focussing Mirror," *Journal of the Royal Asiatic Society of Bengal*, 15 (1949): 25-40.

[22]See note 10 above.

[23]See figure IV.21 accompanying experiment IV.1 of Ptolemy's *Optics* (in Smith, *Ptolemy's Theory*, 194-195), where Ptolemy draws the same conclusions that Alhacen does in book 5, proposition 32: namely that, depending on where the object-point is located on the line of incidence and where the center of sight is located on the line of reflection, the resulting image may appear behind the mirror, on the mirror's surface (i.e., when the cathetus of incidence and the line of reflection are parallel or when the image lies at the center of sight itself), between the center of sight and the reflecting surface, or behind the center of sight. Even a cursory comparison of Ptolemy's and Alhacen's approach to this issue and the diagrams upon which their accounts are based shows that Alhacen was following Ptolemy's analysis quite closely.

[24]Sabra echoes this point in *Optics*, xlii, where, in discussing Alhacen's two treatises on spherical and paraboloidal burning mirrors, he observes that they are "concerned with the behavior of solar rays . . . as agents of combustion, not of vision." See also Sven Dupré, "Optics, Pictures and Evidence: Leonardo's Drawings of Mirrors and Machinery," *Early Science and Medicine*, 10 (2005): 211-236.

[25]For elaboration on this point, see A. Mark Smith, "Reflections on the Hockney-Falco Thesis: Optical Theory and Artistic Practice in the Fifteenth and Sixteenth Centuries," *Early Science and Medicine*, 10 (2005): 163-185.

[26]See Smith, *Alhacen on the Principles*, lxxxix.

MANUSCRIPTS AND EDITING

Textual Issues in the Manuscripts: The Latin text in this current edition is based on the same seven manuscripts as the Latin text in the previous edition of books 4-5: i.e., *F*, *P1*, *S*, *E*, *L3*, *O*, and *C1*. Having already explained in detail both how and why I chose this particular group from the seventeen extant manuscripts containing all or most of the *De aspectibus*, I will not repeat the rationale here.[1] Suffice it to say, I am still satisfied that those seven manuscripts yield an appropriately critical text.

Unlike that of books 4 and 5, the text of book 6 presents an anomaly reminiscent of the one encountered in chapter 3 of the third book. In three of the seventeen manuscripts just mentioned (i.e., *O*, *L3*, and *C2*), chapter 6 on convex conical mirrors is interrupted early on by a clearly demarcated text that opens with the title *capitulum sextum de fallaciis que accidunt in speculis piramidalibus convexis erectis* ("chapter six [literally, "the sixth chapter"] on misperceptions that occur in right convex conical mirrors"). Then follows an extremely brief résumé of the misperceptions arising in such mirrors, which concludes with the phrase *ad huius autem demonstrationem premittamus hanc propositionem* ("and for the purpose of this demonstration let us set forth this proposition"). The theorem that follows is essentially the same as proposition 20, lemma 6 (Latin, pp. 48-50; English translation pp. 196-198), which occurs before the interpolation. From that point on, all the manuscripts continue with proposition 22 (Latin, pp. 52-58; English, pp. 199-204), whose opening phrase *hoc ergo declarato* ("now that this has been demonstrated") presumably refers to the introductory lemma in proposition 20.

Quite clearly, then, this swatch of interpolated text represents an alternate version of the introductory résumé and initial theorem of chapter 6 as it opens in all seventeen manuscripts, including the three anomalous ones. But its placement in those three manuscripts is puzzling. Whereas it should have been inserted right after proposition 20, it actually follows proposition 21, which draws on proposition 20 to support its argument. Equally puzzling is that roughly the first half of proposition 22 (paragraphs 6.20-6.30) recapitulates proposition 21, albeit in a somewhat different narrative form.[2]

Aside from its odd placement, the interpolated text has some peculiar features that set it apart from everything that precedes it. For a start, there is its title. Up to that point, the chapter headings, when they are given, are designated by *pars* rather than *capitulum*. In addition, only one such chapter—chapter 4—includes a descriptive subtitle: *pars quarta in speculis spericis* ("chapter four on spherical mirrors"). Chapters 5 and 6 have no heading at all. Also, the introductory résumé of the alternate version of chapter 6 is longer and commensurately more detailed than that of the initial version contained in all the manuscripts. Finally, there are marked differences in style and vocabulary between the two versions, as witness the shift from *pars* to *capitulum*. In the initial version, for example, the term *sectio* is used to denominate "conic section," whereas in the second the replacement term is *sector*. These alterations in vocabulary and format are matched by a noticeable shift in narrative style, that of version 2 being choppier than that of version 1.[3]

Taken as a whole, these incongruities in style and vocabulary suggest strongly that two distinct translators were at work here. More to the point, the entire text from the beginning of book 6, proposition 22, to the very end of book 7 bears all the stylistic and terminological hallmarks of translator 2—which means that from the point of insertion to the very end of book 7, the text forms a continuous whole produced by translator 2. That being the case, the two puzzling features mentioned earlier are easily explained. First, the recapitulation of proposition 21 in the initial part of proposition 22 makes sense if we view proposition 22 as an amalgam of two distinct theorems, the first of which constitutes proposition 21 and the second of which constitutes proposition 22 proper. Within its context in version 2 of the text, therefore, proposition 22 with its two parts forms the appropriate sequel to proposition 20. Second, the placement of version 2 after proposition 21 of version 1 can be accounted for by assuming that translator 1 was suddenly replaced at this point by translator 2, who took it upon himself to retranslate the opening portion of chapter 6 for the sake of continuity.

The situation described here has an obvious parallel in the one encountered in book 3, chapter 3, where paragraph 3.13 is suddenly interrupted by an interpolation unique to six of the seventeen manuscripts containing all or most of the *De aspectibus* (i.e., *P1*, *F*, *S*, *O*, *V2,* and *Vat*).[4] On close examination, the inserted text proved to be a rough, often maladroit translation of the concluding portion of chapter 2 and the opening section of chapter 3 up to paragraph 3.13. It also proved to be continuous in style with the remainder of book 3 as it appears in all the manuscripts. I therefore concluded that there was an abrupt switch of translators at the point of interpolation, the master (translator 1) yielding to the apprentice (translator 2). I also concluded that the section of version 2 that overlaps with version 1 represented a sort of practice run for translator 2 to prepare him to take over in earnest at paragraph 3.13 of book 3. On that basis I decided to relegate the overlapping portion of version 2 to an appendix rather than include it as an integral part of the critical text. Following the same rationale, I have placed the overlapping portion of version 2 of book 6, chapter 6, in the appendix on pp. 330-335.

While there is no doubt that both cases involve a change in translators, there may be more at play than that. For instance, as remarked in the analysis of book 3, the Latin text from paragraph 3.13 to the end of the book represents less a translation than a paraphrase of the Arabic counterpart established by A. I. Sabra.[5] This of course could be due to the ineptitude or haste with which translator 2 rendered that portion of the text into Latin. But it could also be due to his having used an Arabic version that falls outside the manuscript tradition upon which Sabra based his edition.

This possibility is borne out, at least to some extent, by the second version of book 6, chapter 6. The introductory résumé in both versions is a case in point. In the first it consists of a single paragraph (6.1, p. 48 [Latin] and p. 196 [English]), whereas in the second it consists of three (Appendix, pp. 330 and 331). It could of course be argued that version 1 is a mere distillation or paraphrase of version 2, but to argue that is to ignore the omission in version 1 of pertinent and significant details provided in version 2.[6] Moreover, although there are obvious disparities between the two versions of proposition 20, those disparities are not easily explained on the basis of paraphrasing. For one thing, both versions are roughly the same length. For another, the structure of argument is noticeably different in both, even though each argument leads to the same conclusion along a valid train of logic. If both translators were working from the same text, then surely they would have followed the same train of logic, particularly when that train is as rigidly controlled as it is in a mathematical proof.[7] Finally, there is the issue of the heading and subtitle for chapter 6. Even if he was paraphrasing, why did translator 1 omit it entirely if it was actually there? After all, in the overlapping portion of the second version of chapter 3, book 3, translator 2 (who presumably *was* paraphrasing) included such a heading for chapter 3 (i.e., *de modis quibus error accidit visui*), and he continued to provide such headings (with a few exceptions) throughout the rest of the book.

If nothing else, this raises the possibility that the Latin translation of the *De aspectibus* was based on at least two different, perhaps markedly different, Arabic versions of the text; that at least two, but perhaps no more than two translators were at work on it; and that each translator was drawing on a different Arabic version. That in turn raises the possibility that the manuscript tradition of the Arabic text was more complex and variegated than current scholarship might lead us to suppose. As of now, only five manuscript copies of the Arabic text are known to exist. Of those only one contains the entire treatise, and that one is quite late, dating from 1493-94. The earliest known copy (dating from 1083-84) lacks books 4 and 5; the next in chronological order (dating from 1239) contains books 4 and 5 alone; the third in chronological order (dating from the fourteenth century) has only segments of books 5-7; and the last in chronological order (dating from 1509) is limited to books 1-3.[8] Given the paucity and defective state of these manuscript copies, it would be rash to conclude that they represent the sole, authoritative witnesses to either the text itself or its dissemination in the Arabic world. It would be equally rash, therefore, to conclude that the disparities between the Latin text and the Arabic version established on the basis of these five manuscripts (in disparate groups of three, no less) is due solely to the Latin translators.

The Critical Text: The topical organization of book 6 is articulated clearly at the beginning, where the number of chapters (nine) and a brief description of their content is given explicitly in most of the manuscripts. Yet despite this indication, six of the seventeen aforementioned manuscripts add chapter-breaks at various points in the text. One case has already been discussed: i.e., the break at the beginning of the alternate version of chapter 6 that is found in *O*, *L3*, and *C2*. In addition, *S* and *Er* introduce a chapter-break at the beginning of proposition 22, and *M* introduces one in the middle of chapter 7. On the other hand, most of the manuscripts omit various chapter-breaks where they are appropriate or else signal them weakly.[9] Nevertheless, the topical structure of book 6 is obvious in the organization of the text itself.

For the most part, book 6 follows the organizational structure of book 5, the lion's share of it being devoted to a mathematical analysis of the seven mirrors in order from plane (chapter 3), through convex spherical, cylindrical, and conical (chapters 4-6), to concave spherical, cylindrical, and conical (chapters 7-9). Since that structure is explicit in book 6, I had no need to impose it, as I did for book 5. As to the remaining levels of organization, which I did have to impose, I followed the pattern established earlier for book 5, breaking the text into propositional elements (38 in all), which on occasion I further subdivided into cases. As with the Latin text of book 5, so with this one, I have inserted fairly strong spacing-breaks between propositions and weaker ones between cases. Each proposition is further demarcated by a numerical designation (e.g., [PROPOSITIO 1]). At the next, lower level of organization, I have imposed a paragraph structure in order to make the text easier to follow, each paragraph being numbered according to its chapter (e.g., [6.1], which designates the first paragraph of chapter 6). At the lowest level of organization I have punctuated as I thought appropriate in order to ease the modern reader's way through the narrative, most of which consists of mathematical argument.

Diagrams: Here, again, I followed the pattern established for book 5, paring the diagrams in the manuscripts down to what I consider to be a canonical set. In this case that set consists of only 36, as opposed to 83 for book 5. As in book 5, so in this one, I traced the figures to accompany the Latin text directly from diagrams scanned for the most part from manuscript *O*, although I have relettered them with a modern font and have occasionally reoriented them. As before, I have designated the resulting text-diagrams in capital letters according to the format "FIGURE 6.7.31," the number-series indicating book (6), chapter (7), and proposition (31). For a detailed description of the criteria I followed in choosing and representing the appropriate diagrams, see *Alhacen on the Principles*, cxi-cxv.

The Critical Apparatus: For the conventions used in the critical apparatus of this edition, I refer the reader to *Alhacen's Theory*, clxxii-clxxiv.

The Translation and Commentary: The general guidelines I followed in translating book 6 are those discussed in *Alhacen's Theory*, clxxiv-clxxvi and *Alhacen on the Principles*, cxv-cxvii. Accordingly, I have reinforced the organizational breaks discussed earlier by interpolating parenthetical headings, such as [CASE 1], to clarify the analytic structure of the text. I have also provided external and internal citations at appropriate points in the translation, inserting them parenthetically and setting them in brackets. As before, I have inserted brief explanations set off in brackets at spots in the text so as to streamline the commentary, which I have relegated to endnotes. Unlike the diagrams that accompany the Latin text, those that are matched to the translation are meant to reflect as faithfully as possible the actual conditions specified in the constructions and proofs. In order to distinguish these diagrams from their counterparts in the Latin text, I have designated them according to the lower-case format "figure 6.7.31," the number-series indicating book, chapter, and proposition. As in the previous edition, so in this one, I have placed the figures that go with volume 2 at the end of volume 1 and vice-versa so that the reader can match the logical flow of each propositions with its appropriate diagram(s) without flipping back and forth between proposition and diagram within the same volume. The reference-aids provided in this edition are the same as those provided in the previous two: i.e., a Latin-English index keyed to technical terms in both Latin text and English translation; an English-Latin glossary for cross-referencing to that index; and a general index keyed primarily to the introduction and commentary.

NOTES

[1]For a full description of the seventeen manuscripts, See Smith, *Alhacen's Theory*, clv-clxi. For a discussion of the criteria I used for sorting these manuscripts into families and family-representatives, see ibid., clxi-clxvii. And for the rationale behind my choice of manuscripts upon which to base the text of books 4-5, see Smith, *Alhacen on the Principles*, cvii-cviii.

[2]The points made in this paragraph are illustrated in the schematic below. The left-hand column gives the format of chapter six from the introductory résumé to the end of proposition 22 as it appears in all the manuscripts except *O*, *L3*, and *C2*. The right-hand column gives the format for those three manuscripts, with the interpolated text in gray. Ending with proposition 20, lemma 6, that interpolation mirrors sections A and B in all the manuscripts, so logic would demand that it be placed right before proposition 21 (section C in all the manuscripts), which is based in part on proposition 20. Furthermore, the first part of proposition 22 (section D in all the manuscripts) is clearly redundant insofar as it recapitulates proposition 21 in section C.

All Except O, L3, and C2	O, L3, and C2
A. Introductory résumé	A. Introductory résumé
B. Proposition 20, lemma 6	B. Proposition 20, lemma 6
C. Proposition 21	C. Proposition 21
	Title
	A. Introductory résumé
	B. Proposition 20, lemma 6
D. Proposition 22	D. Proposition 22
1. Part 1 (Prop. 21)	1. Part 1 (Prop. 21)
2. Part 2 (Prop. 22')	2. Part 2 (Prop. 22')

[3]See note 87, p. 248 below for a more detailed accounting of these terminological and stylistic differences.

[4]See Smith, *Alhacen's Theory*, clxi and clxviii-clxix, for a discussion of this interpolation. Note, by the way, that *O* is the only manuscript containing both interpolations, a fact that may indicate its privileged status as a witness to the original formation of the Latin versions of the *De aspectibus*.

[5]Cf. Sabra, *Optics*, 250-367, and Smith, *Alhacen's Theory*, 588-627.

[6]The main differences in detail are as follows: 1) Version 1 likens the misperceptions that occur in convex conical mirrors to those occurring in convex spherical mirrors; version 2 likens them more appropriately to the misperceptions that occur in convex cylindrical mirrors. 2) Version 2 notes that images in convex conical mirrors take on the shape of those mirrors; version 1 does not. 3) Version 1 remarks

that, as visible lines approach the vertical with respect to such mirrors, their images appear less curved, whereas the closer they approach the horizontal, the more curved their images appear; version 2 makes no mention of this fact. 4) Version 2 points out that the images of lines posed horizontally with respect to such mirrors are noticeably shortened; version 1 says nothing on this matter. More significant than its inclusion of more information, however, is that, in providing that particular information, version 2 offers a far more insightful summary of chapter 6 than does version 1.

[7]The most obvious difference between the two versions of proposition 20 lies in the order of steps taken in the respective constructions. In both versions, for instance, BFZ (figures 6.6.20 and 6.6.20alt, pp. 129 and 153, respectively) is identified as the relevant conic section quite early in the first paragraph of the proof (i.e., paragraph 4, p. 330, of version 2 and paragraph 6.2, p. 48 of version 1). As soon as it is so identified in version 2, we are instructed to drop normal ED from E and then draw tangent TQ to point Z. At the same point in version 1, on the other hand, we are instructed to pass a plane through Z to form circle GBZ; the instruction to draw tangent TQ comes considerably later, in paragraph 6.5, p. 49. Such differences are far too numerous to detail here, but even a fairly superficial comparison of the two texts will reveal significant disparities between the two at the procedural level. It is of course possible that one or both of the translators were playing somewhat fast and loose with the narrative structure of the same Arabic text.

[8]See Sabra, *Optics*, lxxx-lxxxiii. Just how problematic the Arabic manuscript tradition is comes clear from the schematic Sabra provides on p. lxxxii. If we designate the five manuscripts according to chronological order from A (earliest) to E (latest), then the entire text of the Arabic treatise breaks down as follows: books 1-3, represented by A, D, and E; books 4-5, represented by B, C, and D (C consisting of excerpts only); and books 6-7, represented by A, C, and D. Thus books 1-3 exist in only three manuscripts; books 4-5 in only three, of which one is defective; and books 6-7 in only three, of which one is defective.

[9]For an overview of the addition and omission of chapter-breaks in book 6, see Table 6 of Appendix 2 in Smith, *Alhacen's Theory*, 660.

ALHACEN'S
DE ASPECTIBUS

LATIN TEXT

LIBER SEXTUS

Liber iste in novem partes partitur. Pars prima, titulus libri; secunda, quoniam error accidit visui propter reflexionem; tertia, in errore evenienti in speculis planis; quarta, in errore qui oritur in speculis spericis exterioribus; quinta in speculis columnaribus exterioribus; sexta, in piramidalibus exterioribus; septima, in spericis concavis; octava, in columpnis concavis; nona, in piramidalibus concavis.

PARS PRIMA

[1.1] Patuit ex libris superioribus modus adquisitionis formarum in speculis per visum, situs linearum reflexionis vel accessus, situs ymaginum et loca ipsarum. Verum per reflexionem non semper comprehenditur forme veritas. In concavis enim speculis apparet ymago faciei distorta, et occultatur visui dispositio ipsius vera, unde planum errorem incidere in comprehensione formarum per reflexionem. Huius erroris modum et modi causam propositum est in libro presenti explanare et secundum diversitates speculorum disquirere varietates errorum.

1 liber sextus *om. FP1SO* 2 liber *om. E*/iste: sextus *R*/partitur: dividitur *R*/pars prima *om. FP1*/*post* prima *add.* est *R*/libri *corr. ex* pun *C1* 3 quoniam: quod *R*/accidit: accidat *R*/reflexionem *corr. ex* reflexonem *L3*/in: de *R* 4 evenienti: venienti *L3E*; eveniente *R*/*post* planis *scr. et del.* concavis *L3*/in[2]: de *R*/qui *corr. ex* que *C1*/*post* qui *scr. et del.* in *L3* 5 *post* quinta *add.* de errore *R* 6 *post* sexta *add.* de errore *R*/*post* septima *add.* de errore *R* 7 *ante* in[1] *add.* de errore *R*/columpnis: columpnaribus *R*/*post* nona *add.* de errore *R* 8 pars prima *om. FP1*; *transp. L3*; *inter. a. m. E*; prooemium libri capitulum primum *R* 9 ex: in *O*/libris superioribus *transp. R* 10 vel: et *R* 12 concavis enim speculis: speculis enim concavis *L3ER*/*post* speculi *add.* an *S*; *add.* aliquando *corr. ex* ali *O* 13 occultatur: occulta *L3E*/ipsius: eius *L3R* 14 *post* planum *add.* est *OR*; *inter.* est *a. m. C1*/in *om. S*/per: propter *L3ER* 15 huius *rep. et del. L3*/libro *corr. ex* loco *F* 16 presenti: presente *R*/post presenti *add.* determinare *P1*/disquirere: discurrere *P1R*; *corr. ex* discurrere *E* 17 errorum *corr. ex* eorum *L3*

PARS SECUNDA

[2.1] Comprehensionem formarum in visu directo liber secundus docuit, et singula que propter egressum a temperantia in visu illo errorem inducunt liber tertius diligenter exposuit. Fit autem comprehensio formarum per reflexionem sicut et directe, et quorum fit adquisitio in directione fit etiam in reflexione, utpote lucis, coloris, figure, magnitudinis, distantie, et similium.

[2.2] Et quemadmodum in directione rerum prefixarum et cognitarum ad alia fit collatio, et inde oritur coniecturatio, et sumitur iudicium in anima, similiter accidit in reflexione. Unde quecumque temperamentum egressa in visu directo errorem efficiunt, in reflexione similiter inducunt. Et secundum singula maior accidit error in reflexione propter lucem debilem quam debilitat ipsa reflexio.

[2.3] Ut autem generaliter loquamur, non potest in reflexione comprehendi veritas forme sicut potest in directione propter triplex impedimentum reflexioni speciale. Primum est quoniam in reflexione apparet rei forma pre oculis visui opposita, cum non sit re vera. Secundum quoniam lux et color corporis visi miscentur cum colore speculi, quam mixturam visus percipit non verum rei vise colorem vel lucem. Tertium quoniam ipsa reflexio, ut in superioribus est assignatum, lucem et colorem debilitat, quare in reflexione latebit visum veritas lucis et coloris plus quam in directione.

[2.4] Amplius superiora docuerunt quoniam quantitas temperamenti eorum que in visu directo errorem inducunt fortitudinem lucis et coloris respicit, fortiore enim luce vel colore erit maior, debiliore minor. Cum autem per reflexionem debilitentur lux et color, erit latitudo temperamenti singulorum errorem inducentium minor in reflexione quam in directione, et temperantie diminuta latitudo pluralitatem erroris inducit. Preterea quedam minutie corporum com-

18 pars secunda *om. FP1L3*; *inter. a. m. E*; quod error accidat visui propter reflexionem capitulum secundum *R* 19 formarum in visu *corr. ex* in visu formarum *C1*/liber secundus *transp. C1* 21 autem *corr. ex* esse *L3* 23 coloris *corr. ex* corporis *C1*/*post* coloris *add.* et *L3* 26 alia: alias *OL3C1R*/collatio *corr. ex* collectio *L3* 27 iudicium: in deum *S*/*post* similiter *scr. et del.* inducunt et secundum singula *S*/quecumque: quicumque *O* 28 in: secundum *O* 29 inducunt: indocuit *F*; docuit *P1* 32 potest *om. ER* 33 reflexioni *corr. ex* reflexionis *F*; reflexionis *S*/speciale: speculi *E*/*post* primum *add.* enim *C1E*/quoniam: quod *R* 35 quoniam: quod *L3R*; quem *C1* 36 *post* speculi *inter.* propter *O* 37 quoniam: quod *R*/in *inter. O* 38 lucem . . . debilitat *inter. O* 40 docuerunt: docuerit *FP1*/quoniam: quam *FP1E*; quod *R* 42 enim *inter. O*/*post* luce *scr. et del.* erit *E*/debiliore: debiliorem *O* 43 debilitentur: debilitantur *E* 45 pluralitatem *om. O* 46 erroris: errorem *O*/*post* minutie *scr. et del.* me *E*

prehendi poterunt per directionem que nullatenus comprehensibiles sunt per reflexionem. Palam ergo quod directionem superat reflexio in maioritate errorum et numero.

PARS TERTIA
[In speculis planis]

[3.1] In singulis speculis erronea formarum accidit comprehensio, sed iuxta varietatem speculorum fit varietas errorum. In speculis planis minor accidit error quam in aliis. In hiis etenim comprehenditur veritas figure, situs, et quantitatis, sicut in directione, quod per probationem patebit.

[3.2] **[PROPOSITIO 1]** Proponatur speculum planum [FIGURE 6.3.1, p. 304], et sit AB linea in superficie illius speculi communis superficiei speculi et superficiei orthogonali super superficiem speculi. Sint H, Z duo puncta in superficie illa orthogonali, E centrum visus, et a puncto H ducatur perpendicularis super superficiem speculi, que sit HL. Et producatur ut LG sit equalis LH. Similiter, producatur perpendicularis ZF ut DF sit equalis FZ.

[3.3] Planum ex superioribus quoniam H refertur ad E a puncto speculi, et locus ymaginis ipsius est G, tantum distans a superficie speculi quantum H. Similiter, Z refertur ad E, et locus ymaginis est D.

[3.4] Ducta autem linea ZH, et similiter linea GD, quodcumque punctum linee ZH refertur ad E. Locus ymaginis eius est tantum distans a superficie speculi quantum ipse punctus, et ita quilibet punctus linee ZH tantum videtur distare quantum distabit. Unde, si

47 poterunt *corr. ex* potuit *F*/comprehensibiles *om. E*/comprehensibiles sunt (48) *transp. R* 48 *post* sunt *inter.* comprehensibiles *a. m. E* 49 errorum: erroris *E* 50 pars tertia *om. FP1L3*; *mg. a. m. E*; de errorre qui accidit in speculis planis capitulum tertium *R* 51 comprehensio: comprehenso *O* 52 *post* varietatem *scr. et del.* formarum accidit comprehensio *E*/*post* varietas *scr. et del.* eo *C1* 53 accidit: accidet *FP1*/etenim: enim *C1* 54 situs: sicut *P1E*; *corr. ex* sicut *a. m. L3*; *om. R*/et *om. P1*/quantitatis: quantitas *C1*; *corr. ex* quantitas *L3*/*post* sicut *add.* et *ER* 55 probationem *corr. ex* propositionem *S* 57 speculi: circuli *S*; *corr. ex* circuli *a. m. L3*/superficiei . . . et (58) *om. O*/*post* superficiem *add.* scilicet *E*/speculi²: sibi *S*; *mg. a. m. L3*; *inter. a. m. E* 59 H Z: L F *R*/post ortogonali *scr. et del.* est *L3* 60 H: L *R* 61 HL: BL *S*; LH *R*/ut . . . producatur *om. FP1*/LG: HG *R*/producatur²: producitur *C1*/producatur perpendicularis (62) *transp. L3* 62 ZF: FZ *R*/ut: et *S*/FZ: ZF *R* 63 planum: palam *R*/post planum *add.* est *S*/ex *corr. ex* et *SO*/quoniam: quod *R*/H: L *R*/a: ab *R* 64 *ante* puncto *add.* aliquo *R*/ymaginis: ymaginum *FP1OL3E*/ipsius *om. R* 65 H: L *R*/Z: F *R* 66 ymaginis: ymaginum *L3* 67 ZH: FL *R*/et *om. P1*/linea² *om. R*/quodcumque: quodque *E* 68 ZH: FL *R*/est *inter. O*/est tantum *transp. O* 69 ipse punctus: ipsum punctum *R* 70 punctus: punctum *R*/ZH: FL *R*/distabit: distat *ER*

linea ZH fuerit recta, erit linea DG recta. Si fuerit arcus, erit DG arcus et eiusdem curvitatis, quare linea ZH apparebit eiusdem quantitatis, eiusdem figure cuius fuerit, quod est propositum.

[3.5] Verum, si in punctis linee ZH fuerit varietas colorum minutim variata, forsan non discerneretur variatio; sed una pretendetur visui coloris confusio. Unde erit error in luce et in colore, et hoc in numero propter reflexionem. Illa etenim colorum et lucium varietas forsan comprehendi posset directe, sed egressus est color a temperantia respectu reflexionis, non respectu directionis. Similiter, minutie occultantur aut confunduntur in reflexione que discerni possent in directione.

[3.6] Et propter debilitationem lucis vel coloris ex reflexione accidit error in longitudine qui quidem non accideret directe.

[3.7] In situ manifeste accidit error ex sola reflexione, in ymagine enim sinistra comprehendimus ea que in corpore viso, si esset in loco ymaginis, dextra videremus. Cum enim aliquid alii opponitur, contrarius est eis adinvicem situs, quod enim uni fuerit dextrum alii erit sinistrum. Igitur quod rei vise dextrum est ymagini sinistrum, et sinistrum in ymagine dextrum erit videnti, sed comprehenditur in ymagine sinistrum.

[3.8] Et generaliter in modo lucis, vel coloris, vel situs error semper accidit ex sola reflexione. In hiis et in aliis que errorem inducunt directe inducunt similiter in reflexione, et facilius, quoniam temperamentum singulorum minus est visui reflexo quam in directo. Horum omnium unum apponatur exemplum, et idem in ceteris intelligatur.

[3.9] In visu directo, cum fuerit corpus visum remotum ab axibus visualibus, accidit ipsum videri duo; idem evenit in speculis re visa ab axibus elongata.

71 ZH: FL *R*/erit DG arcus *om. FP1* 72 ZH: HZ *L3E*; LF *R* 74 in *inter. a. m. C1*/ZH: FL *R*/minutim *corr. ex* minutum *L3* 75 forsan: forsitan *FP1E*/discerneretur: discernetur *FP1ER* 76 erit: erat *L3*/erit error *transp. R*/et^1 *om. SL3E*; *inter. O*/in^2 *om. FP1C1R*/*post* et^2 *scr. et del.* in *L3*/hoc in *corr. ex* in hoc *E*/in^3 *inter. L3*/*post* in^3 *scr. et del.* no *F* 77 numero: termino *FP1* 78 comprehendi *corr. ex* comprehenderi *L3* 79 respectu^1 *rep. et del. F*/reflexionis *corr. ex* directionis *C1*/non *om. P1*/*post* similiter *add.* particule *ER* (*inter. a. m. E*)/minutie: minute *ER* 80 confunduntur *corr. ex* confundantur *P1*/possent: posset *S* 82 debilitationem: debilitatem *R* 83 qui *om. FP1*; *inter. L3*; que *O*/directe *om. P1* 84 manifeste *corr. ex* manifefeste *F*/*post* manifeste *scr. et del.* quod *L3*/sola reflexione *transp. R* 85 si *om. S*/*post* si *scr. et del.* o *P1* 86 aliquid: quid *E* 87 contrarius: concursus *S*/est eis *transp. E*/est . . . situs: eis situs est adinvicem *R*/adinvicem *corr. ex* advicem *a. m. C1*/alii . . . dextrum (88) *inter. E* 88 *post* est *add.* erit *P1*/ymagini . . . erit (89) *inter. a. m. L3* 89 *post* dextrum *scr. et del.* est *F*/erit: est *R*/sed . . . sinistrum (90) *om. P1R* 91 error semper *transp. R* 92 *post* reflexione *add.* et *FP1*/in hiis *om. R*/hiis et in *om. FP1*/inducunt directe (93) *om. FP1* 93 inducunt: inducit *E*; *om. C1*/et *om. FP1*/quoniam: quam *P1* 94 *post* est *add.* in *R*/visui: visu *ER*/reflexo: reflexio *P1*; *corr. ex* reflexio *L3*; *corr. ex* reflectio *O*/in *scr. et del. O* 95 apponatur: proponatur *ER* 97 idem: item *L3E*/re *corr. ex* rei *L3*

[3.10] In speculis ab aliqua longitudine videbitur corpus minus quam sit, quod forsan directe a tanta longitudine videretur minus quam esset in veritate, sed non adeo minus. Et hoc minoritatis additamentum in speculis provenit propter minus longitudinis temperamentum.

[3.11] In figura non numquam accidit error in speculis per causas per quas in directo, sed maior et frequentior propter situm.

[3.12] Si aliquid ab aliqua longitudine opponatur speculo, et eius capita non percipiantur a visu, ut funis vel aliquid tale, videbitur forsan continuum speculo. Idem accidit in visu directo. Si opponatur funis aliquis foramini et non videantur capita funis, non apparebit distantia inter funem et foramen, licet magna sit, et est propter situm. Si autem alterum capitum videatur, aliud vero non, videbitur fortassis illud caput continuum. Et in singulis ubi directe accidit similiter in reflexione.

PARS QUARTA
In speculis spericis [exterioribus]

[4.1] Universitas errorum in speculis planis accidentium evenit similiter in spericis exterioribus, et preter hoc, in spericis speculis res visa videtur minor quam sit. Et generaliter in hiis speculis nichil ex re visa comprehenditur in veritate preter ordinationem partium, que talis apparet in speculo qualis est in corpore viso.

[4.2] **[PROPOSITIO 2]** Quod autem semper videatur res minor in hoc speculo quam ipsa sit probatur.

[4.3] Sit AB [FIGURE 6.4.2, p. 304] linea visa, ZP speculum, D centrum circuli, E centrum visus. A reflectatur ad E a puncto H, B a

99 videbitur: videtur *S* 100 *post* sit *add.* corpus *C1*/*post* videretur *add.* etiam *R*/minus *inter. a. m. E* 101 *post* minus *add.* sed *L3E*/et: sed *O*/*post* et *add.* in *S* 102 provenit: pervenit *C1*/*post* minus *add.* in *R*/longitudinis: longitudine *ER* 104 per: propter *R* 105 per: propter *R*/*post* in *add.* visu *R* 106 et eius . . . speculo (108) *mg. a. m. L3* 107 percipiantur: percipitur *L3*/a visu *inter. a. m. E*/funis *corr. ex* finis *P1*/forsitan: forsan *SOL3C1* 108 *post* continuum *add.* in *L3C1*/visu: viso *C1* 109 videantur *corr. ex* videtur *a. m. E* 110 funem *corr. ex* finem *F*/sit *om. O* 111 aliud: alium *FP1*; alterum *R* 112 ubi *inter. a. m. E*/*post* directe *add.* error *R*/*post* accidit *scr. et del.* i *O*; *add.* error *L3C1E* (*inter. a. m. L3E*) 113 in *inter. a. m. E*/*post* reflexione *add.* pax *F*; *add.* par *P1* 114 pars . . . spericis (115) *om. FP1SL3*; *mg. a. m. E*; de errore qui accidit in speculis spericis convexis capitulum quartum *R* 117 in *inter. O*/in spericis speculis *om. R*/ spericis speculis *transp. O*/speculis *inter. O* 118 hiis speculis *transp. S* 120 corpore viso: imagine *R* 121 semper . . . res: res semper videatur *R*/videatur res *transp. E* 122 in hoc speculo *om. R*/post speculo *scr. et del.* qualis est in corpore viso quod *P1*/ipsa *om. R* 123 ZP: ZX *R* 124 *post* centrum *inter.* scilicet *a. m. E*/circuli *om. R*/circuli E *inter. a. m. E*/centrum[2]: punctum *R*; *corr. ex* punctus *a. m. E*/a puncto H B *inter. O*/ B a *inter. a. m. E*/*post* B *scr. et del.* D *P1*

puncto N. Linea AB producta aut transibit per centrum speculi, aut non.

[4.4] Transeat. Et ducatur a puncto N linea contingens circulum, que sit NL; a puncto H contingens HM. Et ducantur linee reflexionis BN, EN, AH, EH, et producantur linee EH, EN donec cadant in perpendicularem, que est AD, et puncta casus sint T, Q. Palam quoniam T est locus ymaginis A; Q est locus ymaginis B. Dico quoniam AB maior est QT.

[4.5] Patet ex superioribus quoniam proportio AD ad DT sicut AM ad MT. Similiter, proportio BD ad DQ sicut proportio BL ad LQ. Sed AD maior BD, et DT minor DQ. Erit igitur maior proportio AM ad MT quam BL ad LQ.

[4.6] Secetur AM in puncto F ut proportio FM ad MT sit sicut BL ad LQ. Erit igitur minor BM ad MT quam BL ad LQ. Secetur MT in puncto K ut proportio BM ad MK sit sicut BL ad LQ. K necessario cadet inter M et Q, quoniam LQ minor MQ, et BL maior BM. Cum igitur FM ad MT sicut BL ad LQ et sicut BM ad MK, erit proportio FB ad KT sicut BL ad LQ. Sed BL maior LQ. Igitur FB maior KT, quare AB maior QT, quod est propositum.

[4.7] Si vero linea AB producta non perveniat ad centrum, ducatur a puncto A [FIGURE 6.4.2a, p. 304] linea ad centrum, que sit AG, et sit G centrum, et a puncto B ducatur linea BG. Locus ymaginis A sit punctus D, locus ymaginis B sit E, et ducatur linea ED, que quidem est ymago linee AB. Dico quoniam AB maior est ED, quoniam ED aut est equidistans AB aut non.

[4.8] Si fuerit equidistans, planum quoniam est minor. Si non fuerit equidistans, producatur usquoque concurrat cum ea. Sit concursus Z, et a puncto E ducatur equidistans AB, que sit EH. Angulus EDH aut est acutus, aut rectus, vel maior.

125 *post* N *add.* ducatur *P1*/linea *om. L3ER*/aut[1] *inter. a. m. E* 127 N *om. O*/*ante* linea *add.* et ducatur *O* 128 *post* contingens *add.* circulum *ER*/ducantur: ducatur *S*/*post* linee *add.* accessus et *R* 129 et . . . EH *mg. F*/in *om. O* 130 sint: sunt *O*/quoniam: quod *R* 131 est[1]. . . Q *om. P1*/Q: quod *S*/Q . . . ymaginis[2] *mg. O*/dico quoniam *transp. L3*/quoniam: quod *R*; *scr. et del. L3* 133 patet: palam *R*/quoniam: quod *R*/DT: TD *FP1* 135 BD: DB *R*/erit . . . maior: ergo maior est *R*/igitur *inter. a. m. C1* 137 secetur . . . LQ[2] (138) *scr. et del. F*; *om. P1*/linea *scr. et del. F*/MT[2] *om. S* 138 *post* minor *add.* proportio *R* 139 sit *om. FP1*/necessario cadet (140) *transp. R* 140 cadet *corr. ex* cadat *a. m. E*/et[1] *om. FSL3*; *inter. OL3*/quoniam: quia *R* 142 FB: FL *SO* 144 ducatur . . . centrum (145) *om. P1* 145 *post* linea *add.* a puncto *C1*/AG: AD *R* 146 G: D *R*/BG: BD *R*/*ante* locus *add.* et *R* 147 punctus D: punctum G *R*/E: P *R*; *corr. ex* EE *F*/*post* linea *scr. et del.* et *E*/ED: GP *R*; *corr. ex* ZE *L3* 148 *post* dico *scr. et del.* quod *P1*/quoniam[1]: quod *R*/ED: GP *R* 149 ED: GP *R* 150 quoniam: quod *R* 151 usquoque: usquequo *FSL3*; quousque *R* 152 Z: EZ *P1*; *corr. ex* EZ *F*/E: P *R*/ducatur: producatur *ER*/EH: PH *R* 153 EDH: PGH *R*/vel: aut *R*

[4.9] Si rectus vel maior, erit latus EH maius ED. Sed EH minus AB, et ita propositum.

[4.10] Si fuerit acutus, poterit accidere quod forma sit maior ipsa re cuius est forma, quam licet excedat raro accidet. Et cum acciderit, forsan comprehendetur forma a longitudine tali quod minor videbitur quam sit, quoniam ipsum corpus ab hac longitudine forsan videtur minus.

[4.11] **[PROPOSITIO 3]** Quod autem forma in hiis speculis aliquando videatur maior re visa, scilicet cum maior fuerit, et comprehendatur a tali longitudine a qua certa eius quantitas possit discerni declarabitur.

[4.12] Sit A [FIGURE 6.4.3, p. 305] centrum speculi, et superficies sumatur reflexionis que secabit speculum super circulum. Sit circulus ille EDB, ED dyameter illius circuli, et producatur dyameter ED usque ad Z ut multiplicatio EZ in ZD non sit maior quadrato AD, quod planum, cum sit possibile dyametro ED talem addi lineam ut ductus totalis in partem additam sit equalis quadrato AD. Et dividatur linea ZD in partes equales in puncto H. Erit igitur AH medietas EZ. Ductus ergo AD in HD non erit maior quarta parte quadrati AD, et quoniam ductus AH in HD maior est quadrato HD, sit ductus AH in HT equale quadrato HD.

[4.13] Fiat circulus secundum quantitatem AH, et a puncto H ducatur corda equalis medietati linee HD, que sit HQ. Et producantur linee QA, QT, et supra punctum Q fiat angulus equalis angulo QAH, qui sit HQN. Cum ergo hiis duobus triangulis hii duo anguli sint equales, et unus communis, scilicet QHA, erit tertius tertio equalis, scilicet AQH angulo HNQ. Et erunt trianguli similes, et erit proportio AH ad HQ sicut HQ ad HN. Igitur quod fit ex ductu AH in HN equale quadrato HQ.

154 EH[1, 2]: PH *R*/maius: maior *FP1*/ED: PG *R* 155 *post* ita *add.* est *R* 156 poterit: potest *R*/quod: ut *R* 157 est *inter. a. m. C1*/accidet: accidit *L3*/cum: si *R* 158 comprehendetur: comprehendentur *L3*/a *inter. O* 159 ab . . . minus (160) *mg. a. m. L3*/forsan *corr. ex* forsian *F*/videtur: videbitur *R* 161 aliquando videatur maior (162): videatur maior aliquando *C1* 162 maior fuerit et *om. R*/comprehendatur: comprehenditur *R* 163 a qua *om. L3*/certa *corr. ex* terra *a. m. C1*/certa eius *transp. L3ER*/*post* quantitas *add.* non *R* 166 *post* secabit *scr. et del.* circulum *O* 168 multiplicatio: multiplicato *FP1*/ZD *corr. ex* D *E*/non sit *corr. ex* sit non *E*/non sit maior: sit equalis *R*/quadrato: quam *S* 169 *post* planum *add.* est *P1R*/planum cum sit *rep. S* 171 *post* H *add.* AD *F* 172 AD[1]: AH *L3C1ER*/*post* HD *scr. et del.* sit ductus H *L3*/non erit maior: erit equalis *R*/parte: parti *R* 173 HD[1] *corr. ex* KD *mg. a. m. F; corr. ex* HT *L3*/sit . . . HD (174) *scr. et del. E* 174 equale: equalis *L3ER* 175 H *corr. ex* HS *F* 176 ducatur: producatur *SL3C1ER*/corda *inter. a. m. L3*/*post* corda *scr. et del.* eius *E*/HQ: BQ *O* 177 supra: super *R* 178 HQN *corr. ex* HNQ *F*/*post* HQN *scr. et del.* et *E*/*post* ergo *add.* in *R* 179 sint: sunt *L3C1*/scilicet *om. L3*/tertio *om. P1* 180 trianguli similes *transp. P1;* triangula similia *R* 181 AH *corr. ex* QH *E*/HQ[2] *corr. ex* AD *F*/ex *inter. O* 182 HN: NH *C1*/*post* HN *add.* erit *SL3C1*/*post* equale *add.* erit *O; add.* est *R*

[4.14] Sed quadratum HQ est quarta pars quadrati HD, cum HQ sit medietas HD. Igitur multiplicatio AH in HN equalis est quarte parti multiplicationis AH in HT, quare HN est quarta pars HT. Igitur N cadit inter H et T. Restat ut ductus HT in TN sit tres quarte quadrati HT.

[4.15] Verum angulus QHD acutus, et equalis angulo HQA, quia respiciunt equalia latera in maiori triangulo. Igitur angulus QHN equalis angulo HNQ, et ita HQ equalis QN.

[4.16] Et angulus HNQ acutus, quare angulus QNT obtusus. Quadratum igitur TQ superat quadratum QN et quadratum TN ductu linee TN in NH, quoniam, ut dicit Euclides, quadratum lateris oppositi obtuso superat quadrata duorum laterum quantum est quod fit ex ductu unius lateris bis in partem ei adiunctam procedentem usque ad locum casus perpendicularis a capite alterius lateris ducte. Et si a puncto Q ducatur perpendicularis super lineam HT, cadet in puncto medio linee HN, et ductus TN in medietatem HN bis equipollet ductui TN in HN.

[4.17] Igitur quadratum TQ superat quadratum QN et TN ductu TN in NH. Sed ductus HN in NT cum quadrato NT equalis est ductui HT in TN. Igitur ductus HT in TN est excessus quadrati TQ supra quadratum HQ.

[4.18] Amplius, sit proportio AI ad AH sicut QT ad QH [FIGURE 6.4.3a, p. 305]. Erit quadratum ad quadratum sicut quadratum ad quadratum, et erit proportio excessus quadrati AI super quadratum AH ad quadratum AH sicut ductus HT in TN ad quadratum

183 sed quadratum: per quantum *F*/quadrati: que *S*; *mg. a. m. L3*/*post* HD *scr. et del.* igitur *S* 184 AH: HA *FP1*/in HN *corr. ex* ad HN *C1* 185 est *inter. O* 186 H et T: HZT *S*/TN *corr. ex* TL *E*/sit: sint *R*/quarte: quadrate *S*; in quantitate *P1*; *corr. ex* quantum *F*/quadrati: quadrate *S*; quantum *alter. in* quarti *F* 187 QHD: QHA *R*/*post* acutus *add.* est *R*/et *om. C1*/*post* equalis *inter.* est *a. m. C1* 188 equalia latera *transp. FP1*/maiori triangulo *transp. R*/angulus *corr. ex* angulo *F* 189 *post* equalis[1] *add.* est *O*/equalis angulo HQN *rep. F*/*post* HQN *add.* quia utraque equatur HQA *P1*/*post* et *scr. et del.* L *L3* 190 HNQ *corr. ex* HQN *a. m. E*/angulus[2] *om. FP1*/QNT: GNT *P1S*; *corr. ex* GNT *mg. a. m. F* 191 quadratum[1]: quantum *FP1S*/igitur *om. FP1*/superat quadratum QN *om. FP1*/*post* QN *scr. et del.* i *L3*/quadratum[3]: quantum *FP1*/ductu *corr. ex* ducti *O* 192 NH: HN *R*/lateris: laterum *SL3*/oppositi: opposita *FP1*; opponi *S* 193 obtuso *corr. ex* obtusa *F*/quadrata *corr. ex* quadrato *L3*/laterum: alterum *FP1* 194 lateris: laterum *FP1SO*/procedentem: precedentem *FP1SE* 196 et: nam *R*/post puncto *scr. et del.* medio linee HN *C1*/cadet: cadent *FP1* 197 *post* puncto *scr. et del.* per ZE pri *E*/ medio: medium *R*/linee *om. O*/equipollet: equipollens *FP1*; *corr. ex* equipoll *O* 198 ductui *corr. ex* ductu *O*/*post* ductui *scr. et del.* p *C1* 199 TQ superat quadratum *mg. F*/quadratum[2]: quadrata *RL3* (*alter. in L3*)/et *om. FP1SOL3ER*; *inter. a. m. C1*/TN: in *SO* 200 TN *rep. S*; tamen *P1*/*post* NH *inter.* et quadratum NT *a. m. O*/HN *corr. ex* HNM *L3*/HN in NT: TN in HN *R*/in *inter. O*/NT[2]: TN *R*/ductui: actui *P1* 201 TN *corr. ex* ITN *F*/quadrati: quantitati *FP1*/supra *mg a. m. F*; super *SOL3C1E* 202 quadratum *corr. ex* quantum *mg. a. m. F* 203 AI: AL *L3E* 204 erit *om. S*/quadratum[1]: quadrati *FSOL3C1E*; quantitati *P1*/quadratum[2]: quantum *FP1*; quantitatum *O*/sicut . . . quadratum[1] (205) *inter. a. m. E*/quadratum[3]: quadrati *FSOL3C1E*; quantitati *P1*/ad[2] *om. S*/*post* ad[2] *add.* d *P1* 205 quadratum[1]: quantitatum *FP1*; *om. S*/quadrati: quantitati *P1*/super: supra *R*/quadratum[2]: quantum *P1* 206 ad quadratum AH *mg. F*; *rep. O*

QH. Et quoniam quadratum QH quater sumptum efficit quadratum HD, et ductus HT in TN quater sumptus efficit triplum quadrati HT, erit ductus HT in TN ad quadratum QH sicut tripli quadrati HT ad quadratum HD.

[4.19] Sit autem HC tripla ad HT. Erit ductus CH in HA triplus ad quadratum HD, sed quoniam proportio AH ad HD sicut HD ad HT, erit HT ad HA sicut quadratum HT ad quadratum HD. Verum proportio CH ad HA sicut ductus CH in HT ad ductum HA in HT, et ita CH ad HA sicut tripli quadrati HT ad quadratum HD. Sed hec erat proportio excessus quadrati AI super quadratum AH ad quadratum AH. Igitur CH ad HA sicut excessus quadrati AI super quadratum AH ad quadratum AH. Igitur coniunctim proportio CA ad AH sicut quadrati AI ad quadratum HA, excessus enim quadrati AI super quadratum HA cum quadrato HA efficit quadratum AI.

[4.20] Igitur IA erit media in proportione inter CA et HA, cuius rei conversam paulo ante tetigimus. Igitur proportio CA ad IA sicut IA ad HA, et eadem erit proportio residui ad residuum, id est CI ad IH, et cum IA maior HA, erit CI maior IH.

[4.21] Amplius, ductus AH in HD minor quarta parte quadrati AD. Igitur HD est minor quarta parte linee AD. Igitur est minor quinta parte AH. Cum ergo AH sit maior quam quintupla ad HD, et ductus eius in HT efficiat quadratum HD, erit HT minor quinta parte HD, et ita HT erit minor vicesima quinta parte HA. Sed proportio CI ad IH sicut IA ad HA, ut dictum est. Igitur, coniunctim erit CH ad IH sicut IA cum AH ad AH. Igitur tertia primi ad secundum sicut tertia tertii ad quartum.

211 HC: HCA *F*; HAT *P1*; HE *C1*; HO *R*/CH: HO *R*/HA: HQ *S*; HT *R* 212 HD[1]: HT *R*/*post* HD[2] *add.* est *ER*/HT *corr. ex* HA *E* 213 HA: AH *L3ER*/quadratum: quadrati *FP1SOL3C1E*/ quadratum HD *transp. deinde corr. C1*/verum *inter. a. m. L3* 214 CH[1, 2]: OH *R*/HA[1, 2]: AH *R*/*post* HA[1] *add.* est *E*/HA[2] *corr. ex* HT *E*/ita: proportio *R* 215 CH: OH *R*/*post* sicut *add.* proportio *R*/erat *inter. a. m. L3* 216 super: supra *R*/ad quadratum AH (217) *inter. a. m. E* 217 CH: OH *R*/HA: AH *L3ER*/super: supra *R* 218 *post* AH[2] *scr. et del.* excessus enim quadrati AI *C1*/igitur . . . HA (219) *mg. a. m. L3*/proportio *rep. S*/CA: OA *R* 219 HA: AH *L3C1ER*/enim *inter. L3*/super: supra *R* 220 HA[1]: AH *C1R*; *corr. ex* AH *E*/quadrato: qua *L3*/HA[2]: AH *R* 221 CA: OA *R*/HA: AH *R*/cuius . . . tetigimus (222) *om. R* 222 CA: OA *R*/sicut IA *mg. F* 223 CI *inter. E*; OI *R* 224 et . . . IH *om. R* 225 AH: AD *R*/ *post* HD *inter.* est *a. m. C1*/*post* minor *add.* est *R* 226 igitur[1] *om. FP1*/*post* igitur[2] *add.* HD *R* 227 ergo *corr. ex* erga *L3*/quintupla: cum tripla *L3*; quadrupla *P1*; quadra *alter. in* quadrupla *F*/*post* ad *add.* AH *S*; *scr. et del.* AH *C1* 228 eius *inter. O*/HD *rep. FP1*/HT minor *transp. deinde corr. O* 229 HD: HT *L3*/vicesima: et *FP1*; *om. S*; 3 *E*/quinta: quarta *L3*/*post* quinta *scr. et del.* parte quinta *C1*/parte *inter. a. m. O*/CI: OI *R* 230 IH[1]: CH *S*/sicut IA *om. L3*; *inter. a. m. C1*/ IA: IH *SE*/ad[2]. . . IA (231) *om. FP1*/HA: AH *R*/coniunctim: coniunctinctim *S*/CH: OH *R*/IH: FH *S* 231 IA *corr. ex* IH *a. m. C1*/ad AH *om. FP1OL3*; ad HA *C1*/primi: prime *R*/secundum: secundam *R* 232 tertii: tertie R/post ad *scr. et del.* quantum *F*/quartum: quartam *R*

[4.22] Sed HT est tertia pars linee CH. Igitur TH ad IH sicut tertia pars linee IA cum AH ad lineam AH. Igitur TH ad IH sicut due tertie linee AH cum tertia linee IH ad lineam AH. Sed quoniam CI maior IH, erit IH minor medietate CH, et erit tertia IH minor sexta parte CH, et ita tertia IH erit minor medietate TH. Igitur due tertie AH cum minori parte medietate HT se habebunt ad AH sicut TH ad IH. Igitur IH ad HT sicut AH ad duas sui tertias cum minori parte medietate HT.

[4.23] Sed HT minor vicesima quinta AH, et eius medietas minor medietate vicesime quinte. Sed linea AH in viginti quinque partes divisa; due eius tertie cum medietate vicesime quinte non efficiunt octodecim eius partes. Igitur proportio IH ad HT maior quam sit proportio viginiti quinque ad octodecim.

[4.24] Item, cum HT sit minor vicesima quinta AH, erit AT maior viginti quattuor vicesime quinte AH. Sed linea IH minor medietate CH, et ita minor HT cum medietate HT, et ita minor una et dimidia viginti quinque partium AH, et ita IA minor viginti sex et dimidie partis sumptis partibus secundum divisionem HA in viginti quinque. Ergo proportio IA ad AT sicut minoris viginiti sex et dimidii ad maius viginti quattuor. Igitur proportio IA ad AT minor quam viginti sex et dimidii ad viginti quattuor. Sed IH ad HT maior quam viginiti quinque ad octodecim. Igitur IH ad HT maior quam IA ad AT.

[4.25] Sit proportio IM ad MT sicut IA ad AT. Cadet quidem M inter I, H. Item, maior erit proportio IM ad MH quam IA ad AT, et ita

233 CH: OH *R*/CH . . . linee (234) *mg. a. m. E*/*post* IH *add.* est *ER*/tertia *om. FP1* 234 *post* cum *scr. et del.* tertia linee AH *C1*; *add.* tertia parte *R*/*post* ad[1] *scr. et del.* a *S*/IH: IA *R* 235 linee: linea *FP1*; line *S*/*post* quoniam *add.* linea *R*/CI: OI *R*/*ante* maior *add.* est *R* 236 erit IH *om. L3*/IH[2] *corr. ex* CH *E*/CH: OH *R* 237 CH: OH *R*/TH *corr. ex* HT *E* 238 minori: minore *R*/parte *inter. a. m. E*/medietate: quam sit medietas *R*/*post* IH *scr. et del.* sicut TH ad IH *F* 239 minori: minore *R*/parte *om. R*/medietate: quam sit medietas *R* 241 sed HT *om. S*/vicesima quinta: vicesimo quinto *S*; viginti quinque *FP1L3*/AH: HA *C1* 242 medietate: quam medietas *R*/vicesime quinte: viginti quinque *FP1L3C1*/*post* quinte *add.* partis *R*/sed *inter. a. m. C1*/viginti quinque: vicesime quinte *E*/partes: parte *FP1* 243 divisa: commissa *L3*/eius *inter. E*; *om. R*/*post* medietate *add.* quadrate *FP1SE*/vicesime quinte: viginti quinque *FP1*; *om. S*/*post* quinte *add.* partis *R* 244 octodecim *om. FP1*; octo *L3E*/eius partes *transp. FP1*/ad *rep. P1*/*post* HT *inter.* est *O*/*post* maior *add.* est *R* 245 *post* proportio *scr. et del.* quam sit proportio *C1*/viginti quinque: vicesime quinte *E* 246 vicesima quinta: vicesimo quinto *SC1*; vicesime quinte *E*/*post* quinta *add.* parte *R* 247 viginti quattuor: vicesima quarta *E*/*post* quattuor *add.* partibus quarum AH est viginti quinque *R*/vicesimis quintis: viginti quinque *S*; vicesima quinta *L3E*/vicesimis . . . AH *om. R*/*post* minor *add.* est *R* 248 CH: OH *R*/HT cum *om. R*/cum medietate HT *mg. F*/HT *mg. P1* 249 viginti quinque: vicesima quinta *E*/ita IA *transp. R*/viginti sex: vicesima sexta *E* 250 partis *om. R*/partibus *om. FP1*/AH: HA *FP1*/in . . . quinque *om. R*/viginti quinque: ZH *FP1*; vicesime quinte *E* 251 IA . . . proportio (252) *om. FP1*/*post* AT *scr. et del.* minor *C1*/*post* minoris *add.* linee *R*/viginti sex: viginti octo *S*/maius: maiorem *R* 252 AT *corr. ex* ADT *FP1*/*post* minor *add.* est *R* 253 *post* sed *add.* proportio *R*/ad[1] *om. FP1*/*post* maior *add.* est *R* 254 *post* igitur *add.* proportio *R*/*post* maior *add.* est *R* 255 IM: NM *S*/IA: NL *FP1L3E*; IN *S*/cadet *corr. ex* cadat *E* 256 *post* I *add.* et *OR* (*inter. O*)/IM: MI *L3*; LM *C1*

maior quam IA ad AH. Sit igitur proportio IL ad LH sicut IA ad AH. Cadet quidem L inter M et I.

[4.26] Amplius, a punctis L, M ducantur contingentes LB, MG, et ducantur linee IB, HB, IG, TG, AB, AG, que ultime producantur usque ad exteriorem circulum.

[4.27] Et habebis ex quinta quinti libri quod angulus IBZ sit equalis angulo HBA, cum enim sit proportio IL ad LH sicut IA ad AH, erit H locus ymaginis in reflexione a puncto B. Et si dicatur contrarium ut sumatur alius locus ymaginis, improbabis per impossibile, sumpta impossibilitate a proportione quam verum est esse IA ad lineam a puncto A ad locum ymaginis sicut IL ad lineam a puncto L ad locum ymaginis.

[4.28] Cum igitur H sit locus ymaginis, et LB sit contingens super AB, producta HB faciet angulum reflexionis equalem sibi collaterali, et quoniam LB perpendicularis super ABZ, restabit angulus IBL equalis angulo LBH. Eodem modo erit angulus IGZ equalis angulo TGA, et cum MG sit perpendicularis, erit angulus IGM equalis angulo MGT.

[4.29] Amplius, producatur a puncto H ad lineam AB linea equidistans IB, que sit HP, et a puncto T equidistans IG, que sit TR. Erit angulus IBZ equalis angulo HPB. Sed angulus IBZ equalis, ut dictum est, angulo HBA, et ita duo anguli HBA, HPB sunt equales, quare duo latera HB, HP sunt equalia. Similiter, TR equalis TG. Verum angulus HPB acutus, cum sit equalis angulo reflexionis; erit angulus HPA obtusus, et erit HA maior HP, et ita maior HB. Similiter, erit TA maior TG.

[4.30] Amplius, quoniam HP equidistans IB, erit proportio IA ad AH sicut AB ad AP; erit similiter proportio IA ad AT sicut GA ad AR,

257 IL *om. FP1*/LH: IH *FP1*/IA[2]: LA *E*/AH[2] *corr. ex* HA *E* 258 M et I: MTI *FP1*/I: L *L3* 259 MG: ING *F* 260 ducantur: ducant *L3*/*post* que *add.* due *R* 262 habebis: habebit *SOL3E*; habebitur *R*/quinta: quarta *S*; quinque *L3*; *om. FP1R*/*post* quinta *add.* figura *C1*/quinti libri: quinto libro *P1*; quarto libro *R*/angulus *om. S*/sit . . . enim (263) *rep. S* 263 enim: igitur *R*/IL: AL *L3*/LH: HL *FP1* 264 *post* ymaginis *add.* I dum *R*/in reflexione: reflectitur *R* 265 improbabis: I probabis *R*/*post* improbabis *add.* quod *FP1* 266 *post* quam *add.* enim *L3E*; *add.* non *R*/verum: necessarium *O*/verum est *transp. R*/IA . . . A (267) *inter. O*/a[2]: ab *O* 267 puncto[1] *om. O*/A ad locum *om. R*/*post* ymaginis *add.* ductam ad punctum A *R*/IL: ID *FP1* 269 sit contingens: contingat *R*/super AB (270): circulum in B *R* 270 HB: AB *R*/*post* angulum *add.* LBZ *R*/reflexionis *om. R*/equalem *om. P1*; *corr. ex* qualem *O*/sibi: suo *R*/collaterali: collaterabit *FP1* 271 LB: IB *E*/*post* LB *add.* est *P1*; *inter.* est *O*/restabit: restabat *SL3*/IBL: BL *L3E* 272 *post* LBH *add.* faciet *S*/IGZ: IG *L3E* 273 TGA: TAG *E*/*post* perpendicularis *add.* super AGZ *R*/IGM: IGMS *F* 275 producatur: ducatur *R* 276 *ante* IB *scr. et del.* IB *E*/*post* IG *add.* ad lineam AG *R* 277 IBZ[1] *corr. ex* IBH *S*/IBZ[2]: QBZ *L3*/*post* equalis[2] *add.* angulo HBA *R* 278 angulo HBA *om. R*/et . . . HBA[2] *mg. F*/HPB *om. L3*; HP *alter. in* HPA *E*/sunt . . . HPB (280) *om. S*/equales *corr. ex* equalia *E* 280 *post* HPB *add.* est *R*/acutus: acucutus *S*/*post* angulo *add.* IBZ *R*/reflexionis *om. R*/*post* erit *add.* igitur *R* 281 et[1] *om. O*/HA: AH *R*/et[2] . . . HB *om. P1L3R* 282 TG *inter. a. m. E* 283 equidistans: equidistat *L3ER* (*alter. in a. m. L3*)/IB *corr. ex* HP *S*/proportio *om. R*/ad *om. P1* 284 AT *corr. ex* AH *E*/GA: AG *R*

et erit proportio AH ad AI sicut AP ad AB. Sed IA ad AT sicut AB ad AR, cum AB sit equalis AG. Igitur a primo erit proportio AH ad AT sicut AP ad AR.

[4.31] Verum, cum angulus HPA obtusus, quadratum HA excedet quadratum HP et quadratum AP cum multiplicatione AP in lineam ductam a puncto P usque ad locum perpendicularis ducte a puncto H bis. Sed perpendicularis ducta a puncto H cadet in puncto medio linee PB, cum HB, HP sint equales, et ita quadratum HA excedet quadratum HP et quadratum AP in multiplicatione AP in PB. Et ita quadratum AH excedit quadratum HP in multiplicatione AB in AP, quoniam ductus AP in PB cum quadrato AP valet ductum AB in AP. Similiter, quadratum AT excedit quadratum TR in ductu AG in AR, sive AB in AR, quod idem est.

[4.32] Ducatur ergo linea AB in duas lineas AP, AR, et provenient duo excessus. Igitur proportio excessus ad excessum sicut AP ad AR, quare proportio excessus quadrati AH super quadratum HP ad excessum quadrati AT super quadratum TR sicut AH ad AT. Et cum HP equalis HB, et TR, TG, erit proportio excessus quadrati AH super quadratum HB ad excessum quadrati AT super quadratum TG sicut AH ad AT.

[4.33] Sed multiplicatio AH in HU equalis est quadrato HB. Igitur multiplicatio AH in AU erit excessus quadrati AH super quadratum HB. Igitur proportio AH ad AT sicut multiplicationis AH in AU ad excessum quadrati AT super quadratum TG. Et si due linee AH, AT ducantur in AU, erit proportio AH ad AT sicut multiplicationi AH in

285 AH: AB *FP1*/AI: AL *L3*; IA *R*/AB: AH *FP1E* 286 a primo *om. R*/AH *corr. ex* AG *L3*/AT: AI *alter. in* AR *a. m. E* 287 AR *corr. ex* AI *E* 288 *post* HPA *add.* sit *R*/excedet *corr. ex* excedit *E* 289 cum: in *O*; *om. R*/lineam ductam (290): linea ducta *L3E* 290 puncto[1] *corr. ex* puncta *F*/P *om. E*/ducte *corr. ex* dute *F* 291 bis *om. L3*/bis . . . H *om. FP1*/puncto[2] *om. R*/medio: medium *R* 292 PB: PH *L3E*/*post* quadratum *scr. et del.* HP et quadratum *S* 293 HP: AP *O*/et[1] *inter. O*/et[1] . . . HP (294) *om. L3*/quadratum[2] *om. O*/AP[1]: HO *O*/AP[2] . . . multiplicatione (294) *mg. O* 294 excedit: excedet *S*/in[1]: cum *L3*/AB: APB *E* 296 AT . . . quadratum *mg. F*/quadratum: quadraturam *FP1* 297 AB *om. FP1*/AR: AT *S*/in . . . idem *inter. a. m. E* 298 *post* AB *scr. et del.* et *F*/duas: dua *S*/*post* AP *add.* et *R* 299 excessus *corr. ex* accessus *S*/*post* proportio *scr. et del.* AD *F*/ad excessum *mg. F* 300 AR: R *S*/*post* AR *scr. et del.* quare proportio excessus ad excessum sicut AP ad AR *E*/quare: erit ergo *R*/*post* proportio *add.* hoc sequitur quia dictum est quod proportio AH ad HT est sicut proportio *FP1*/quadrati *mg. a. m. E*; qua *L3*/super: supra *R*/*post* HP *add.* et *L3*/ad: ED *L3* 1 super: supra *R*/TR *corr. ex* HB *S*/AT[2]: HT *L3* 2 *post* HP *add.* sit *R*/HB: MB *L3E*/AH: HA *S*/super: supra *FP1R* 3 HB . . . quadratum *mg. C1*/super: supra *R* 5 sed . . . AT *om. S*/HU: HB *FP1OC1*/*post* multiplicatio *add.* EH in HD est equalis quadrato linee a puncto H ad circulum DBE contingenter ducte et erit minor HB et ita multiplicatio EH in HD minor est quadrato HB et fiat ductus *R*/equalis est *transp. L3E*/quadrato: quadrati *L3*/*post* HB *add.* ergo HU minor est HA et quadratum AH est equale multiplicationi AH in AU et HU *R* 6 AU: AHU *FP1*/AH[2]: HA *ER*/super: supra *R* 7 AH[1] *corr. ex* HA *E*/*post* sicut *add.* proportio *R*/AU: AB *L3*/ad[2] . . . AU[1] (10) *mg. C1* 8 super: supra *R*/TG: AG *FP1*; TQ *L3E* 9 erit . . . AU[1] (10) *om. FP1*/*post* sicut *add.* proportio *R*/multiplicationi: multiplicationis *R*

AU ad multiplicationem AT in AU. Igitur multiplicatio AT in AU est excessus quadrati AT super quadratum TG. Igitur erit multiplicatio AH in HU equalis quadrato HB, et multiplicatio AT in TU equalis quadrato TG.

[4.34] Amplius, arcus BG dividatur per equalia in puncto O, et ducantur tres perpendiculares super lineam HA, scilicet BF, OY, GK, et a puncto G ducatur linea equidistans HA, que sit GS, et a puncto B ducatur perpendicularis super AG, que sit BX. Hoc quidem BX, si produceretur usque ad circulum, divideret linea AG ipsam per equalia, et arcum cuius esset corda. Et ita secaretur alius arcus equalis arcui BG, quoniam alium arcum respiceret angulus GBX, et ita angulus GBX est medietas anguli supra centrum respicientis eundem arcum, secundum Euclidem. Igitur angulus GBX est medietas anguli BAG quem dividit linea OA per equalia. Igitur angulus GBX est equalis angulo OAG. Duo autem anguli BSG, BXG recti.

[4.35] Si intelligatur circulus super BG transiens per S, transibit per X, et fiet arcus SX super quem cadent duo anguli XBS, XGS. Igitur hii duo anguli sunt equales. Sed angulus GAY equalis angulo XGS propter equidistantiam linearum, et ita angulus GAY equalis angulo XBS. Et ut dictum est, angulus GBX equalis angulo OAG. Erit angulus OAY equalis angulo GBS, et erit triangulus OAY similis triangulo GBS. Igitur proportio GB ad BS sicut OA ad AY.

[4.36] Amplius, cum angulus AHB sit acutus, quadratum AB minuit ex quadratis AH, HB quantum est illud quod fit ex ductu AH in HF bis, secundum quod dicit Euclides. Igitur quadratum AH cum quadrato HB superat quadratum DA, que est equalis AB, in ductu

11 super: supra *R*/TG: DG *O*/igitur erit *transp. R* 12 AH: HA *L3ER*/HU: HB *S*/HB . . . quadrato (13) *om. L3*/*post* et *add.* p *F* 14 *post* dividatur *scr. et del.* dimidiatur *E*/*post* O *add.* et ducatur AO *R* 16 linea *om. R* 17 *post* ducatur *scr. et del.* linea *P1*/sit *mg. F*/BX[1]: BG *SL3E*; BC *R*/BX[2]: LG *L3*; BG *E*; BC *R* 18 produceretur: producantur *FP1*; producatur *L3E*; producantur *alter. in* producatur *O*/divideret: dividet *FP1C1*; dividat *L3E*; dividat *alter. in* dividet *O*/AG: AH *P1*/ipsam: ipsum *L3* 19 cuius *inter. O*/*post* ita *scr. et del.* secavi *F* 20 BG *inter. O*/quoniam: quem *C1*/alium: illum *R*/GBX: GLG *L3*; GBG *E*; CBG *R* 21 GBX: GLG *L3*; GBG *E*; CBG *R*/supra: super *R* 22 Euclidem: eundem *FP1*; *corr. ex* eundem *E*; Euclid *O*/GBX: GBG *L3E*; CBG *R*/BAG: GAB *R* 23 quem: quoniam *L3*; *corr. ex* quoniam *E*/OA: EA *S*; IA *O*; GA *L3E*; AO *R*/*post* angulus *scr. et del.* angulo *E*/GBX: GBG *L3*; GBE *E*; CBG *R*/est *inter. a. m. E*/est equalis *transp. E* 24 BSG: BLG *FP1SL3E*/*post* BSG *add.* et *FP1*/BXG: BEG *E*; BCG *R*/*post* recti *add.* sunt *R* 25 *post* si *add.* igitur *R*/*post* per *add.* centrum *P1*/transibit: transiens *FP1* 26 X: Q *FP1*; E *E*; C *R*/fiet: fiat *FP1*/SX: Q *FP1*; SE *E*; SC *R*/cadent: cadunt *FP1*/XBS: XLS *L3*; EBS *E*; CBS *R*/XGS: EGS *E*; CGS *R* 27 hii *om. FP1*/equales: equale *L3*/*post* equalis *add.* est *OR* (*inter. O*)/XGS: EGS *E*; CGS *R* 28 propter . . . et (29) *mg. a. m. E*/*post* GAY *add.* est *L3* 29 XBS: GS *FP1*; GBS *SO*; EBS *E*; CBS *R*/et *inter. C1*/GBX: GBQ *FP1*; GBE *E*; GBC *R*/equalis . . . OAY (30) *om. P1*/*post* angulo *scr. et del.* T *L3* 30 triangulus: angulus *FP1*; triangulum *R*/similis: simile *R* 31 igitur *om. L3*/GB: BG *C1*/BS: US *L3E*/AY *corr. ex* TA *E*/*post* AY *add.* et proportio GB ad GS sicut OA ad OY *R* 32 *post* amplius *inter.* est *O*; *scr. et del.* est *C1*/angulus *inter. O*/quadratum: quam *FP1* 33 minuit ex: minus est *R*/HB: LB *FP1* 34 bis *inter. O*; hoc *L3*/quadratum: quantum *FP1* 35 superat: super AT *L3*/DA: DH *S*; AD *R*

AH in HF bis, et ita in ductu AH in HD bis et AH in DF bis. Sed multiplicatio AH in HD bis cum quadrato AD est equalis quadrato AH cum quadrato HD. Et ita ablato communi quadrato AB [cum multiplicatione AH in HD bis], restabit quadratum HD cum ductu AH in FD bis equalis quadrato HB.

[4.37] Sed multiplicatio AH in HT equalis est quadrato HD, et multiplicatio AH in HU equalis quadrato HB. Erit ergo multiplicatio AH in HU equalis multiplicationi AH in HT et multiplicationi AH in DF bis. Subtracto ductu AH in HT, quem communem ponimus utrique multiplicationi, restabit multiplicatio AH in TU equalis multiplicationi AH in DF bis. Igitur TU est dupla DF.

[4.38] Amplius, cum angulus ATG sit acutus, secundum predictum modum erit quadratum AT cum quadrato TG equale quadrato AD cum ductu AT in TK bis, et ita cum ductu AT in TD bis, et in DK bis. Et probatur modo predicto quod quadratum TG equale est quadrato TD cum ductu AT in DK bis. Sed ductus AT in TU equalis quadrato TG, et ita equalis quadrato TD cum ductu AT in DK bis.

[4.39] Sit ductus AT in TE equalis quadrato TD. Restat ergo ut ductus AT in EU sit equalis ductui AT in DK bis, per ablationem communis, quod est ductus AT in TE. Igitur EU est dupla DK. Sed iam dictum est quod TU est dupla DF. Restat ergo TE duplum FK.

[4.40] Amplius, proportio AH ad HT sicut AH ad HD duplicata, HD enim media est in proportione inter illas, cum eius quadratum sit equale ductui AH in HT. Et similiter proportio AT ad TE sicut AT ad TD duplicata. Sed maior est proportio AT ad TE quam AH ad HD. Igitur maior est proportio AT ad TE quam AH ad HT, et cum AH maius AT, erit HT maior TE. Sed TE dupla FK.

36 HD: HA *L3* 38 AB: AH *FP1OE*; AD *R*/multiplicatione: ductu *R* 39 restabit: restat *FP1*/AH: AB *S*/AH . . . multiplicatio (41) *mg. C1* 40 bis *inter. E*/equalis: equale *R*/HB: B *S* 41 sed . . . HB (42) *om. L3*/HT: HD *S*/equalis est *transp. S*/est *om. O* 42 *post* HU *inter.* est *O* 43 AH^2 . . . multiplicationi^2 *om. L3* 44 bis *corr. ex* bbis *P1*/*post* subtracto *add.* que *R*/quem *om. OL3*; quam *C1E* 45 restabit: restat *FP1*/equalis *scr. et del.* est *E* 47 *post* acutus *add.* erit *R*/predictum: supradictum *L3E* 48 erit *om. R*/equale: equalis *L3E*; *corr. ex* equales *C1* 49 TK . . . in^2 *om. L3*/AT in TD: ACN TD *FP1*/in *om. S*/TD: DT *C1*; *corr. ex* T *O*/TD bis et *om. S*/DK: TK *L3* 50 et . . . bis (51) *om. S*/probatur: probabitur *L3C1ER*/equale est *transp. FP1* 51 cum *rep. C1*/AT^1: A *FP1*/DK: TK *L3*/sed: sit *FP1*/TU: TE *F*; TR *P1*/*post* equalis *add.* est *R* 52 TG: TD *FP1* 53 sit *corr. ex* si *O*/*post* sit *add.* autem *R*/TE: TO *SO* 54 EU: OU *FC1*; TU *P1*; GU *S*; *corr. ex* GU *O* 55 quod: qui *R*/TE: TO *FP1SC1*; *alter. ex* T *in* TO *O*/EU: IOU *FP1*; OU *SC1*; GB *O*/sed . . . dupla (56) *mg. a. m. E*/DK: KD *R* 56 DF: TF *L3E*/TE: TO *FP1SOC1*/duplum: dupla *R*/FK: FQ *FP1*; KF *R* 57 *post* HT *add.* est *L3R* 58 HD: HA *L3*/enim *om. FP1*/quadratum: quam *L3* 59 ductui *corr. ex* ductu *O*/TE: TO *FP1SOC1* 60 sed *inter. O*/*post* maior *add.* AT ad DO quam AH ad HD igitur maior *FP1*/TE: TO *FP1*; TD *SOL3C1ER* (*inter. O*)/HD *alter. in* HT *a. m. E* 61 igitur . . . HT *scr. et del. E*; *om. R*/TE: TO *FP1SOC1*/*post* quam *add.* AH ad HD igitur maior est proportio AT ad CO quam *FP1*/et: ergo *inter. O*/*post* AH *add.* sit *R* 62 maius: maior *R*/TE^{1,2}: TO *FP1SOC1*/sed TE *om. L3*/*post* dupla *add.* ad *R*/FK: KF *R*/*post* FK *add.* ergo HT maior est quam dupla ad KF *R*

[4.41] Item, ut dictum est, proportio BG ad GS sicut OA ad OY. Erit proportio BG ad OA sicut GS ad OY. Sed OA equalis BA, et GS equalis FK. propter equidistantiam. Erit proportio BG ad BA sicut FK ad OY.

[4.42] Amplius, IH minor medietate CH, et CH tripla HT. Erit IH minor HT et medio ipsius. Sed HT minor quinta HD. Igitur IH minor TD multo, quare IH multo minor ND, quare MI minor ND. Et palam per hoc quod I cadet inter H et Z.

[4.43] Amplius, quod fit ex ductu EZ in ZD non est maius quadrato AD; igitur quod fit ex ductu EM in MD est minus quadrato AD. Sed quoniam MG est contingens, quod fit ex ductu EM in MD est equale quadrato MG, secundum quod dicit Euclides. Igitur MG est minor AD; igitur MG est minor AG.

[4.44] Amplius, duo trianguli AGM, MGK habent unum angulum communem, et uterque eorum habet angulum rectum. Igitur sunt similes, quare proportio MK ad KG sicut MG ad GA, et ita MK minor est KG. Et cum OY sit maior GK, erit HD minor OY.

[4.45] Amplius, AH ad HD sicut HD ad HT; erit ergo sicut medietas HD ad medietatem HT. Et ita AH ad HD sicut QH ad medietatem HT, cum QH sit medietas HD, et ita AH ad QH sicut HD ad medietatem HT, et ita QH ad AH sicut medietas HT ad HD. Sed medietas HT maior FK, et HD minor OY. Erit igitur medietas HT ad HD maior quam FK ad OY, quare erit QH ad AH maior quam FK ad OY.

[4.46] Amplius, linea AQ secat circulum EBD. Sit punctus sectionis Q, et ducatur linea DQ, que erit equidistans QH. Eritque proportio QH ad HA sicut QD ad DA, et ita QD ad DA maior quam FK ad

63 ut dictum *om. L3*/*post* proportio *scr. et del.* proporti *S*/*post* sicut *scr. et del.* CK *FP1*/OA: CA *FP1S*; EA *L3C1E* 64 proportio *om. R*/OA: GA *FP1*; GOA *S*/OA: CA *FP1S*; EA *L3E* 65 FK: SK *L3* 67 *post* amplius *add.* quia *R*/minor *om. FP1*/*post* minor *add.* est *R*/CH[1, 2]: OH *R*/et CH *om. FP1*/HT: HTD *FP1*; TH *R* 68 medio: medietate *R*/*post* quinta *add.* parte *R* 69 minor[1]. . . IH *rep. S*/*post* minor[1] *add.* est *R*/TD: ID *L3E*/quare IH multo *mg. a. m. E*/multo minor *transp. deinde corr. C1*/MI *scr. et del. E*/*post* MI *add.* multo *R* 70 per: propter *O*/hoc: hec *S*/cadet: cadit *R*/Z: S *L3E* 71 EZ: Z *FP1*; ES *L3E*/ZD: SD *L3E*/non est maius: est equale *R* 72 AD igitur *inter. O*/igitur . . . AD[2] *mg. a. m. E*/minus: maius *FP1* 73 sed *om. FP1*/*post* MG *add.* circulum DBE *R*/contingens: continens *L3E*; contingit *R*/contingens . . . est (75) *mg. a. m. O*/EM: AM *FP1SOL3E* 74 quadrato *inter. E*/secundum: scilicet *S* 75 igitur: et ita *O*/MG *om. R*/est minor *transp. R*/AG: AS *L3* 76 duo *om. R*/trianguli: triangula *R*/AGM *corr. ex* AMGM *L3* 77 uterque: utrunque *R*/habet: habent *S*/*post* habet *add.* unum *L3C1ER* 78 similes: similia *R*/minor est (79) *transp. L3* 79 HD: HG *L3* 80 *post* amplius *add.* quia *R*/ergo *om. R*/*post* ergo *add.* AH ad HO *FP1*; *inter.* AH ad HD *O*/sicut: sic *R*/*post* sicut *add.* HD ad HT *L3* 81 *post* QH *scr. et del.* ad *E* 82 cum . . . HT[1] (83) *mg. a. m. E*/HD[1]: HT *S*/AH: HA *L3*/HD[2]: HT *L3* 83 *post* HD *add.* sed medietas HT ad HD *P1* 84 *post* maior[1] *add.* est *R*/*post* et *add.* ita *P1*/HD[1]: HB *FP1*/OY: COY *FP1*; AY *O*/*post* igitur *add.* proportio *R*/*post* HT *scr. et del.* maior FK *F* 85 *post* erit *add.* proportio *R* 86 AQ: AK *FP1*/punctus: punctum *R* 87 Q: quod *FP1*; que *O*; X *SC1*; Œ *R*/DQ: DX *SC1*; DŒ *R*/que erit *transp. deinde corr. S*/eritque: erit OS *FP1*; erit et *L3* 88 QD[1]: ZD *FP1*; XD *SC1*; Œ *R*/et . . . DA[2] *om. FP1*/*post* ita *add.* proportio *R*/QD[2]: XD *SC1*; Œ *R*

OY. Sed FK ad OY sicut GB ad BA. Erit igitur maior QD ad DA quam BG ad BA, et ita QD maior BG, et arcus QD maior arcu GB.

[4.47] Amplius, producatur AQ usque ad punctum S ut sit AS equalis AI, et ducatur linea SI, que erit equidistans QH, et erit proportio SI ad QH sicut IA ad AH. Sed supra positum est quod IA ad HA sicut TQ ad QH; erit igitur SI equalis TQ.

[4.48] Amplius, mutetur figura ad evitandam linearum intricationem multiplicem et propter defectum litterarum ad distinctionem nominum. Cum ergo IA sit equalis linee quam diximus AS, fiat circulus secundum quantitatem ipsarum [FIGURE 6.4.3b, p. 306]. Et loco S ponamus nomen N, et producantur AG, AB usque ad hunc circulum, et sint ABC, AGR; loco littere Q ponamus F. Dictum est quoniam arcus DF maior est arcu BG. Sit arcus BM equalis arcui DF, et ducatur linea AMU et linee IM, NM, et linea QM, que producatur usque ad exteriorem circulum. Et cadat in punctum Z, et ducantur linee ZA, ZG.

[4.49] Cum arcus BM sit equalis arcui DF, addito communi, erit arcus MF equalis arcui DB. Erit angulus NAM equalis angulo IAB, et latera lateribus equalia; erit MN equalis IB. Et cum positum sit supra, AQ equale AH, erunt AQ, AM equalia HA, AB, et angulus angulo. Erit QM equalis HB, et erit angulus QMN equalis angulo HBI, quoniam duo eius latera duobus illius equalia. Et basis que est IH equalis basi NQ, et angulus NMU equalis angulo IBC.

[4.50] Sed angulus IBC equalis angulo HBA, et angulus HBA equalis angulo QMA; erit angulus NMU equalis angulo QMA. Et quoniam QMZ est linea recta, ut posuimus, erit angulus QMA equa-

89 *post* maior *add.* proportio *R*/QD: quod D *F*; XD *SC1*; QZD *L3E*; ŒD *R*/DA *corr. ex* ADA *S* 90 BG[1]: GB *C1*/QD[1]: quod D *FP1*; XD *SC1*; ZQD *L3E*; ŒD *R*/*ante* maior[1] *scr. et del.* ad *S*/*post* maior[1] *add.* est *O*/BG[2]: G *FP1*/QD[2]: OD *FP1*; XD *SC1*; ZQD *L3E*; ŒD *R* 92 *post* equalis *add.* arcui *P1*/AI: AR *FP1*/proportio *om. R* 93 AH: HA *R*/positum: dictum *FP1* 94 HA: AH *SOC1ER* (*alter. in E*)/igitur *inter. E* 95 mutetur: mittetur *S*/evitandam *corr. ex* vitandam *E*/intricationem: intritionem *S* 96 et *inter. C1*/distinctionem: instinctionem *O* 97 nominum: linearum *R* 99 S: scilicet *L3*/ponamus: ponatur *R*/nomen: litera *R*/hunc circulum (100) *transp. R* 100 AGR: AGK *L3E*/loco: loct *E* /Q: quod *FP1S*; que *O*; quam *L3E*; X *C1*; Œ *R*/dictum: deinde *L3E* 101 quoniam: quod *R*/arcus *om. FP1*/sit *corr. ex* sicut *C1* 102 *post* linee *add.* IB IG *R*/IM: YM *S*/producatur *corr. ex* producantur *O* 103 Z: S *L3E* 105 *post* cum *add.* autem *R*/arcus: arcu *FP1*/communi: quoniam *FP1* 106 erit: eritque *R*/IAB: RAB *FP1* 107 latera: latria *S*/*ante* lateribus *add.* et *S*/equalis *om. FP1*/*post* IB *add.* et angulus NMA equalis angulo IBA et angulus NMU angulo IBC *R*/positum: positus *L3* 108 AQ[1]: AR *P1*/*post* AM *add.* latera *R* 109 angulus *inter. a. m. E*/QMN: QMA *R*/HBI: HBA *R*/*post* HBI *add.* et QMN equalis angulo HBI *R* 110 que: qui *L3*/IH: QN *R*/*ante* equalis *add.* est *R* 111 NQ: HI *R*/NMU: MNR *FP1*/IBC: TBC *FP1*/IBC sed angulus (112) *om. SL3E* 112 sed: et *R*/angulus[1] *om. R*/angulo *om. L3* 113 QMA[1]: quia *S*/*post* QMA[1] *add.* et quoniam QMS est linea recta ut posuimus *mg. a. m. E*/erit: ergo *R*/angulus *om. R*/erit . . . QMA[2] *inter. O*/NMU: MNB *P1*; MNS *E*/angulo[2] *om. R* 114 quoniam *om. O*/*post* quoniam *add.* ut posuimus *R*/QMZ: QMS *L3E*/*post* recta *scr. et del.* QMA *C1*/ut *inter. O*/ut posuimus *om. R*

lis angulo UMZ, quare punctus N refertur ad Z a puncto M, et locus ymaginis ipsius Q. Hoc tamen deest probationi ut pateat MZ totam esse extra circulum, quod sic patebit.

[4.51] Palam quoniam contingens ducta a puncto B cadet inter I et H, et remotio puncti B a puncto H quanta est puncti M a puncto Q, et IH equalis NQ. Igitur contingens ducta a puncto M cadet inter N et Q. Igitur QM secat circulum, quare tota MZ extra circulum, et ita propositum.

[4.52] Amplius, quoniam angulus NMU equalis angulo UMZ, erit arcus NU equalis arcui UZ. Erit angulus NAU equalis angulo UAZ. Sed iam patuit quod angulus NAU equalis est angulo IAC. Erit angulus IAC equalis angulo ZAU.

[4.53] Angulus BAG aut erit equalis angulo GAM, aut minor, aut maior. Sit equalis. Si igitur ab angulo IAB subtrahatur angulus BAG, et ab angulo ZAU angulus MAG, remanebit angulus IAG equalis angulo ZAG. Erit IG equalis ZG, et triangulus triangulo, et erit angulus IGA equalis angulo ZGA. Restabit angulus IGR equalis angulo ZGR. Sed angulus IGR equalis angulo TGA. Erit angulus TGA equalis angulo ZGR. Si igitur TG producatur, veniet ad Z, quare TGZ linea recta. Igitur I a puncto G refertur ad Z, et locus ymaginis eius T.

[4.54] Sit ergo Z visus. Reflectentur ad ipsum duo puncta N, I a duobus punctis M, G, et loca ymaginum puncta T, Q. Igitur linea TQ erit ymago linee IN, et probatum est supra quoniam TQ equalis est IN, et ita potest accidere in hiis speculis ymaginem esse equalem rei vise.

[4.55] Si vero angulus BAG fuerit maior angulo GAM, erit angulus ZAG maior angulo IAG. Sit angulus KAG equalis angulo IAG.

115 UMZ: MZ *FP1*; MNG *L3*; NMS *E*/punctus: punctum *R*/refertur: reflectitur *R*/Z: S *L3E* 116 ipsius *om. P1*/MZ: MS *L3*; *alter. ex* MG *in* MS *E* 117 extra *inter. E* 118 palam: pam *P1*/quoniam: quod *R*/B: H *L3*/cadet: cadat *R*/I: L *E* 119 *post* et^{2} *add.* tanta est *R*/remotio: remoto *L3*/Q: quod *FP1* 120 IH *corr. ex* H *P1*/NQ: NH *L3*/cadet *om. FP1* 121 MZ: MC *L3*; MS *E*/*post* MZ *add.* est *R*/et *om. FP1*/et ita propositum (122) *om. R* 123 *post* angulus *scr. et del.* minor *C1*/*post* equalis *add.* est *R*/UMZ: UMS *L3*; NMS *E* 124 *post* arcui *scr. et del.* UR *F*/UZ: AS *L3*; NS *E*/*post* UZ *add.* et *R*/UAZ: NAZ *FP1S* ; NAS *L3E* 125 patuit: placuit *L3*/equalis *om. F*/equalis est *transp. P1*/est *inter. E*/angulo IAC *transp. C1*/erit: igitur *R*/angulus^{2} *om. R*/angulus IAC (126) *om. L3E* 126 *post* IAC *add.* erit *R*/ZAU: SAU *L3E*; UAZ *R* 127 *ante* angulus *add.* amplius *C1*/angulus: amplius *L3E*/*post* angulus *add.* vero *R*/aut maior (128) *om. P1* 128 ab: BA *L3E*/IAB *corr. ex* IBA *a. m. E*; IAC *R* 130 ZAG: SAG *L3E*/*post* ZAG *add.* et *R*/*post* IG *add.* equalis angulo ZAG erit IG *FP1*/ZG: SG *L3E*/et^{1} *inter. a. m. C1*/triangulus: triangulum *R* 131 ZGA: SAG *L3E*; ZAG *R*/*post* restabit *add.* igitur *R*/IGR: IGT *C1*; *alter. ex* AGT *in* IGT *S*/angulo^{2} . . . equalis^{1} (132) *om. P1*/ZGR: SGI *L3E* 132 sed: fiat igitur *R*/angulus^{1}: angulo R/angulus IGR: equalis EGR *F*/angulo^{1}: angulus *R*; *om. L3*/TGA^{1}: IGA *P1*; *corr. ex* TAG *E*/*post* TGA^{1} *add.* ut patuit in precedenti figura longe ante *FP1*/TGA^{2} *corr. ex* TAG *E* 133 ZGR: SGR *L3E*/Z: S *L3E*/TGZ: TGS *L3E* 134 refertur: reflectitur *R*/Z: S *L3E*/*post* eius *add.* est punctum *R* 135 *ante* sit *add.* si *R*/sit ergo Z: ergo Z sit *R*/ipsum *mg. F*/*post* ipsum *inter.* visum *a. m. C1*/N I *transp. R* 136 igitur linea TQ *om. FP1* 137 *post* linee *scr. et del.* e *C1*/et *om. R*/probatum: propriatum *L3E*/*post* probatum *add.* autem *R*/quoniam: quod *R*/TQ: RQ *C1* 139 GAM: GABA *FP1* 140 sit: si *S*/IAG: VAG *FP1*

Quoniam punctus K dimissior puncto Z, et punctus M dimissior G, linea KG secabit lineam ZM. Secet in puncto L, Igitur, existente visu in puncto L, refertur N ad ipsum a puncto M, et locus ymaginis Q; refertur ad ipsum I a puncto G, et locus ymaginis T, secundum priorem probationem. Et ita TQ ymago IN, quod est propositum.

[4.56] Si vero angulus BAG fuerit minor angulo GAM, erit angulus ZAG minor angulo IAG. Sit angulus OAG equalis angulo IAG, et producatur linea OG. Palam quoniam I refertur ad O a puncto G. Linea OG aut secabit lineam ZMQ extra circulum speculi aut non.

[4.57] Si secet extra, et in puncto sectionis fuerit visus, refertur ad ipsum duo puncta I, N, et loca ymaginum T, Q, et ita redit propositum.

[4.58] Si forsan linea OG secabit lineam ZMQ intra circulum, nec poterit aptari predicta probatio. Sed dico quoniam extra hanc totalem superficiem erit invenire punctum ad quod refertur duo puncta I, N a duobus punctis speculi, et ymago TQ.

[4.59] Verbi gratia, palam quoniam angulus NAZ duplus ad angulum IAB, et angulus IAO duplus ad angulum IAG, secundum predicta. Et angulus NAZ non excedit angulum IAO in angulo maiori angulo NAI. Et duo anguli OAI, ZAN maiores tertio, quod est IAN, et duo OAI, IAN maiores tertio NAZ, et duo ZAN, NAI maiores tertio IAO. Habemus ergo tres angulos, quorum quilibet duo maiores tertio.

[4.60] Igitur ex illis est facere angulum corporalem. Fiat angulus ille super A, et sit linea SA erecta super A, et angulus IAS sit equa-

141 punctus[1, 2]: punctum *R*/*post* K *inter.* est *O*/dimissior: demissius *R*/puncto Z: punctorum *FP1SL3E*/M: ZM *SL3E*/*ante* G *add.* puncto *R* 142 *post* linea *add.* G linea *F*/*post* secabit *scr. et del.* KG secabit *C1*/lineam *om. C1*/*post* L *scr. et del.* refertur *F* 143 refertur: reflectetur *L3ER*/a *om. L3*/Q . . . ymaginis (144) *inter. E*/*post* Q *add.* similiter *R* 144 refertur: reflectetur *E*/refertur . . . I: I reflectetur ad ipsum *R*/*post* ipsum *add.* vel refertur *E*/a puncto G *om. R*/ymaginis: ymaginum *S*/*post* ymaginis *add.* est *R*/T *om. P1* 145 probationem: proportionem *OL3E*/*post* TQ *inter.* est *O*/*post* ymago *add.* est *R* 146 angulus BAG *transp. FP1* 147 IAG[1]: YAG *S*/*post* IAG *scr. et del.* et ita *F*; *add.* et *P1*/sit *mg. F*/OAG: SAG *FP1*; EAG *SL3E*/*ante* equalis *scr. et del.* equalis *E* 148 producatur: ducatur *R*/OG: CG *FP1*; LG *L3*; TG *E*/quoniam: quod *R*/refertur: reflectitur *R*/*post* O *scr. et del.* a puncto *E* 149 OG: EG *FP1*; TG *L3E*/ZMQ: ZMG *P1S* 150 in . . . visus: visus fuerit in puncto sectionis *R*/refertur: reflectentur *R*; *corr. ex* refen *O*; *alter. in* referuntur *E* 151 I N *transp. R*/*post* ymaginum *add.* erunt *R*/*post* redit *scr. et del.* ad *E* 153 si: sed *OC1*/secabit: secet *R*/ZMQ: ZMG *FP1*/intra: inter *L3*/nec: nichil *L3E*; non *R* 154 aptari: applicari *R*/quoniam: quod *R*/ hanc: N *FP1*/*post* totalem *scr. et del.* circulum *O* 155 erit: licebit *R*/ad quod *inter. a. m. E*; a quo *O*/refertur: referuntur *L3C1E*; reflectantur *R* 156 punctis *om. L3E*/punctis speculis *transp. R*/speculi: speculis *E*/*post* ymago *add.* erit *R* 157 quoniam: quod *R*/*post* duplus *add.* est *OR* (*inter. O*) 158 IAB: CAB *R*/IAO: NLO *FP1*/IAG: YAG *S* 159 angulo: alico *O*/maiori: maiore *R* 160 *post* angulo *inter.* angulo *O*/NAI: NAR *P1*/OAI . . . duo[1] (161) *om. R*/ZAN: IAN *E*/quod: qui *O*/quod . . . tertio[1] (161) *mg. a. m. E*/est *om. FP1* 161 duo[1]: dico *S*/*post* duo[1] *inter.* anguli *a. m. C1*/OAI: IAO *R*/IAN: ZAN *C1*; IAM *P1*; *corr. ex* IAM *F*/*post* tertio *add.* qui est *R*/NAZ . . . IAO (162) *om. P1*/ZAN *corr. ex* NAZ *E* 162 IAO *om. F*/*post* IAO *add.* et duo NAZ IAO maiores tertio NAI *R*/*post* maiores *add.* sunt *R* 163 *post* tertio *add.* et omnes simul quattuor rectis minores *R* 164 est: licet *R*/fiat *corr. ex* fiet *a. m. C1* 165 SA erecta: SAE recta *FP1*

lis angulo IAO, angulus NAS equalis angulo NAZ. Angulus NAI manebit immotus, et fiet linea AS equalis lineis AN, AI, que omnes sunt equales.

[4.61] Et producantur linee TS, QS. Palam quoniam angulus TAS equalis angulo TAO, et duo latera duobus lateribus. Erit basis TS equalis basi TO, et triangulus triangulo, et ita angulus GTA equalis angulo STA. Similiter, angulus QAS equalis angulo QAZ, et latera lateribus. Erit triangulus equalis triangulo, et angulus MQA equalis angulo SQA.

[4.62] Dividatur angulus TAS per equalia per lineam AY; sit Y punctus in quo linea illa secabit lineam TS. Palam, cum angulus IAG sit medietas anguli IAO, erit angulus TAG equalis angulo TAY, et angulus GTA equalis angulo YTA, et unum latus commune, scilicet TA. Erit TG equalis TY, et triangulus triangulo, et erit AY equalis AG, et ita Y in superficie spere. Igitur angulus IAG equalis angulo IAY, et latera lateribus. Erit triangulus triangulo equalis, et erit AGI equalis angulo AYI. Linea IY producta erit equalis IG.

[4.63] Et producatur AY extra speram usque ad punctum P. Restabit angulus IGR equalis angulo IYP. Verum, cum TS sit equalis TO, et TY equalis TG, restat GO equalis YS. Igitur AY, YS equalia AG, GO, et basis AS equalis basi AO. Erit triangulus equalis triangulo; erit angulus AYS equalis angulo AGO. Restat angulus SYP equalis angulo OGR. Igitur duo anguli IGR, OGR equales sunt duobus angulis IYP, SYP.

[4.64] Verum linea AS secat speram. Sit punctus sectionis O. Igitur tria puncta O, Y, D sunt in superficie spere, quare linea OYD est pars circuli spere, et est linea communis superficiei spere et superficiei ITASP, quare punctus I refertur ad punctum S a puncto Y, et locus ymaginis T.

166 IAO . . . angulo[2] *mg. F*/*post* IAO *add.* et *ER*/angulus[2]: angulo *S* 167 fiet: fiat *R*/lineis: linee *R*/*post* AN *add.* vel *R* 169 producantur: producuntur *L3*/*post* TAS *add.* est *R* 170 duobus lateribus *transp. R*/erit: ER *FP1* 171 TO: TOM *FP1*/triangulus: triangulum *R*/GTA *corr. ex* GAT *E* 172 QAZ: NAZ *L3* 173 erit: et *R*/triangulus equalis: triangulum equale *R*/MQA *alter. in* MAQ *a. m. E* 175 AY: AI *P1O*/*post* AY *add.* et *FP1* 176 punctus: punctum *R*/illa *om. FP1* 178 angulo *om. R* 179 TG *corr. ex* CG *O*/TY . . . equalis[2] *mg. O*/triangulus: triangulum *R* 180 Y *inter. E*/spere: speculi *R*/igitur: erit etiam *R* 181 latera lateribus *transp. deinde corr. S*/erit: et *R*/triangulus: triangulum IAG *R*/*post* triangulo *scr. et del.* equlis *P1*/equalis: equale *R*/*post* erit[2] *add.* angulus *R* 182 *post* AYI *add.* et *R*/IY: AY *L3*; TY *E*/erit *om. ER* 185 *post* TG *scr. et del.* reg *L3*/equalia: equalis *FP1*/AG GO: AGO *L3* 186 AO *corr. ex* A *O*/triangulus equalis: triangulum equale *R*/equalis triangulo *transp. FP1*/*post* triangulo *add.* et *C1R* (*inter. C1*)/angulus *om. P1* 187 SYP: SIP *S*; *corr. ex* SIP *O*/OGR: EGR *L3E* 188 anguli *inter. O*/sunt *om. C1* 189 secat: secabit *ER*/sit: si *S*/*ante* punctus *scr. et del.* punctus *F*/punctus: punctum *R*/O *inter. O*; E *ER* 190 puncta: punctu *O*/O: E *R*; *inter. O*/Y: I *L3*/quare: quia *FP1*/OYD: O *FP1*; EYD *R*; *corr. ex* YD *O*/est *inter. O* 191 superficiei[2]: speciei *S* 192 *ante* ITASP *add.* reflexionis *R*/ITASP: REASP *F*; TSP *R*; RASP *O*; *alter. ex* RESP *in* REASP *P1*/punctus *mg. F*; punctum *R*/refertur: reflectitur *R* 193 *post* ymaginis *add.* est *R*

[4.65] Similiter, diviso angulo NAS per equalia per AZZ, probabitur predicto modo quoniam QZ equalis QM, et AZ equalis AM, et ZS equalis MZ, et duo anguli NZZ et SZZ equales duobus angulis NMU, ZMU. Et ita N refertur ad S a puncto Z, et locus ymaginis Q, et ita TQ ymago IN, quod est propositum.

[4.66] Amplius, si a puncto I ducatur perpendicularis super NA, cadat inter N et Q, non extra N, cum angulus INA acutus, quoniam equalis angulo NIA, et si caderet perpendicularis illa extra N, esset acutus maior recto. Faciet ergo perpendicularis illa angulum rectum super NQ, quem angulum respiciet linea IN, quare linea IN maior illa perpendiculari, quare perpendicularis illa minor TQ.

[4.67] Punctus linee NQ in quem cadit perpendicularis reflectitur ad punctum S, et ymago eius cadet in linea NA supra punctum Q, quia quanto remotiora sunt puncta que reflectuntur tanto loca ymaginum magis accedunt ad centrum circuli, ex decima quinti huius.

[4.68] Et quecumque linea ducatur a puncto T ad aliquod punctum NQ supra Q erit maior TQ. Igitur ymago perpendicularis erit maior ipsa perpendiculari. Eodem modo, quecumque linea ducatur a puncto I ad NQ inter hanc perpendicularem et IN, erit ymago ipsius maior ipsa.

[4.69] Verum determinentur hec certius. Punctus N refertur ad Z a puncto M, et locus ymaginis Q. Linea QM secat circulum in puncto quod sit E. Contingens ergo ducta a puncto Z ad circulum cadet super punctum aliquod arcus ME, et contingens illa cadet supra Q, quoniam punctus in quem cadet erit finis contingentie et finis ymagi-

194 per[1] *inter. O*/AZZ: AZ *FP1*; AZI *S*l ATZ *L3E*; AX *R*/probabitur: probatur *O* 195 predicto modo *transp. L3ER*/quoniam: quod *R*/QZ: QX *R*/*post* QZ *add.* est *OL3*/equalis[1] *mg. a. m. C1*/*post* equalis[1] *add.* est *ER*/AZ: AX *R*/ZS: ZZS *FP1*; XS *R* 196 MZ *corr. ex* MZZ *FP1*/*post* anguli *scr. et del.* MZSS *L3*/NZZ: MZZ *L3E*; NXÆ *R*/et SZZ *mg. F*/SZZ: SXÆ *R*/equales: equalis *FSL3C1E*/NMU *corr. ex* MUN *E* 197 *ante* ZMU *add.* quam *FP1E*/ZMU: CZMU *FP1*; sed MU *L3*; *mg. a. m. E*/ita *om. L3*/refertur: reflectetur *R*/S: SZ *FP1*/Z: X *R* 198 IN: YZ *FP1*; YN *SOL3C1E* 200 cadat: cadet *OER* (*alter. in OE*)/*post* INA *add.* sit *OR* (*inter. O*) 201 N: M *FP1* 202 illa angulum *transp. deinde corr. C1* 203 quem: quoniam *FP1OE*/quem . . . IN[1] *om. L3*/angulum *corr. ex* angulus *OC1*/respiciet: respicit *R*/quare linea IN *om. S*/*post* maior *add.* est *R*/*post* illa *add.* est *mg. a. m. E*/illa *mg. a. m. E* 204 *ante* perpendiculari *scr. et del.* ri *E*/perpendiculari quare *mg. F*/perpendicularis: perpendiculari *P1* 205 punctus: punctum *R*/*post* punctus *add.* reflexionis *P1*; *add.* igitur *R*/linee *om. SOL3C1E*/quem: quod *R* 206 S: ? *L3*/et *om. L3ER*/*post* ymago *add.* vero *R* 207 quanto: quantum *FP1* 208 magis *corr. ex* MG *O*/ex . . . huius *om. R*/decima: tertia decima *O*/quinti: quarti *FP1*; quadrati *L3*/huius *om. SO* 209 ducatur: ducetur *R* 210 Q *inter. E*/TQ . . . maior (211) *rep. S* 211 ducatur: ducetur *R* 212 ipsius . . . ipsa (213) *corr. ex* maior ipsa ipsius *C1* 214 hec certius: huius tertius *FP1*/punctus: punctum *R*/*post* N *add.* quia *R*/refertur: reflectitur *R*/*post* ad *scr. et del.* C *F*/Z: S *L3E* 215 a . . . Z (216) *mg. O*/*post* ymaginis *add.* est *R*/QM: que *L3E*; ZMQ *R* 216 sit: est *R*/E: 3 *R*/*ante* ergo *scr. et del.* ergo *F*/ergo *om. P1* 217 ME: MZ *R*/*post* ME *add.* si vero caderet in punctum 3 secaret peripheriam non tangeret cadit igitur in peripheriam M3 *R*/contingens: communis *L3E*; continens *FP1SO* 218 punctus: punctum *R*/quem: quod *R*/cadet: cadit *ER*/finis contingentie et *om. FP1*/ymaginum: ymaginis *C1*

num, et puncta sub puncto illo qui est finis contingentie non poterunt reflecti; superiora poterunt.

[4.70] Igitur perpendicularis ducta a puncto I, si ceciderit supra punctum qui est finis contingentie, refertur punctus in quem cadit, et erit ymago perpendicularis maior perpendiculari. Si vero perpendicularis cadat in punctum contingentie aut infra, non refertur punctus in quem cadit, quare nulla erit ymago perpendicularis. Verumptamen, quoniam finis contingentie est infra N, erunt inter finem contingentie et N infinita puncta quorum quodlibet reflectetur, et ymago cuiuslibet supra NQ. Et cuiuslibet linee ducte a puncto I ad aliquod illorum punctorum erit ymago maior linea cuius fuerit ymago.

[4.71] Igitur accidit in hiis speculis ymaginem aliquando equalem rei vise, aliquando maiorem, quod erat explanandum. Huius autem rei explanationem nec scriptam legimus nec aliquem qui dixisset aut intellexisset audivimus

[4.72] Amplius, in hiis speculis linee recte videntur curve, ut in pluribus curvitate quidem speculum non respiciente sed ei adversa. Similiter, curve apparebunt in hiis speculis curve, et si curvitas speculum respexerit, contrario situ apparebit, et hoc quidem intelligendum non in omnibus sed in pluribus, ad cuius rei explanationem necesse est quedam antecedentia premittere, unum quorum est:

[4.73] **[PROPOSITIO 4]** Si fuerint duo puncta eiusdem longitudinis a centro speculi et inequalis longitudinis a centro visus, ymago puncti remotioris a centro visus erit remotior a centro spere quam propinquioris, et finis contingentie remotioris remotior a centro fine contingentie propinquioris, sive puncta illa sint in eadem superficie cum centro visus, sive in diversis.

219 qui: quod *R*/*post* contingentie *scr. et del.* refertur punctus in quem cadit *S* 221 I *corr. ex* Q *E*; *om. FP1*/*post* I *add.* super NQ *R*/ceciderit *mg. C1* 222 qui: quod *R*/*post* refertur *scr. et del.* igitur *C1*/refertur: reflectetur *R*/punctus in quem: punctum in quod *R* 224 aut infra *inter. O*/aut . . . est (226) *om. S*/refertur: reflectetur *R*/punctus: punctum *R* 225 erit *inter. a. m. C1*/perpendicularis *inter. O* 226 est *om. L3E*; *inter. O*/infra: intra *FP1SOC1*; inter *L3E*/erunt *corr. ex* erit *O*/inter *corr. ex* intra *S* 227 N *om. FP1*; *alter. in* MF *O*/infinita: infra *FP1SOL3* (*inter. O*); ? *E*/quodlibet: quolibet *S*/*post* quodlibet *scr. et del.* puncta quorum *S*/*post* et^{2} *add.* erit *R* 228 supra: super *R*/NQ: PQ *C1*; *corr. ex* N *E*/aliquod: quodlibet *ER* 229 fuerit: fuit *C1* 231 *post* maiorem *add.* esse *R* 232 rei *corr. ex* re *O*/nec: non *FP1*/aut intellexisset (233) *om. FP1* 234 linee recte videntur *om. S*/ut: et *R* 236 similiter curve *transp. O*/curve *inter. O*/speculum: speculis *L3E* 237 apparebit: apparebunt *S* 238 *post* cuius *scr. et del.* e *S* 239 antecedentia *corr. ex* ante *O*; accidentia *S*; acerva *L3E*/unum: unus *FP1*/unum quorum *transp. FP1R* 240 *post* si *scr. et del.* vero *C1* 241 *post* longitudinis *scr. et del.* a centro speculi et inequalis longitudinis *F* 242 spere: speculi *C1* 243 remotioris: remotionis *L3*/*post* remotioris *add.* erit *R*/remotior *om. P1*/centro fine *transp. L3*/*post* centro *add.* speculi quam *R*/fine: finem *S*; finis *R* 244 contingentie: continente *P1O*; contingente *alter. in* continente *F*; continen *S*/eadem *corr. ex* eodem *L3* 245 *post* cum *add.* a *FP1*

[4.74] Probatio: Sint T, D [FIGURE 6.4.4, p. 307] duo puncta equaliter a G centro speculi remota, E centrum visus. Superficies DGT secabit speculum super circulum qui sit AB. Et sit angulus EGD equalis angulo TGZ, angulus EGT equalis angulo TGH, et sumatur in circulo punctum a quo T refertur ad Z, quod sit Q. Dico quoniam T non refertur ad H ab aliquo puncto BQ.

[4.75] Palam enim quoniam non a puncto Q. Si autem sumatur punctum quodcumque in BQ, linea ducta a puncto H ad illud punctum secabit lineam QZ. Igitur ad illud punctum sectionis refertur T a puncto sumpto in BQ, et ad idem punctum sectionis refertur a puncto Q. Igitur T refertur ad idem punctum a duobus punctis illius circuli, quod est impossibile in hiis speculis, ut in libro quinto patuit.

[4.76] Restat ut T reflectatur ad H ab aliquo puncto QA. Sit illud M, et a puncto M ducatur contingens circulum usque ad lineam GT, que sit MN. Erit N finis contingentie T respectu H.

[4.77] Et a puncto Q ducatur contingens que sit QO, que quidem necessario cadet sub MN. Producatur ZQ usque dum cadat super GT in puncto C. Erit C locus ymaginis Z. Erit igitur proportio GT ad TO sicut GC ad CO. Igitur maior erit proportio GT ad TN quam GT ad TO. Ergo multo maior GT ad TN quam GC ad CN. Sit ergo GT ad TN sicut GL ad LN. Erit GL maior GC, et L locus ymaginis H.

[4.78] Sint ergo linee HG, EG, ZG equales, GF equalis GC, GS equalis GO. Cum igitur angulus EGD sit equalis angulo TGZ, et remotio D a puncto E sicut Z a puncto T, erit ymago D respectu G tantum elevata in linea GD quantum ymago T in linea GT. Igitur ymago D in puncto F. Et similiter, finis contingentie D respectu E erit eius-

246 duo puncta *transp. C1* 247 equaliter: equalia *L3*/a G *inter. a. m. E*/*post* visus *add.* et D propinquius visui quam T *R*/*post* superficies *add.* communis sectionis *R* 248 DGT: DTG *FR*; DG *P1*/super circulum *om. P1* 249 angulus . . . TGH *mg. O* 250 refertur: reflectatur *R*/*post* refertur *scr. et del.* ad *F*/Z: S *L3E*/quoniam: quod *R* 251 refertur: reflectitur *R*/aliquo: alio *FP1SO*; a *L3E*; *corr. ex* alio *C1*/BQ: HQ *L3* 252 enim *om. FP1R*; *inter. E*/quoniam: quod *R*/non *om. FP1* 253 *post* ad *scr. et del.* al *S*/illud: illum *FP1O* 254 QZ: quia *FP1*; QS *L3E*/illud *corr. ex* aliud *P1*/refertur: reflectitur *R*/a: ab *R* 255 *ante* puncto[1] *add.* aliquo *R*/BQ: HG *S*/punctum sectionis *transp. R*/refertur: reflectitur *R*/a puncto *rep. F* 256 refertur: reflectitur *R*/punctis *corr. ex* punctum *C1* 257 est *om. SOL3C1ER*/ut *inter. a. m. E*/patuit *om. L3* 258 *post* restat *add.* ergo *R*/ab *om. S*/aliquo: alio *FP1SL3C1E*/puncto *mg. F*/illud: illum *FP1* 259 et *inter. P1*/ad: a *O* 260 N: enim *F*/*post* T *scr. et del.* respectu *F* 261 *post* Q *scr. et del.* erit *S*/QO: QA *SL3E* 262 sub: super *FP1E*/*post* MN *add.* et *R*/ZQ: SQ *L3E*/GT: GR *O* 263 C[1, 2]: P *R*/Z: S *L3E*/proportio . . . erit (264) *om. P1*/TO: PG *R* 264 sicut: sunt *L3*/GC: TO *R*/CO: OP *R*/TN: CQ *FP1*; TA *SL3*/ad[3] *inter. O* 265 GT[1]: TG *L3*/quam: quoniam *FP1*/GC: GP *R*/CN: PN *R*/sit . . . TN (266) *mg. F* 266 maior: minor *E*/GC: GP *R*/*post* et *add.* erit *R*/L *inter. E* 267 sint: sicut *FP1*/linee . . . ZG: HG EG ZG linee *R*/EG *corr. ex* EH *L3*/ZG: SG *L3E*/*post* GF *scr. et del.* EG *P1*/GC: GP *R*/*post* GS *scr. et del.* G *C1* 268 GO *inter. a. m. E*/TGZ: TGS *L3E* 269 Z: S *L3E*/G: E *R*/tantum *corr. ex* deinde *E* 270 elevata: elevatum *FP1*; elevavata *S*/*post* in[1] *scr. et del.* medio *C1*/GD: G *FP1*/T: Z *FP1SOC1*; S *L3E*/*post* T *add.* respectu Z *R*/*post* GT *scr. et del.* eri *P1*; *add.* erit *R* 271 eiusdem *om. C1*; *corr. ex* eius *O*/eiusdem altitudinis (272) *transp. ER*

dem altitudinis cuius est finis contingentie Z, quare finis contingentie D in puncto S.

[4.79] Verum, quoniam angulus EGT equalis angulo TGH, et HG equalis EG, erit L ymago T respectu E, sicut est respectu H. Et N finis contingentie respectu E, quare ymago puncti remotioris ab E remotior a centro ymagine propinquioris, et finis contingentie remotioris remotior a centro fine propinquioris, quod erat propositum.

[4.80] **[PROPOSITIO 5]** Amplius, proposita linea AB [FIGURE 6.4.5, p. 307], et divisa in punctis G, D ut sit proportio AB ad BD sicut AG ad GD, si a punctis sectionis ducantur tres linee concurrentes in punctum unum, scilicet GE, DE, BE, et a puncto A ducatur linea secans illas tres lineas, dico quoniam linea illa divisa erit secundum predictam proportionem.

[4.81] Probatio: Ducatur linea AT secans tria latera GE, DE, BE in tribus punctis Z, H, T. Dico quoniam proportio AT ad TH sicut AZ ad ZH.

[4.82] Ducatur a puncto H equidistans AB, que sit HQ. Palam quoniam proportio AB ad BD constat ex proportionibus AB ad HQ et HQ ad BD. Sed quoniam QH equidistans AB, erit triangulus TQH similis triangulo BTA, et erit proportio AB ad QH sicut AT ad TH. Similiter, triangulus QEH similis triangulo BED. Igitur erit proportio QH ad BD sicut HE ad ED. Ergo proportio AB ad BD constat ex proportionibus AT ad TH et HE ad ED.

[4.83] Producatur QH usque cadat super EG in puncto M. Proportio AG ad GD constat ex proportionibus AG ad HM et HM ad GD. Sed cum angulus EMH sit equalis angulo ZGD, erit angulus HMZ

272 *post* contingentie[1] *add.* puncti T respectu *R*/Z . . . contingentie[2] *mg. F*/Z: S *L3E*/*post* Z *add.* et *P1*/*post* quare *add.* erit *R* 273 D *om. FP1* 274 *post* equalis *add.* est *R* 275 sicut . . . E[1] (276) *scr. et del. E*/est: E *L3*/*post* respectu[2] *add.* puncti *R*/N *inter. O* 276 respectu . . . contingentie (277) *om. S*/*post* E[1] *add.* sicut est respectu puncti H *R*/remotioris *corr. ex* remotionis *C1* 277 *ante* a *add.* est *R*/ymagine: ymaginis *FP1*/remotioris *corr. ex* remortioris *F* 280 ut . . . GD (281) *rep. et del. C1*/sit *inter. a. m. E*/ad *om. FP1* 281 AG: AD *L3E*/sectionis: sectionum *R*/concurrentes: currentes *L3E* 282 punctum unum *transp. E*/*ante* scilicet *scr. et del.* unum *E*/DE BE *transp. E*/*post* linea *scr. et del.* AT secans tria latera GE DE BE *S* 283 quoniam: quod *R* 284 proportionem: probationem *FP1L3* 285 probatio *om. OR*/AT: AC *R*/BE: LE *E*/tribus punctis (286) *transp. C1* 286 Z: S *L3E*/T: C *R*/quoniam: quod *R*/AT: AC *R*/TH: CH *R*/AZ: AS *L3E*; *corr. ex* AIZ *F*/ZH: SH *L3E* 287 ducatur . . . H *om. S*/palam . . . HQ (288) *mg. C1* 288 quoniam: quod *R*/proportio: probatio *E*/AB[2]: AH *FP1*/ad[2] *rep. F* 289 HQ: BQ *O*/equidistans: equidistat *P1*/*post* equidistans *add.* equalis *L3*/triangulus: triangulum *R*/TQH: IQH *L3E*; CQH *R* 290 similis: simile *R*/BTA: HTA *L3C1E*; CAB *R*/erit *om. C1*/AT: AC *R*/TH: CH *R* 291 triangulus: triangulum *R*/QEH: EQH *C1*/QEH . . . triangulo *mg. O*/similis: simile *R*/igitur erit *transp. S* 292 sicut . . . BD[2] *mg. a. m. E*/*post* ED *scr. et del.* producatur QH *C1*/AB *rep. et del. F* 293 AT: AC *R*; *corr. ex* AG *S*/TH: CH *R* 294 *post* usque *add.* dum *OR*/cadet: cadat *OE* 295 *ante* AG[1] *add.* igitur *R*/HM: ? *FP1*/*post* GD[2] *add.* sicut dictum est *S* 296 ZGD: SGD *L3E*/angulus[2]: triangulus *C1*/HMZ: HMS *L3E*

equalis angulo ZGA, et erit triangulus AZG similis triangulo HZM, et erit proportio AZ ad ZH sicut AG ad HM.

[4.84] Sed triangulus HEM similis triangulo GED. Erit proportio HM ad DG sicut HE ad ED. Igitur proportio AG ad GD constat ex proportionibus AZ ad ZH et HE ad ED, et eadem est AG ad GD que est AB ad BD. Igitur illa eadem constat ex proportionibus AT ad TH et HE ad ED, et similiter constat ex proportionibus AZ ad ZH et HE ad ED. Igitur eadem est proportio AT ad TH que est AZ ad ZH, et ita propositum.

[4.85] Eadem erit probatio quecumque linea ducatur a puncto A secans illas lineas tres concurrentes. Et si ducantur alie tres linee a tribus punctis G, D, B ad aliud punctum quam E concurrentes, et a puncto A ducatur linea quecumque secans eas, dividetur secundum predictam proportionem. Et ita quocumque modo concurrant tres linee, et si tres linee EG, ED, EB producantur ultra tria puncta B, D, G ex alia parte, et a puncto A ducantur linee secantes eas ex illa alia parte, numquam ille linee dividantur secundum predictam proportionem.

[4.86] **[PROPOSITIO 6]** Amplius, data linea AB predicto modo divisa, si a puncto A ducatur alia linea, velut AT, que dividatur iuxta eandem proportionem, et a punctis divisionum AB ducantur linee ad puncta divisionum AT, que quidem non sit equidistans, dico quoniam ille tres concurrent in uno eodem puncto.

[4.87] Probatio: Sit proportio AT ad TH sicut AZ ad ZH. BT, DH non sunt equidistantes; igitur concurrent in puncto quod sit E. Linea GZ aut concurret ad idem punctum, aut non. Si ad illud, habemus

297 ZGA: SGD *L3E*; EGO *C1*/triangulus: triangulum *R*/AZG: ASG *L3E*/similis: simile *R*/HZM: AZM *P1*; HSM *L3E*; HMZ *R* 298 AZ: AS *L3E* (*alter. ex* TIS *E*)/ZH: SH *L3E*; HZ *C1*; ZB *O* 299 triangulus: triangulum *R*/HEM: BEM *O*/similis: simile est *R*/*post* erit *add.* igitur *R* 300 DG: GD *C1R*/HE *inter. a. m. E*/ED: DE *C1*/GD: DG *FP1* 1 proportionibus: proportione *R*/AZ: AS *L3E*/ZH: SH *L3E*; ZB *O*/et HE: ZHE *S*/GD: DG *C1* 2 est *om. SOER*/AT . . . proportionibus (3) *om. R* 3 et[1] *om. FP1*/AZ: AZG *FP1*; AG *S*/ZH: SH *L3E* 4 est[1]: erit *E*/AT: AC *R*/TH: CH *R*/AZ: AE *L3E*/ad[3] *inter. O*/ZH: SH *L3E*/*post* ita *add.* est *R* 6 erit probatio *transp. C1*/probatio: proportio *FP1L3*/A: D *S*; *corr. ex* H *E*; *inter. O* 7 illas: alias *FP1*/*post* illas *scr. et del.* duas *C1*/illas lineas *transp. R*/alie tres *transp. L3* 8 B: H *S*/aliud: alium *FP1*; illud *L3* 9 secans . . . ita (10) *mg. a. m. E*/eas *inter. O*/dividetur: divideretur *FP1*; *corr. ex* dividatur *C1* 10 proportionem: probationem *FP1SL3E*/quocumque *corr. ex* quecumque *a. m. E*/concurrant: currant *E* 11 et . . . linee *om P1*/ED: AD *E* 12 illa *mg. a. m. C1* 13 *post* parte *scr. et del.* alius *O*/numquam: in quam *FP1SOC1*/ille *inter. O*/ille linee *transp. O*/dividantur: dividuntur *E*; dividentur *R*/proportionem: probationem *FP1SL3C1E*; *corr. ex* probationem *O* 16 alia *om. C1*/velut *om. P1*/AT *corr. ex* TAT *P1*; AC *R* 17 proportionem: probationem *L3E*/divisionum: divisionis *L3*/AB *corr. ex* BAB *L3* 18 AT: AC *R*/sit equidistans: sint equidistantes *R*/quoniam: quod *R* 19 *post* uno *add.* et *R* 20 probatio *om. R*/AT: AC *R*/TH: CH *R*/AZ: AS *L3E*/ZH: SH *L3E*/BT: BC *R*/DH: BH *L3* 21 *post* in *add.* aliquo *R* 22 GZ: GS *L3E*/*post* non *add.* et *FP1*/illud: illum *FP1*; idem *R*

propositum. Si non, ducatur linea EG. Secabit quidem lineam AT in alio puncto quam Z. Sit illud punctum L. Erit igitur proportio AT ad TH sicut AL ad LH, iuxta priorem probationem. Sed positum est AT ad TH sicut AZ ad ZH, et ita impossibile.

[4.88] Similiter, si ponatur quod linea GZ concurrat cum DH ad punctum E, probabitur hoc modo quod linea BT concurret ad idem. Similiter, si ponatur quod GZ, BT concurrant ad punctum E, convincetur quod DH concurret ad idem.

[4.89] **[PROPOSITIO 7]** Amplius, divisa AB secundum hanc proportionem, si fuerint linee GZ, DH, BT equidistantes, et ducatur AT dividens illas, erit AT divisa secundum hanc proportionem.

[4.90] Probatio: Cum DH sit equidistans GZ, erit proportio AZ ad ZH sicut AG ad GD, et cum BT sit equidistans DH, erit AB ad BD sicut AT ad TH. Sed AB ad BD sicut AG ad GD; erit AT ad TH sicut AZ ad ZH, et ita propositum. Hiis premissis, accedamus ad propositum.

[4.91] **[PROPOSITIO 8]** Et primum, de arcu declaretur quomodo in hiis speculis ymago eius sit curva curvitate quidem speculum non respiciente, sed centrum.

[4.92] Verbi gratia, sit AB [FIGURE 6.4.8, p. 308] arcus oppositus speculo, et sit G centrum illius arcus et similiter centrum speculi, D centrum visus. Et ducantur linee DG, AG, BG. Et sumatur E in arcu AB quocumque modo, et ducatur linea EG. Linea DG non sit in superficie ABG. Linea DG aut erit ortogonalis super superficiem ABG, aut declinata.

[4.93] Sit ortogonalis. Erunt anguli DGA, DGE, DGB equales, et latera lateribus, quare bases equales. Igitur omnia puncta arcus AB eiusdem longitudinis erunt a centro visus, quare ymagines omnium

23 *post* propositum *scr. et del.* m *O*/AT: AC *R* 24 Z: S *L3E*/illud: illum *F*/L *om. FP1*/AT: AC *R*/ad TH (25) *om. S* 25 TH: CH *R*/*post* LH *scr. et del.* et ita impossibile *C1*/iuxta . . . ZH (26) *mg. a. m. E*/priorem *om. E*/AT: AC *R* 26 TH: CH *R*/AZ: AS *L3E*/ZH: SH *L3E* 27 quod linea *transp. deinde corr. C1*/GZ: GS *L3E* 28 BT: BC *R*/concurret: concurrat *R*/*post* idem *add.* punctum *P1* 29 similiter . . . idem (30) *rep. FP1* (*mg. F*)/*post* quod *add.* linea *FP1*/GZ: GS *L3E*/BT: BC *R*/convincetur: coniuncte *FP1O*; probabitur *R* 32 proportionem: probationem *L3E*/GZ: GS *L3E*/DH: AD *L3*/BT: BC *R* 33 AT[1] *inter. a. m. C1*; AC *R*/dividens . . . AT[2] *om. P1*/*post* illas *scr. et del.* AT *C1*/AT[2]: AC *R*/divisa *corr. ex* dividens *C1*/hanc *om. O*/proportionem: probationem *L3* 34 probatio *om. R*/GZ: GS *L3E*/AZ: AS *L3E* 35 ZH: SH *L3E*/BT: HT *FP1*; BC *R* 36 AT[1, 2]: AC *R*/ad[4] *mg. F*/TH[1, 2]: CH *R* 37 AZ: AS *L3E*/ZH: SH *L3E*/*post* ita *add.* patet *R* 38 *post* propositum *add.* primum *FP1* 39 et *om. L3ER*/primum: primo *C1*; *om. O*/declaretur: declaratur *L3*; declaremus *ER* 40 eius *rep. et del. C1*/curva: concurva *FP1* 42 oppositus: opponens *C1* 44 *post* linee *scr. et del.* EG linea *C1*/AG: HD *FP1*/BG: AG *F*; AH *P1*; *corr. ex* B *O* 45 *post* linea[2] *add.* vero *R* 46 *post* linea *add.* igitur *R*/aut erit ortogonalis *corr. ex* ortogonalis aut erit *C1* 48 *post* equales *add.* quia recti *C1* 50 eiusdem: eiudem *P1*/erunt *inter. O*/*post* omnium *add.* punctorum *R*

eiusdem longitudinis a centro.

[4.94] Sint Q, M, L ymagines A, E, B. Erit igitur GQ equalis GM et GL, quare QML erit arcus, et convexitas ipsius respectu centri non respectu speculi, sive reflexionis loci, quod est propositum.

[4.95] Si vero linea DG non fuerit perpendicularis super superficiem AGB, ducta perpendiculari a puncto D super hanc superficiem, cum illa perpendicularis sit minor omnibus lineis ductis a puncto D ad hanc superficiem, erit angulus quem respicit hec perpendicularis supra G minor quolibet angulo supra punctum G intellecto quem respiciat alia linea a puncto D ad superficiem ducta, et linea ducta a puncto D ad superficiem quanto remotior erit a perpendiculari, tanto maior erit, et respiciet maiorem angulum. Si igitur hec perpendicularis non cadat in arcu AEB, sed ex parte una, erunt omnes linee ducte a puncto D ad hunc arcum declinate in partem unam, et remotiores maiores et maiorem angulum respicientes.

[4.96] Sit ergo, et sumantur tria puncta in arcu, scilicet E, C, B. Finis contingentie puncti B sit L; finis contingentie puncti C sit M, quoniam, cum C propinquius D quam B, erit M propinquior G quam L, et ita CM maior BL.

[4.97] Q sit ymago C, T ymago B, et ducatur TQ. Et ducantur linee CB, ML, que quidem producte concurrent, si enim a puncto M duceretur equidistans CB, secaret ex GB lineam equalem CM. Concurrant in puncto O.

[4.98] Et quoniam proportio GC ad CM sicut GQ ad QM, similiter, BG ad BL sicut GT ad TL, linea QT concurret cum lineis CB, ML. Et sit concursus in puncto O.

[4.99] Finis contingentie puncti E sit N [FIGURE 6.4.8a, p. 308]. Quoniam punctus N dimissior est puncto M, erit EN maior CM.

51 eiusdem longitudinis *transp. C1*/*post* longitudinis *add.* sunt *R*/*post* centro *scr. et del.* TM quare ymagines omnium eiusdem longitudinis a centro *O* 52 sint: sicut *SL3*; sintque *R*/*post* ymagines *add.* ipsorum *R*/igitur *om. C1*/GQ *corr. ex* GL *C1*/GM: GLN *FP1* 53 et[1] *om. R*/QML: QM *E* 54 sive: fine *FP1*/reflexionis loci *transp. R*/loci: BOCI *FP1* 55 super . . . perpendiculari (56) *mg. a. m. E* 56 AGB *corr. ex* ABGB *L3*/ducta . . . D *om. P1* 57 perpendicularis: perpendiculari *SL3*/sit: sicut *L3*/D: O *S* 58 quem: quam *FP1*/respicit: continet *R* 59 supra[1, 2]: versus *R* 60 respiciat: respiciet *FP1*; respicit *C1*; continet *R*/*post* ad *add.* hanc *R* 61 *post* D *add.* ducta *SE*/*post* ad *add.* hanc *R*/quanto: quando *S*/a *om. S* 62 *post* erit *scr. et del.* OE *C1*/respiciet: continebit *R*/*post* angulum *add.* versus G *R*/igitur hec *transp. L3* 63 arcu: arcum *R*/una *corr. ex* unam *P1* 64 in: ad *R* 65 respicientes: continentes versus G *R* 66 *post* B *add.* finis contingentie puncti C sit M *mg. O* 67 sit[1] *om. FP1*/finis . . . M *om. O*/C: GET *S*; GTC *L3*; GET *alter. in* GCT *E*/M: LM *S* 68 cum: igitur *R*/propinquior: propinquius *R*/C *inter. O*/propinquius *corr. ex* propinquioris *S*/D *om. FP1* 69 CM: tantum *S*; *corr. ex* C *O* 70 et[2] *om. S*; *inter. a. m. E* 71 CB: CP *FP1*; CH *SL3E*/enim *rep. et del.* F 72 ex GB *om. S*; *inter. a. m. E* 74 CM *corr. ex* GM *C1*; *corr. ex* QN *E* 75 BG: GB *SL3ER*/TL: D *S*/*post* TL *add.* ergo *R*/concurret: concurrent *SL3*; *corr. ex* concurrent *O*/et *om. R* 76 sit *om. O*/*post* O *scr. et del.* et quoniam proportio *C1* 77 puncti: in *S* 78 punctus N dimissior: punctum N dimissius *R*

Ductis ergo lineis EC, NM, concurrent. Sit concursus in puncto P, et ducatur linea QP, et procedat donec cadat super EG in puncto F. Et ducatur linea TQ usque ad EG, et cadat in puncto K.

[4.100] Palam quoniam K erit supra F. Verum, cum proportio GC ad CM sicut GQ ad QM, et a punctis divisionum ducantur tres linee concurrentes in aliam partem producte, secabunt lineam EG secundum predictam proportionem, quare proportio GE ad EN sicut GF ad FN. Sed N finis contingentie, quare F locus ymaginis. Igitur linea FQT erit ymago arcus ECB, et erit linea curva non recta, quoniam TQK est recta, et curvitas linee non ex parte speculi.

[4.101] Similiter, si perpendicularis a puncto D cadat ex alia parte arcus, similis erit probatio. Si vero cadat perpendicularis in medio arcus AB, linee a puncto D ex diversis partibus ad arcum ducte equaliter distantes a perpendiculari erunt equales, et equales angulos respicient supra G. Et ymagines earum equaliter a G distabunt, et finis contingentie similiter, et erit probare predicto modo de utraque parte arcus per se secundum quod dividitur a perpendiculari quod eius ymago sit linea curva modo predicto, quod est propositum.

[4.102] **[PROPOSITIO 9]** Amplius, sumatur circulus cuius non sit centrum centrum speculi; verumptamen, sit in eadem superficie cum centro speculi. Dico quoniam, si in hoc circulo exteriori sumatur arcus ex parte centri speculi, id est propinquior ei, erit eius ymago curva.

[4.103] Dato enim hoc arcu, ducatur linea a centro speculi ad centrum exterioris circuli, et producatur hec linea usque ad arcum datum. Linea ducta a centro speculi ad hunc arcum, que est pars dyametri maioris circuli, erit brevior omnibus lineis ductis ab eodem centro speculi ad illum arcum. Et a centro speculi possunt duci ad

79 EC *inter. E*/NM: MN *R*/*post* NM *scr. et del.* N *C1* 80 QP *corr. ex* Q *O* 81 puncto: punctum *R* 82 quoniam: QM *S*; quod *R* 83 punctis *corr. ex* punctus *S* 84 lineam *mg. a. m. E* 85 GE *corr. ex* EG *L3*/EN: EM *FP1*; EF *SC1E* 86 *post* N *add.* est *OR* (*inter. O*)/*post* locus *add.* est *R* 87 ECB: ACB *SL3*; QCB *E*/non recta quoniam: quoniam non recta *C1*/quoniam . . . recta (88) *om. P1* 88 *post* non *add.* est *ER*/*post* speculi *add.* S *FP1* 89 cadat: cadet *C1* 90 probatio: proportio *SL3E*/in: a *L3E*/medio: medium *R* 91 linee: lineis *S*/D: B *FP1*/ducte: deducte *O* 92 distantes: equidistantes *F*; equidistat *P1*/a *om. S*/equales[1] *corr. ex* superficies *O* 93 respicient: respicientes *FP1*; respiciant *L3*; continebunt *R*/supra: versus *R*/earum *om. R*/equaliter a G distabunt: a G equaliter distabunt *R* 94 finis: fines *OL3C1ER* (*alter. in E*)/similiter *inter. O*/erit: licebit *R* 95 quod[1] *om. L3* 96 eius: est *SL3E*/sit: super *C1* 97 amplius *om. FP1*/*post* cuius *add.* centrum *C1*/non sit centrum (98): centrum non sit *R* 98 centrum[2] *mg. F*; *inter. O*; *om. L3C1* 99 quoniam: quod *R*/exteriori: exteriore *R* 100 id est: vel *E*; *om. R*/ei *om. FP1*/*post* ei *add.* secundum medium eius punctum *R*/eius ymago *transp. R* 102 *post* enim *scr. et del.* dato *F*/ducatur *inter. O* 103 centrum: arcum *SL3E*/hec: B *FP1*; *om. OC1* 104 ducta *corr. ex* data *C1*/a centro: ad centrum *FP1*/que: qui *SL3E* 105 circuli *om. FP1*/lineis *mg. F* 106 illum arcum: illuminatum *L3*

arcum datum due equales a diversis partibus huius brevis, que quidem maiores illa brevi. Et si secundum alteram illarum fiat circulus cuius centrum centrum speculi, transibit per capita harum duarum linearum arcus excedens arcum datum.

[4.104] Et palam quod ymago huius arcus excedentis erit linea curva, secundum predicta. Et ymagines punctorum huic arcui et arcui dato communium eedem, et medius punctus arcus excedentis remotior a centro puncto arcus dati quod ipsum respicit, quare eius ymago propinquior centro quam ymago puncti arcus dati illum respicientis. Et ita cuiuslibet puncti arcus exterioris ymago propinquior centro ymagine puncti arcus dati quod illud respicit. Quare ymago arcus dati curvior quam ymago arcus exterioris, quare ymago arcus dati curva est, quod est propositum.

[4.105] **[PROPOSITIO 10]** Amplius, quod linee recte ymago in hiis speculis sit curva sic probatur.

[4.106] Sit AB [FIGURE 6.4.10, p. 309] linea visa, G centrum speculi. Ducantur linee AG, BG. Aut sunt equales, aut non. Si equales, fiat circulus cuius G centrum ad quantitatem illarum, qui sit AEB. Cadet quidem linea AB intra circulum. Palam ex predictis quoniam ymago arcus AEB erit curva. Sit ergo ymago eius ZTH. Ymago A sit Z; ymago B sit H; ymago E sit T.

[4.107] Et ducatur linea GE secans AB in puncto C. Palam quoniam E est in eadem linea cum C remotior a centro quam C. Erit eius ymago propinquior centro quam ymago C. Sit ergo M. Palam ergo quod linea ZMH est ymago linee AB, et est linea curva, quod est propositum.

[4.108] **[PROPOSITIO 11]** Si vero linee AG, BG fuerint inequales, linea AB protracta aut secabit speculum, aut non. Sit quod non secet,

107 *post* due *add.* linee *R* / equales: quales *C1* / a: ad *S* / huius: huiusmodi *FP1* 108 *post* maiores *add.* erunt *R* / si secundum alteram: in C alter utram *FP1* 109 *post* centrum[1] *add.* sit *R* / centrum[2] *om. FP1S*; *inter. O* / centrum speculi *transp. R* / *post* per *scr. et del.* equalia *C1* / harum *inter. O* 112 ymagines: ymaginis *SL3E* / et arcui (113) *om. OL3* 113 communium: contrarium *FP1O* / eedem: eodem *SOL3E*; *corr. ex* eodem *mg. C1* / et *om. O* / medius punctus: medium punctum *R* / excedentis *corr. ex* excedentes *O* 114 *ante* remotior *add.* est *OR* / remotior: remotius *R* / *post* centro *add.* speculi quam *R* / puncto: punctum *R* 115 *post* propinquior *add.* est *R* / illum: illi *E* 117 *ante* centro *add.* est *R* / illud: ipsum *R* 118 curvior . . . dati (119) *om. S*; *mg. a. m. E* / quare *corr. ex* qurare *F* 120 quod *inter. O* / in *rep. et del. C1* 121 sic probatur *transp. SL3ER* 123 linee: linea *FP1*; *om. L3* / *post* BG *add.* hee *R* 124 ad: secundum *R* / AEB: EAB *FP1* 125 linea *om. O* / intra: inter *SL3* / quoniam: quod *R* 126 sit[1]: si *S*; *corr. ex* si *O* / ZTH: STH *SL3E*; *corr. ex* ZH *C1* 127 Z: S *SL3E* 128 et: ed *FP1O* / ducatur: producatur *SE* / linea *om. R* / C: F *R* / quoniam: quod *R* 129 E *inter. O* / C[1]: F *R* / *post* C[1] *inter.* cum C sit *O* / quam *om. R* / C[2]: G *R* / *post* erit *add.* ergo *R* 130 *post* quam *add.* F *R* / C: O *S*; *om. R* 131 ZMH: SMH *SL3E* 134 *ante* linea *scr. et del.* in *O* / sit: si *S*; *corr. ex* si *O* / sit quod *corr. ex* si *a. m. C1*

et sit AG maior BG [FIGURE 6.4.11, p. 309], et fiat circulus supra G ad quantitatem AG, qui sit AEQ, et producatur AB usque cadat in circulum ex parte B. Cadat in puncto Q.

[4.109] Patet ex superioribus quoniam ymago arcus AE est curva. Punctus ymaginis A sit Z; punctus ymaginis E sit M. Erit ZM ymago arcus AE, et quoniam ymago puncti B remotior a centro quam ymago puncti E, erit ymago linee AB curva, quod per puncta media arcus AE et linee AB poterit probari, quod est propositum.

[4.110] Nota quod in priori figura, si secetur a linea AB ex parte A pars quedam, et ex parte B secetur pars ei equalis, residuum linee habebit ymaginem curvam, et erit eadem probatio que est de linea AB. Et in hac figura secta alia parte linee AB ex parte B, de residuo erit eadem probatio que est de linea AB.

[4.111] **[PROPOSITIO 12]** Si vero linea AB tangit speculum, aut secabit, aut continget. Sit contingens [FIGURE 6.4.12, p. 309]. G sit centrum speculi, et ducantur linee AG, BG. Superficies ABG secat speculum supra circulum communem, qui sit SEZ. Palam quoniam linea AB continget speculum in hoc circulo. Contingat in puncto E. Protrahatur ergo AB usque ad E. D sit centrum visus. Superficies in qua sunt linee DG, AG secat speculum supra circulum communem superficiei et speculo. Et sit arcus illius circuli ZP. Similiter, linea communis superficiei in qua sunt DG, BG et circulus arcus illius circuli sit HP.

[4.112] Palam quoniam B refertur ad D ab aliquo puncto arcus HP. Si a puncto illo ducatur contingens, secabit lineam BG, et punctus sectionis erit finis contingentie. Sit punctus ille M.

[4.113] Palam etiam quod, si a puncto M ducatur contingens super circulum SEH, cadet contingens illa citra E, quoniam AB est

135 BG: GB *R*/supra: super *R* 136 AB: AH *FP1*/usque: quousque *R*/cadat: cada *P1* 137 puncto Q: punctum E *R* 138 patet: patebit *FP1O*/quoniam: quod *R*/arcus AE *transp. deinde corr. C1* 139 punctus[1]: punctis *S*; punctum *R*/punctus[2]: punctum *R*/erit: sit *L3*/ZM: MZ *C1* 140 *post* AE *scr. et del.* et quoniam ymago arcus AE *FC1*/*post* remotior *add.* est *OR* (*inter. O*) 141 *post* quod *add.* etiam *R*/AE *corr. ex* EA *O* 142 *post* AB *add.* faciliter *R* 143 nota . . . AB (147) *om. R*/si *om. E* 144 *post* secetur *scr. et del.* a linea AB ex parte A pars equedam *S*/pars[2] *om. L3* 145 probatio: proportio *SL3* 146 et . . . AB[2] *om. L3*/secta: recta *O* 147 que: quod *L3* 148 si . . . AB *om. P1* 149 sit contingens: tangat primo et *R*/*post* G *scr. et del.* ce *P1* 150 secat: secans *FP1*; secabit *SR*; *corr. ex* secabit *a. m. E* 151 supra: super *FP1R*/communem *corr. ex* commeunem *F*/SEZ *corr. ex* SZ *C1*; EHZ *R*/quoniam: quod *R* 152 continget: contingit *S* 153 *post* ergo *scr. et del.* AD *F* 154 secat: secabit *SR*; *corr. ex* secabit *a. m. E*/supra: super *R* 155 *post* superficiei *add.* reflexionis *R*/speculo: speculi *R*/et[2] *om. SL3C1ER*/ZP: SP *E*; speculi *L3*/ZP . . . circuli (156) *om. S* 156 *post* superficiei *add.* reflexionis et speculi *R*/et: est *O*/circulus *om. R* 158 quoniam: quod *R*/B: HP *SL3E*/refertur: reflectitur *R*/aliquo: alio *SL3E* 159 punctus: punctum *R* 160 erit finis *transp. C1*/*post* erit *add.* IT *FP1*/punctus ille: punctum illud *R*/ille *inter. O* 161 si *om. FP1*/ducatur: ducantur *O* 162 super *om. FP1*/super circulum: arcum circuli *R*/SEH: EH *R*/citra: circa *SL3*/est contingens (163): contingit *R*

contingens in puncto E, et punctus B est altior puncto M. Cadet ergo in puncto F, que contingens producta secabit lineam AE. Secet in puncto T. Ex alia parte secabit lineam AG. Secet in puncto C.

[4.114] Fiat angulus BGS equalis angulo BGD, et producatur GS usque ad punctum L ad equalitatem linee DG. Erit ergo arcus HS equalis arcui HP, et sicut refertur B ad D a puncto arcus HP, refertur ad L a puncto arcus HS. Et erit reflexio a puncto F sicut in arcu HP est reflexio a puncto a quo ducatur contingens ad punctum M, et illa duo puncta a puncto M eiusdem longitudinis. Ducantur ergo linee BF, LF.

[4.115] A refertur ad D ab aliquo puncto arcus ZP. Verum in triangulo HZP duo arcus HZ, HP sunt maiores tertio, scilicet ZP. Sed HP est equalis HS. Igitur ZP est minor ZS. Rescindatur ZS ad equalitatem in puncto Y, et ducatur linea GY, que producta ad equalitatem GD secabit necessario lineam FL. Secet in puncto X, et sit GXK equalis GD.

[4.116] Palam quoniam sicut A refertur ad D ab aliquo puncto arcus ZP, similiter, refertur ad K ab aliquo puncto arcus ZY. Dico quoniam non refertur ad ipsum nisi a puncto quod est citra F ex parte Z.

[4.117] Si enim dicatur quod potest a puncto F vel aliquo puncto arcus FY, linea ducta a puncto A ad punctum reflexionis secabit lineam BF. Ad illud punctum sectionis reflectitur punctus K, et ad idem punctum refertur punctus L, et ita duo puncta in hiis speculis reflectuntur ad idem punctum ex eadem parte, quod est impossibile. Restat ut punctus A refertur ad K ab aliquo puncto arcus ZF.

[4.118] Si ab illo puncto ducatur contingens, secabit lineam AZ, et cadet inter C et Z, quoniam punctus F dimissior quolibet puncto

163 punctus: punctum *R*/altior: altius *R*/cadet: cadit *O*; cadit *alter. in* cadat *C1*; cadat *R* 164 puncto: punctum *R*/que: Q *L3*/contingens: continens *FP1O*/producta: predicta *FP1* 166 angulus: circulus *P1O* 167 usque *om. C1*/HS: hiis *P1* 168 refertur[1, 2]: reflectitur *R*/ad: at *FP1*/a: ab aliquo *R*/*post* HP *add.* sic *R* 169 ad *om. L3*/a: ab aliquo *R*/HS: BS *E* 170 ducatur: ducitur *SOC1R*/ad punctum: a puncto *S* 171 *post* puncta *add.* sunt *R*/a . . . longitudinis: eiusdem longitudinis a puncto M *R*/longitudinis: lineis *S* 173 *ante* A *add.* item *R*/refertur: reflectatur *R*/aliquo: alio *SL3E*; angulo *O* 174 HP: SP *S*/sunt maiores *transp. R*/scilicet *om. SL3ER* 175 rescindatur: recindatur *FP1*/rescindatur ZS *mg. C1* 177 secabit necessario *transp. R*/FL: BL *FP1O* /GXK: GTK *S*; GX LI *P1* 179 quoniam: quod *R*/quoniam sicut *transp. deinde corr. O*/refertur: reflectitur *R*/puncto . . . axis (144) *om. S* 180 *post* ZP *add.* et *FP1*; *scr. et del.* et *O*/*post* similiter *inter.* S *E*/refertur *inter. E*; reflectitur *R*/K: KAB *P1*/K ab: KAB *F*/quoniam: quod *R* 181 non refertur *transp. FP1*/refertur: reflectetur *R*/citra: circa *L3*; *corr. ex* circa *a. m. E*/F *om. FP1* 182 dicatur quod *rep. P1*/quod *inter. O*/potest: possit *R*/aliquo: alio *L3C1ER* 183 FY: SY *F* 184 *post* BF *add.* et *ER*/ad illud: aliud idem *E*/illud: idem *R*/reflectitur: reflectetur *L3C1ER*/punctus: punctum *R*/punctus . . . et (185) *mg. a. m. E* 185 refertur: reflectetur *R*/punctus *om. FP1*; punctum *R*/L: B *ER*/reflectuntur: reflectetur *L3*; reflectentur *ER* 186 restat *corr. ex* restabit *a. m. E* 187 punctus: punctum *R*/refertur: reflectatur *R*/aliquo: alio *O*/ZF: ZB *O* 188 illo *corr. ex* alio *a. m. E* 189 C et Z: Z et C *R*/punctus F dimissior: punctum F dimissius est *R*

arcus ZF, et ita contingens a puncto F altior aliis a punctis arcus ZF ductis. Cadat ergo contingens illa in puncto N, et ducatur linea NM, que quidem linea, cum transeat per acumen trianguli BMT et producta dividat angulum, necessario secabit BT. Secet in puncto Q, et ducatur linea GQ.

[4.119] Sit autem I ymago puncti A; O sit ymago puncti B; U sit ymago puncti Q. Palam, cum B sit propinquior puncto G quam A, erit O remotior a puncto G quam I. Ducatur ergo linea IO. Palam etiam quod proportio AG ad AN sicut GI ad IN, et proportio BG ad BM sicut GO ad OM. Cum ergo linee AG, BG dividantur secundum hanc proportionem utraque in tribus punctis, et a punctis divisionum ducantur linee quarum due, scilicet AB, MN, concurrant ad idem punctum, scilicet ad idem punctum Q, tertia necessario concurret ad idem illud punctum.

[4.120] Igitur IO producta cadet supra Q, quare IOQ recta linea. Igitur IOU non erit recta. Sed IOU est ymago linee AQ, quare ymago linee AQ erit curva. Posito autem puncto B loco puncti Q, et aliquo puncto linee AB posito loco puncti B, erit eodem modo penitus probare quoniam ymago linee AB est curva, et hoc est propositum.

[4.121] **[PROPOSITIO 13]** Si vero AB [FIGURE 6.4.13, p. 310] secat circulum, secet in puncto E, M finis contingentie linee BG. B refertur ad D ab aliquo puncto arcus HP. Arcus ab illo puncto reflexionis usque ad H aut est equalis arcui HE, aut maior, aut minor.

[4.122] Si equalis (sed palam quoniam arcus ille est equalis arcui HQ), sit Q punctus circuli in quem cadat contingens ducta a puncto M ex parte E. Igitur AE transit per punctum Q, et ita MQ secat AE per punctum E.

[4.123] Si vero arcus ille minor est arcu HE, secabit MQ lineam AE ultra punctum Q, ut efficiatur triangulus EQT.

190 contingens: continens *O*/F: R *P1*/aliis: illis *C1*/a[2] *inter. E*/*post* punctis *scr. et del.* ZF *E* 191 contingens: continens *FP1O*/puncto: punctum *R*/NM: MN *R* 193 *post* dividat *scr. et del.* punctum *C1* 195 *post* puncti[1] *scr. et del.* O *L3*/U: R *R* 196 propinquior: propinquius *R* 197 I: C *R* 200 hanc *mg. F*/proportionem: probationem *L3E*/tribus: duobus *R*/et *om. F*/a punctis *mg. F* 202 ad idem punctum *om. R*/idem *om. O*/necessario *om. C1* 203 idem *om. L3C1E*/illud: illum *E*; *om. R*/*post* punctum *add.* necessario *C1* 204 supra: super *R*/*post* IOQ *add.* est *R* 205 IOU[1]: IOY *P1*; IOR *R*/non: N *L3*/IOU[2]: IOR *R*/quare . . . AQ (206) *inter. a. m. E*/quare: quia *E* 206 *post* autem *add.* a *L3*/*post* puncti *add.* P *FP1*/Q *inter. O*/aliquo puncto (207) *transp. P1* 207 puncto *inter. F*/modo penitus *transp. OL3ER* 208 quoniam: quod *R* 209 vero *inter. O*/secat: secet *ER* 210 *post* linee *add.* contingentis circulum EHZ a puncto F producte ad lineam *R*/*post* B *add.* igitur *R*/refertur: reflectitur *R* 211 aliquo: alio *OL3C1E*/illo *corr. ex* alio *a. m. E*/puncto[2] *mg. a. m. C1* 213 sed *om. R*/quoniam: quod *R*/est equalis *transp. P1* 214 HQ: HF *R*/Q *om. FP1*/punctus: punctum *R*/quem: quod *FR*; quo *P1*/cadat: cadit *R* 217 *post* arcus *scr. et del.* linee *P1*/*post* secabit *add.* quidem *R*/lineam: linea *OL3E*/AE: HE *P1* 218 *post* Q *add.* secet in T *R*/ut . . . Q (220) *mg. C1*/triangulus: triangulum *R*

[4.124] Si vero arcus ille fuerit maior arcu HE, secabit quidem linea MQ lineam AE citra punctum Q.

[4.125] Sive hoc sive illud fuerit, iteretur predicta probatio, et eodem penitus modo probetur quoniam ymago linee AB est curva, quod est propositum.

[4.126] **[PROPOSITIO 14]** Amplius, si in superficie in qua sunt linea visa et centrum spere fuerit centrum visus—superiora enim dicta sunt visu non existente in illa superficie—linea ergo visa recta aut concurret cum circulo communi illi superficiei et speculo, aut non concurret.

[4.127] Si concurret, aut erit perpendicularis super speculum aut declinata. Si perpendicularis, angulus illarum linearum cadet supra centrum speculi, que quidem linea videbitur recta, ymago enim cuiuslibet puncti illius linee apparebit in ipsa linea, et ita ymago illius linee recta.

[4.128] Si vero linea proposita declinata fuerit, aut erit declinatio ex parte visus, aut ex alia parte. Si ex alia parte, sumatur punctus circuli a quo reflectatur aliquid ad visum, et sumatur linea reflexionis. Aliqua linearum declinatarum cadet forsitan super hanc lineam reflexionis, quod si fuerit, non videbitur quidem hec linea declinationis.

[4.129] Protracta a centro visus ad centrum speculi linea, si sumatur in arcu circuli citra hanc lineam punctus a quo refertur ad visum aliquis punctus linee declinationis. Sed ille punctus refertur a puncto prius assignato, qui est terminus linee reflexionis, cum linea declinationis sit supra lineam reflexionis, et ita ille punctus linee declinationis refertur ad visum a duobus punctis arcus, quod est impossibile.

[4.130] Licet autem reflectatur punctus ille a puncto primum sumpto, non tamen videtur, cum sit in linea reflexionis, quoniam oc-

219 fuerit maior *transp. C1*/quidem *om. C1* 220 lineam *om. FP1*; *rep. C1*; linee AM *L3*/citra: circa *L3* 221 predicta *om. ER* 222 probetur quoniam: probabitur quod *R* 224 *post* si *add.* etiam P1; *scr. et del.* etiam *F* 225 et *inter. C1*; *corr. ex* ad *O*/centrum² *om. ER* 226 dicta *corr. ex* ducta *O*/visu non: visuum *FP1*/visu non existente: non existente visu *L3ER*/ergo *om. R* 227 aut: autem *FP1*/communi: cum *P1* 229 concurret: concurreret *L3*; concurrat *R*/aut¹ . . . perpendicularis (230) *mg. C1*; *om. FP1OL3ER* 230 angulus: alia *O*/illarum: earum *L3*/supra: super *R* 232 apparebit: apparet *L3ER* 233 *post* linee *add.* est *R* 234 proposita: posita *FP1* 235 si . . . parte³ *om. P1*/alia parte *transp. O*/punctus: punctum *R* 236 ad *om. R* 237 aliqua *rep. ER*/*post* linearum *scr. et del.* declinata *F*/cadet *corr. ex* cadat *E*/*post* hanc *scr. et del.* superficiem *C1* 238 videbitur: videtur *C1*/declinationis: declinata *R* 239 *post* declinationis *add.* nisi secundum unum punctum *R* 240 *post* protracta *add.* igitur *R*/si *om. R* 241 citra *corr. ex* circa *O*/punctus: punctum *R*/refertur: reflectatur *R* 242 aliquis punctus: aliquod punctum *R*/declinationis: declinate *R*/ille punctus: illud punctum *R*/refertur: reflectitur *R*/a *corr. ex* ad *C1* 243 qui: quod *R*/terminus *corr. ex* tres *O*/declinationis: declinata *R* 244 supra: super *L3*/ille punctus: illud punctum *R*/linee . . . visum (245) *rep. P1*/declinationis: declinata *R* 245 refertur: reflectitur *R* 246 punctus ille: punctum illud *R* 247 tamen: cum *FP1*/in *inter. P1*/quoniam: que *R*

cultatur per precedentia puncta, et ita linea adiacens linee reflexionis non videtur.

[4.131] Si vero sumatur linea declinationis cuius declinatio non ex parte visus iacens quidem sub linea reflexionis et secans ipsam in puncto circuli, dico quod nullus punctus illius linee videbitur.

[4.132] Sumpto enim puncto, si dicatur quod punctus ille potest reflecti ab aliquo puncto arcus interiacentis lineam reflexionis et lineam a centro visus ad centrum speculi ductam, et ducatur linea ab illo puncto ad punctum arcus sumptum, hec secabit lineam reflexionis, et punctus sectionis reflectitur ad visum a duobus punctis arcus, quod est impossibile.

[4.133] Si vero dicatur quod punctus sumptus in linea refertur a puncto arcus circuli qui est sub ipsa linea, erit impossibile, quia ille totus arcus occultatur a linea.

[4.134] Si vero linea sumpta non attingit circulum, poterit quidem videri, sed modicum est. Si vero sumatur linea declinationis predicte inter lineam reflexionis et lineam per punctum reflexionis primo sumptum transeuntem ad centrum, poterit quidem videri hec linea, et minuetur curvitas ymaginis huius linee secundum quod magis accesserit ad lineam transeuntem ad centrum per punctum reflexionis.

[4.135] Si vero sumantur linee inter lineam ad centrum transeuntem per punctum reflexionis, videbuntur quidem sive declinatio earum sit ex parte visus, sive non. Et modus visus earum simili modo visus linearum inter lineam reflexionis et lineam ad centrum transeuntem. Et hec quidem intelligenda sunt de lineis concurrentibus in arcu circuli qui apparet visui, id est in arcu qui interiacet duas contingentes ductas a centro visus ad circulum.

[4.136] Linearum autem concurrentium cum circulo in parte circuli occulta visui aliqua erit equidistans linee reflexionis. Illa quidem non videbitur. Similiter, conterminabilis equidistanti que est sub

248 puncta *scr. et del. E* 249 *post* videtur *scr. et del.* eum *C1* 250 declinationis: declinata *R*/non: sit *R*/*post* non *inter.* est *O* 251 iacens: adiacens *L3*; *rep. et del. F*/quidem *om. ER* 252 nullus punctus: nullum punctum *R*/illius *om. P1* 253 punctus ille: punctum illud *R*/potest: possit *R* 254 aliquo: alio *FP1OL3E*; *corr. ex* alio *C1* 255 speculi ductam *transp. O*/*post* ab *scr. et del.* alio *O* 256 illo puncto *transp. O* 257 punctus: punctum *R*/sectionis *om. L3*/reflectitur: reflectetur *R*/*post* arcus *add.* speculi *R* 259 punctus sumptus: punctum sumptum *R*/refertur: reflectatur *R* 260 *post* puncto *scr. et del.* a circulis *F*/erit impossibile *transp. FP1*/quia: quod *L3*; *corr. ex* quod *a. m. E* 261 totus *alter. in* punctus *O* 262 attingit: attingunt *P1*; attingat *R*/poterit . . . modicum (263) *mg. O* 263 *post* sed *scr. et del.* pdocum *O*/*post* modicum *add.* linea E sub ipsa linea erit impossibile *O*/est *om. FP1O*/declinationis: declinata *R*/predicte: predicta *R* 265 videri: videre *FP1* 266 minuetur: imminuetur *R* 267 reflexionis *corr. ex* ionis *a. m. E* 268 si . . . reflexionis (269) *mg. a. m. E* 269 per: in *FP1*/sive: sine *L3* 270 *ante* earum *scr. et del.* sit *C1*/sit *inter. O*/modus *corr. ex* modo *C1*/simili: similis *L3C1E* 271 inter *corr. ex* in *O*/inter . . . reflexionis *corr. ex* reflexionis inter lineam *O* 273 qui[1]: quia *L3*/id est *mg. C1* 274 contingentes: continens *FP1L3E*; continentes *O* 276 aliqua *om. L3*/*post* reflexionis *add.* et *R*/*post* quidem *inter.* equidistans *O* 277 conterminabilis: conterminalis *OR*

equidistanti occultabitur, sed conterminabilis equidistanti supra ipsam existens poterit videri.

[4.137] Si vero sumatur linea inter equidistantes non conterminabilis aliqui earum, si fuerit eius declinatio ex parte visus, videbitur. Si ex alia parte, aliquando videbitur, aliquando non, quoniam, si a termino eius producatur equidistans linee reflexionis, si fuerit linea illa sub equidistantem, non videbitur; si supra eam videri poterit.

[4.138] Si vero linee non concurrant cum circulo, aut secabunt lineam ductam a centro visus ad centrum speculi, aut equidistabunt ei. Si secet aliquam earum, linea illa aut secabit eam ex parte visus, id est inter visum et speculum, aut ultra speculum. Si ultra, occultabitur linea illa, sed forsan apparebunt eius capita. Si vero secet lineam visualem ex parte visus, apparebit quidem similiter. Si fuerit equidistans linee visuali, poterit videri. Omnium autem harum linearum ymagines curve.

[4.139] Visu autem existente in eadem superficie cum centro speculi et lineis visis, diminuta est apparentia, et que sic manifestius apparet est illa que declinata est maxima declinatione et illa visum respiciente. Pari modo, arcuum in hiis speculis apparentium et in eadem superficie cum centro speculi et visu existentium, ymagines quidem curve curvitate speculum respiciente.

[4.140] Hec autem intelligenda sunt duplici visu existente in eadem superficie cum centro speculi et re visa. Si enim alter visus modicum declinetur quoad ipsum, alio modo res visa comprehendetur. Et visu existente extra superficiem rei vise et centri speculi, certior erit ipsius rei comprehensio quam existente in ea.

[4.141] **[PROPOSITIO 15]** Quod autem ymago rei vise sit curva, visu existente in superficie centri speculi et rei vise, probabitur.

[4.142] Sit D [FIGURE 6.4.15, p. 311] centrum visus, G centrum speculi. HE sit linea visa, que quidem HE non concurrat cum circulo,

278 equidistanti[1]: equidistante *R*/conterminabilis: conterminalis *R*/*post* conterminabilis *add.* quidem *FP1* 279 existens *corr. ex* exterus *O*/existens poterit *transp. C1*/videri *om. C1* 280 sumatur linea *transp. C1*/linea *inter. O*/conterminabilis: terminabilis *E*; conterminalis *R* 281 aliqui: alium *L3*; alicui *E* 282 alia parte *transp. L3*/quoniam si *inter. O* 283 producatur: ducatur *R* 284 illa *om. R*/equidistantem: equidistanti *E*/videbitur: videtur *L3E*/si *inter. a. m. C1*/eam: eum *E* 287 aliquam: alia *OL3E*; *corr. ex* alia *C1*; aliqua *R*/earum: illarum *P1*/linea illa *transp. P1C1*; lineam illam *O*/eam: illam *L3ER* 288 occultabitur: concultabitur *FP1O* 289 eius capita *transp. C1*/*post* secet *scr. et del.* i *C1* 291 visuali: visualis *O*/autem *om. L3* 292 ymagines *corr. ex* ymaginis *O* 293 in *inter. C1* 294 et[2] *om. FP1*/sic: sit *ER*/*ante* manifestius *inter.* que *ER* 295 declinata: declarata *FP1OL3E*/declinatione: declaratione *FP1L3E* 296 hiis *om. FP1* 297 et *inter. O*/*post* ymagines *scr. et del.* circa *F* 298 quidem *om. FP1*/*post* curve *add.* sunt *R* 299 autem: aut *FP1E* 300 et *inter. O* 1 comprehendetur *corr. ex* comprehensa *O* 2 centri: centrum *L3ER* 4 autem *om. O* 7 HE *corr. ex* E *E*/concurrat: currat *E*/cum *om. P1*/*post* circulo *add.* speculi *R*

sed sit equidistans linee DG, vel secet eam ex parte D. Sumatur superficies in qua sunt linea DG et linea HE; circulus communis huic superficiei et speculo sit AB.

[4.143] Producatur linea HG. Z sit ymago H, punctus circuli a quo refertur H ad D sit B, et a puncto B ducatur contingens, que secet lineam HG super punctum T. Erit T finis contingentie.

[4.144] Ducatur linea GB, que producta necessario concurret cum HE, si enim HE fuerit equidistans DG, concurret quidem. Si vero DG concurrat cum HE, multo fortius GB concurret cum eadem. Concursus ille aut erit in linea HE, aut ultra hanc lineam.

[4.145] Sit ultra. Concurrat in puncto M; ymago puncti M sit Q; finis contingentie sit S. Et ducatur linea ZQ, similiter linea TS, et producatur a puncto A contingens AU. Palam quoniam AB est minor quarta, quare D videat ex circulo minus medietate, quare angulus AGB est acutus, et angulus UAG est rectus. Igitur AU concurret cum GB. Concurrat in puncto U. Dico quoniam punctus U cadat supra punctum S.

[4.146] Cum enim punctus M reflectatur ab aliquo puncto arcus AB, et A sit dimissior illo puncto, erit finis contingentie A altior fine contingentie illius puncti. Et ita S dimissior puncto U. Procedat ergo TS donec concurrat cum linea AU, et sit concursus in puncto K.

[4.147] Et ducatur linea GK, que producta concurrat cum HM in puncto C. Punctus C refertur ad D ab aliquo puncto arcus AB. Sit ille punctus F, a quo ducatur linea contingens usque ad GC, que quidem dimissior linea AK, et erit punctus O dimissior puncto K.

[4.148] Sit O finis contingentie. Ducatur linea DF usque cadat super GC. Sit casus in puncto R. Et producatur ZQ usque ad lineam GC, et cadat in puncto L. Dico quoniam L est supra R.

8 sit *inter. O*/sumatur *om. R* 9 *ante* in *add.* incidentie sit *R*/sunt: sint *R*/linea[1]: linee *R*/et linea *om. R*/*post* huic *scr. et del.* S equalis *O* 10 AB: AD *L3* 11 *post* HG *add.* et punctum in ipsa *R*/Z *inter. P1*/H *om. P1*/punctus: punctum *R* 12 refertur: reflectitur *R*/a *inter. a. m. E*/*post* ducatur *add.* linea *R*/contingens: communis *FP1L3E*; *corr. ex* communis *O*/que: qui *L3* 14 GB: BG *R*; *om. P1*/que . . . DG (15) *mg. O*/necessario *om. P1O*/*post* cum *add* linea *O* 15 quidem: equidem *L3* 16 concurrat: concurret *FP1OL3E*; *corr. ex* concurret *a. m. C1*/cum[1] *inter. O*/concurret *om. OL3E*/concurret cum eadem: cum eadem concurret *R* 18 sit[1]: si *C1*/*post* ultra *inter.* et *O* 19 *post* TS *add.* et DG secet circulum in A *R*/producatur: ducatur *R* 20 contingens: continens *O*/AU: aut *FP1*/quoniam: quod *R* 21 *post* quarta *add.* circuli *R*/quare[1]: quarum *L3*; cum *R*; *alter. in* cum *O*/*post* quare[1] *add.* D cum *deinde del.* D *C1*/medietate: mediante *L3E*/angulus *inter. O* 22 AGB: ABG *C1E*/UAG *corr. ex* AND *O* 23 GB: BG *C1R*/concurrat *corr. ex* concurret *a. m. E*/quoniam: quod *R*/punctus: punctum *R*/cadat: cadet *C1R*; cadit *E* 25 punctus *inter. a. m. E*; *om. R*/ab: a *R*/aliquo: alio *OL3*; *alter. ex* illo *in* alio *a. m. E*/aliquo puncto *transp. R*/arcus . . . puncto (26) *mg. a. m. E* 26 dimissior: demissius *R*/A[2]. . . contingentie (27) *om. P1* 27 dimissior: demissius *R* 28 TS: OS *F*; DS *P1*/cum *om. FP1L3*; *inter. E*/K *inter. E* 29 GK: GTK *O* 30 C[1]: O *L3E*/punctus: punctum *R*/punctus C *inter. O*/C[2]: O *L3*/refertur: reflectitur *R*/aliquo: alio *OL3E*/ille punctus (31): illud punctum *R* 31 ad *inter. a. m. C1*/*post* ad *inter.* lineam *a. m. C1*/GC *om. L3*/*post* quidem *add.* erit *R* 32 punctus O dimissior: punctum O demissius *R* 33 usque: quousque *R* 34 *post* GC *scr. et del.* sit punctus *C1*/sit casus: cadat *R*/casus: cadens *L3C1E*/puncto: punctum *R*/ZQ *corr. ex* Q *O* 35 puncto: punctum *R*/dico quoniam *transp. deinde corr. O*/quoniam: quod *R*/L: LF *L3E*; *corr. ex* LF *C1*

[4.149] Linee enim HC, TK, ZL aut sunt equidistantes, aut concurrent. Sint equidistantes. Cum ergo hee equidistantes, secent lineam CG super tria puncta C, K, L, et secent utramque linearum MG, HG. Et proportio HG ad HT sicut GZ ad ZT; similiter, MG ad MS sicut GQ ad QS. Erit proportio eadem GC ad CK sicut LG ad LK.

[4.150] Sed palam quoniam R est ymago C, linea enim DF linea reflexionis concurrens cum CG in puncto R, et O finis contingentie, quare proportio GC ad CO sicut GR ad RO. Sed maior GC ad CK quam GC ad CO, et ita maior GL ad LK quam GR ad RO. Ergo maior OR ad RG quam KL ad LG, et ita maior OG ad RG quam KG ad LG. Sed KG maior OG, quare LG maior RG. Igitur R dimissior puncto L. Sed ZQL est linea recta. Igitur ZQR est linea curva, et ita ymago linee HC est curva. Posito ergo aliquo puncto linee HC loco puncti M et puncto E loco puncti C, erit probare quod ymago HE est curva.

[4.151] Si vero linee HC, TS, ZQ concurrant, aut erit concursus ex parte D, aut ex parte HG. Sit ex parte D [FIGURE 6.4.15a, p. 311], et sit concursus in puncto C. Erit ZQC linea recta, quare ZQR erit curva, et ita ymago linee HE curva, quod est propositum.

[4.152] Si vero proponatur arcus extra speculum, erit de eo probare quod ymago sit curva sicut probatum est visu non existente in eadem superficie cum arcu et centro speculi, et hoc est propositum.

[4.153] Igitur in hiis speculis linee recte apparent curve, et curve similiter apparent curve. Si autem proponatur visui in hiis speculis corpus curvum sed longum, modicum habens latitudinis, apparebit quidem illius corporis curvitas manifeste, cum ipsa discerni possit per ea que supra corpus aut intra. Non enim plane discernitur curvitas nisi magna, ubi occulte fuerint extremitates longitudinis et latitudinis, unde proposito visui corpore convexitatis modice et quantitatis

36 concurrent: concurrunt *R* 37 sint: sunt *FP1*/equidistantes[2] *inter. O*/*post* equidistantes[2] *add.* hec *FP1*/secent: secant *FP1* 38 CG: GC *R*/super: sunt *FP1O* 39 ZT *corr. ex* ZF *a. m. E* 41 quoniam: quod *R*/C: O *FP1* 42 concurrens: concurrit *R*/CG: OG *E*/O *om. FP1* 43 quare: qua *L3*/*post* GC[1] *scr. et del.* sicut *O*/CO *corr. ex* CK *O*/*post* maior *add.* est proportio *R* 44 quam[1]: quoniam *FP1O*/Gl *corr. ex* GCL *E*/*post* ad[2] *scr. et del.* C *C1*/*post* maior[2] *add.* est proportio *R* 45 KL: LK *ER*/LG: BG *FP1*/et . . . LG[2] *om. L3*; *scr. et del. O*/*post* maior *add.* est proportio *R* 46 KG: CG *C1*/*post* maior[1] *add.* est *R*/OG: EG *FP1*; CG *OL3E*; KG *C1*/*ante* quare *scr. et del.* igitur *O*/R *inter. O*/dimissior: demissius est *R*/L: H *FP1* 48 est . . . HC[2] *mg. a. m. E*/aliquo: alio *FP1O*/linee *om. FP1L3*/HC[2]: HE *R* 49 HE: E *O* 50 si . . . propositum (53) *mg. a. m. O* 51 HG: HDG *L3E*; *corr. ex* HDG *C1*/*post* HG *scr. et del.* sicut *F*/sit: si *FP1*/D[2] *om. P1* 52 ZQC: ZQT *R* 53 linee *om. P1*/*post* curva *add.* est *L3* 54 proponatur *corr. ex* proponantur *E*/de eo probare: probare de eo *R* 55 ymago sit *transp. L3*/sit: est *L3* 57 et . . . curve (58) *om. L3*; *mg. a. m. O*/*post* et *add.* similiter *E*/curve similiter (58) *transp. R* 58 similiter *rep. F*/*post* apparent *add.* similiter *R*/proponatur: proportionatur *L3E* 59 curvum: ? *E*/modicum: modicam *R*/latitudinis: longitudinis *L3E* 60 illius corporis *transp. R* 61 corpus *inter. O*/*post* corpus *add.* sunt *R*/intra: infra *L3C1E*/plane *alter. in* plene *O* 62 *post* occulte *scr. et del.* f *F*/et latitudinis *om. P1* 63 corpore *inter. O*/convexitatis *corr. ex* convectans *a. m. E*/*post* quantitatis *scr. et del.* m *F*

magne, non planum discernitur eius convexitatis, licet ymago ipsius sit convexa, cum non appareant termini corporis in longitudine vel latitudine.

[4.154] Amplius, errores in speculis planis accidentes omnes accidunt et in hiis, et preter illos accidit ymagines linearum rectarum esse curvas, quod a speculis planis est remotum.

[PARS QUINTA
In speculis columpnaribus exterioribus]

[5.1] Amplius, in speculis columpnaribus exterioribus errores accidunt idem qui in speculis spericis exterioribus, linee enim recte videntur curve et diminuta apparet rei vise quantitas, ut in hiis, sed longe fortius quam in eis, quoniam in spericis res magna apparebit quidem minor, sed non multum parva, sed in hiis res etiam maxima videbitur minima. Similiter, linea recta apparebit curva in spericis speculis, sed si modice curvitatis in columpnaribus maxime, unde multiplicantur errores columpnaris speculi super errores sperici.

[5.2] Verum in columpnaribus aliquando fit reflexio a linea recta, scilicet a longitudine speculi, aliquando a circulo, aliquando a sectione. Quando linea visa fuerit equidistans longitudini speculi, fiet reflexio a linea longitudinis, et linea visa apparebit recta modice curvitatis. Et hec quidem probabuntur, ad quorum probationem necesse est quiddam premitti, quod hoc est:

[5.3] **[PROPOSITIO 16]** Sumpta columpnari sectione, et sumpto in ea puncto qui non sit punctus reflexionis, si ab illo puncto ducatur linea ad perpendicularem que est a puncto reflexionis ad axem—et linea illa faciat angulum acutum cum perpendiculari—si ducatur a puncto sumpto linea que sit ortogonalis super contingentem illius

64 non *scr. et del. C1*/planum: plane *ER*; *alter. in* plana *C1*; *alter. in* plene *O*/convexitatis: convexitas *OR* (*alter. in O*)/ymago ipsius *transp. FP1* 65 non: enim *L3E*/corporis *inter. O*/vel *inter. O* 67 errores: erroris *FP1*/accidentes omnes *transp. FP1*/accidunt *corr. ex* accedunt *O* 68 et *om. L3C1E* 69 planis *inter. a. m. E* 70 *ante* amplius *add.* de erroribus qui accidunt in speculis columpnaribus convexis capitulum quintum *R*/errores: exteriores *FP1* 71 idem: hidem *C1*/enim *om. P1* 72 diminuta *corr. ex* diminutea *F*/vise *om. R*/quantitas *corr. ex* quantitatis *C1*/ut in hiis *om. R* 73 *post* fortius *add.* in his *R*/in^2 . . . magna: magna res in spericis *C1* 74 quidem *om. FP1*/multum parva: multo minor *R* 75 recta *om. P1*/curva *mg. a. m. E*/spericis *om. P1*/spericis speculis (76) *transp. ER* 76 sed: et *O*/si *om. OL3C1ER*/curvitatis: curvitas *P1*; *corr. ex* curvitas *E*; *corr. ex* curvitates *L3*/*post* maxime *add.* curvitatis *ER* (*inter. a. m. E*)/unde *om. O* 77 sperici: spericis *FP1* 81 a linea: aliena *C1* 82 probabuntur: probantur *E* 83 est *om. OR*; *inter. E*/quiddam *corr. ex* quidam *C1*/premitti: puncti *L3*/hoc: huiusmodi *R* 85 qui: quod *R*/punctus: punctum *R* 87 linea illa *transp.C1*/*post* linea *scr. et del.* recta *F*/faciat *corr. ex* faciet *O*/si: sicut *L3* 88 ortogonalis: ortogonaliter *FP1*/contingentem: continentem *O*/illius puncti (89): illud punctum *R*

puncti, hec linea concurret cum perpendiculari sub axe et sub concursu prioris linee cum perpendiculari.

[5.4] Verbi gratia sit AEB [FIGURE 6.5.16, p. 312] sectio, E punctus datus, N punctus visus, B punctus reflexionis, BD perpendicularis, EDB angulus acutus, QEL contingens.

[5.5] Supra B fiat circulus columpne equidistans basi, scilicet BTO, et ducatur a puncto E linea longitudinis columpne, scilicet ET. Ducatur axis DH, et ducatur linea DC perpendicularis supra BD.

[5.6] Palam quod superficies HDC est ortogonalis super superficiem circuli. Superficies vero contingens columpnam in puncto B erit equidistans huic superficiei, quoniam linea longitudinis ducta a puncto B erit equidistans axi, et contingens supra B erit equidistans CD. Igitur superficies in qua sunt linee LE, ET non est equidistans superficiei HDC. Igitur concurret cum ea. Concurrat in linea LC, et ducatur linea TC, que quidem erit contingens, cum superficies LET sit contingens. Ducta autem linea TD, erit angulus CTD rectus, quoniam TD dyameter.

[5.7] Fiat autem supra E circulus columpne equidistans basi, scilicet ESP. Punctus axis in hoc circulo sit K, et ducatur linea KE. Ducatur etiam linea DL, que quidem secabit superficiem circuli ESP. Secet in puncto F, ubicumque sit punctus extra circulum vel intra, et ducantur linee KF, EF. Et a puncto F ducatur perpendicularis super superficiem circuli BTO que sit FM, et ducatur linea TM.

[5.8] Palam quoniam KD equidistans et equalis FM, et ita KF equidistans et equalis DM. Similiter KD equidistans et equalis ET, et KE equidistans et equalis DT. Erit ergo TE equidistans et equalis FM, et ita EF equidistans et equalis TM.

89 concurret: concurrit *FP1*/*post* perpendiculari *scr. et del.* cu *P1*/sub[1] *corr. ex* cum *a. m. E*/sub[1]. . . perpendiculari (90) *rep. FP1O* (*inter. O*) 90 prioris: minoris *E* 91 sit *om. L3E*/*post* AEB *scr. et del.* et B *O*/punctus: punctum *R* 92 datus: datum *R*; *om. O*/punctus[1]. . . punctus[2] *mg. O*; punctum visum B punctum *R* 93 EDB: EBD *C1*/contingens: continens *O* 94 supra: super *R*/*post* fiat *scr. et del.* a *E*/columpne . . . basi: equidistans basi columpne *R*/equidistans: quidem *L3E* 95 ET *inter. O*; ? *E* 96 *post* ducatur[2] *add.* a *FP1*/DC: DG *R*/supra: super lineam *R* 97 HDT: HDG *R* 98 contingens: continens *P1O* 99 linea longitudinis *transp. E* 100 erit[1, 2]: est *R*/contingens: continens *O*/*post* contingens *add.* circulum *R*/supra: super *R*/*post* equidistans [2] *add.* supra *L3* 101 TD: DG *R*/LE *om. P1*/LE EC *inter. O* 102 HDT: HDG *R*/igitur concurret *transp. R*/concurret: concurrit *FP1*/concurret cum ea: cum ea concurret *C1*/concurrat *corr. ex* concurret *C1*/in linea *rep. P1*/LT: LG *R* 103 TC: TG *R*/LET *corr. ex* LE *C1* 104 sit *corr. ex* si *O*/TCD: GTD *R* 106 autem: quando *FP1*/supra: super *R*/E *mg. C1*/columpne . . . basi: equidistans basi columpne *R*/basi: basis *O* 107 ESP *corr. ex* EZP *O*/punctus: punctum *R*/*post* KE *add.* et *C1* 108 linea DL *transp. FP1*/que *om. FP1*; *inter. E* 109 punctus: punctum *R*/circulum: circumferentiam *R*/intra: ultra *L3* 110 F *om. FP1*; *inter. O* 111 BTO: BRO *L3* 112 quoniam: quod *R*/*post* equidistans *add.* est *R*/FM: CFM *E* 113 KD: FM *R*/ET *inter. O*/et[3] *om. FP1* 114 equidistans et equalis: equalis et equidistans *R*/et equalis DT *inter. a. m. E*/erit . . . FM *om. R* 115 EF: F *O*/*post* EF *add.* erit *L3ER*

[5.9] Verum superficies KDL est ortogonalis super superficiem sectionis BEO, et est ortogonalis super superficiem circuli ESP. Ergo est ortogonalis super lineam communem sectioni et circulo que est EF. Igitur angulus EFK rectus. Similiter angulus TMD rectus.

[5.10] Cum ergo angulus DTC sit rectus, multiplicatio DM in MC sicut TM in FE, sed quoniam FM equidistans CL, erit proportio DF ad FL sicut DM ad MC. Sed DF maior DM; igitur FL maior MC. Igitur maior est multiplicatio DF in FL quam DM in MC, quare, cum TM sit equalis EF, erit multiplicatio DF in FL maior ductu linee EF in FE, quare angulus LED maior recto, si enim esset rectus, cum linea EF sit perpendicularis super LD, esset ductus DF in FL equalis quadrato EF. Restat ergo ut angulus DEQ sit acutus. Ergo ortogonalis ducta a puncto E, ortogonalis in quam super contingentem QL, cadet sub lineam ED et concurret cum perpendiculari BD sub puncto D, quod est propositum.

[5.11] Hiis premissis, accedendum est ad propositum.

[5.12] **[PROPOSITIO 17]** Proponatur columpna [FIGURE 6.5.17, p. 313]; linea equidistans axi sit TH. Erit quidem TH equidistans linee longitudinis columpne.

[5.13] Si ergo visus fuerit in eadem superficie cum axe et linea TH, poterit quidem reflecti linea, et erit reflexio a linea longitudinis columpne, que linea est communis superficiei in qua sunt visus et axis et superficiei columpne, sicut ostensum est in libro quinto. Sic videbitur linea TH linea recta, quoniam quelibet perpendicularis ducta a puncto linee TH erit in eadem superficie cum visu et axe, et probabitur ymaginem linee TH esse rectam, sicut probatur in speculis planis de visis lineis.

[5.14] Sit autem visus extra superficiem linee TH et axis, et TH equidistans axi, qui axis sit ZK. Fiat superficies per visum transiens

116 *post* verum *add.* etiam *C1*/ortogonalis *corr. ex* cor *O* 117 BEO: BET *FP1L3E*; BEA *C1* 118 ortogonalis: perpendicularis *R* 120 DTC: DMT *R*/*post* rectus *add.* et GTD rectus *R*/MC: MG *R* 121 *ante* sicut *add.* erit *R*/equidistans: equidistat *ER*/CL: D *FP1*; TB *L3*; GL *R*/ad FL (122) *om. FP1*/ad . . . DF (122) *mg. a. m. E* 122 MC^{1,2}: MG *R*/sed: si *P1* 123 est *inter. O*/quam . . . FL (124) *mg. a. m. E*/MC: MG *R*/*ante* quare *add.* ergo maior quam TM in FE *R*/TM: OM *O* 124 FL: FB *L3E*/EF: ES *L3* 125 esset rectus *transp. ER* 126 *post* esset *scr. et del.* D *O*/ductus *corr. ex* ductis *E*/DF *alter. ex* DI *in* DS *O*/FL: LF *C1* 128 sub: supra *FP1L3E*; super *OC1* 129 lineam: linea *R* 133 quidem: equidem *FP1*/TH^{2} *om. R* 134 longitudinis: E lineis *FP1* 135 superficie *om. C1* 136 et *inter. C1* 137 *post* columpne *scr. et del.* si ergo visus fuerit in eadem superficie *C1*/linea est *transp. ER*/est communis *transp. C1*/superficiei *corr. ex* superficie *E* 138 sicut *corr. ex* sint *O*/*post* sic *add.* igitur *R* 139 TH: TB *L3E*/linea *om. L3E*/quoniam: quando *L3* 141 sicut: sic *E*/probatur: probabitur *L3C1E*; probatum est *R*/speculis *corr. ex* speculo *F* 142 de visis: tenis *L3*/visis: divisis *FP1*; rectis *R* 143 sit: si *R*/*post* visus *add.* sit *R*/et^{1}. . . TH^{2} *scr. et del. E*/et^{2} *om. L3E*; *inter. C1* 144 equidistans: equidistat *P1*; equidistet *R*/per visum transiens: transiens per visum *C1*

secans superficiem columpne equidistantem basi. Secabit quidem super circulum. Sit circulus ille BF. Aliquis punctus linee HT refertur ad visum ab aliquo puncto huius circuli. Sit a puncto B, et visus sit E.

[5.15] Punctus ille linee TH sit Q, et ducantur linee EB, QB, et ducatur a puncto B linea longitudinis, que sit ABG, et ducatur a puncto B perpendicularis cadens super axem in puncto L, que sit ML. Et ducatur a puncto E linea equidistans ML, que sit EO, et ducatur QB usque dum concurrat. Sit concursus in puncto O.

[5.16] Palam quoniam angulus QBM equalis est angulo EBM, sed angulus QBM equalis angulo BOE, quia LM equidistans OE. Similiter angulus MBE equalis angulo BEO, quia coalternus. Igitur angulus BOE equalis est angulo BEO, quare BO, BE equalia.

[5.17] Sumatur autem alius punctus in linea TH, qui punctus sit T, et ducatur linea TO. Palam quoniam linea TH equidistans linee longitudinis, que est AG. Ergo sunt in eadem superficie, et in illa superficie est linea QBO, quare in eadem erit linea TO. Secabit ergo lineam AG. Secet in puncto G, et ducatur linea EG.

[5.18] Palam etiam quoniam linea AG est perpendicularis super superficiem circuli BF sicut axis cui equidistat, et superficies illius superficies EOBF secans scilicet columpnam equidistantem basi. Igitur angulus GBO rectus, et angulus GBE rectus. Ergo quadratum linee GO valet quadratum linee GB et quadratum linee BO. Similiter quadratum GE valet quadrata GB et BE, et quoniam BE, BO equalia et GB communis, erit GO equalis GE. Igitur angulus GOE equalis angulo GEO.

145 equidistantem: equidistanter *R* 146 super: secundum *R*; *om. SL3E*/super circulum *transp. deinde corr. C1*/aliquis punctus: aliquod igitur punctum *R*/HT: HS *L3*/refertur: reflectitur *R* 147 aliquo: alio *SOL3E*/a *om. R*/puncto: punctum *R* 148 E *om. FP1* 149 punctus ille: punctum illud *R*/ille linee *transp. deinde corr. C1*/QB: QL *FP1*/*post* QB *add.* QE *R* 150 ducatur[1]: ducantur *L3*; *corr. ex* ducantur *C1*/linea . . . B (151) *om. S*; *mg. a. m. E*/ABG *om. FP1*/ducatur[2]: ducantur *L3* 152 *post* linea *add.* longitudinis *P1*/ML: LM *R* 153 QB: BQ *C1*/usque: quousque *R*/dum *om. R*/*post* concurrat *inter.* cum eo *O* 154 quoniam: quod *R*/equalis est *transp. SER*/est *om. L3* 155 *post* equalis *add.* est *R*/OE: OZ *FP1*; EO *C1* 156 angulus: angulis *F* 157 est *inter. a. m. E*/*post* quare *add.* latera *R* 158 alius punctus: aliud punctum *R*/qui: quod *R*/punctus[2] *om. R*/qui . . . TH (159) *om. S* 159 T: TG *FP1*/quoniam: quod *R*/equidistans: equidistat *ER* 160 illa: eadem *C1* 161 linea[2] *om. FP1*/TO: TQ *R* 162 et *om. SL3ER*/et ducatur *corr. ex* educatur *O*/linea *corr. ex* lineam *S* 163 quoniam: quod *FP1R*/AG *rep. et del. F*/est *corr. ex* et *O* 164 cui equidistat *transp. S*/equidistat: equidistabit *S*; *alter. ex* equidistabit *in* equidistabat *E*; *corr. ex* equidistant *C1*/superficies[1]: superficiei *L3*/*post* illius *add.* circuli *R* 165 equidistantem: equidistanter *SC1ER*/basi: bas *S*; basis *L3* 166 GBO *corr. ex* BGO *C1*/*post* GBO *add.* est *R*/*post* GBE *add.* est *R*/et angulus *om. P1*/GBE: GEBE *S*/*post* GBE *inter.* est *E*/rectus[2] *om. S*; *inter. E* 167 GB: BG *ER*; BO *S*/et . . . BO *om. FP1S*; *mg. a. m. E* 168 quadrata: quadratum *L3E*/*post* BE[2] *add.* et *R*/equalia: sunt equales *R*/et[2] *om. C1*/*post* BE[2] *add.* et *SL3E*/et[3] *om. O* 169 communis: communi *O*

[5.19] Ducta autem perpendiculari ZGN, erit equidistans EO, cum sit equidistans MBL. Igitur angulus TGN equalis angulo GOE, et angulus NGE equalis angulo GEO, quare angulus TGN equalis angulo NGE. Cum autem E, O, N, G, Z sint in eadem superficie, et in illa sit G, E, G, T erunt in eadem superficie, et ita in eadem superficie sunt linee EG, NG, TG. Igitur T refertur ad E a puncto G.

[5.20] Sumpto autem in linea TH puncto H eiusdem longitudinis a puncto Q cuius est punctus T, et ducta linea HO, transibit quidem per punctum linee AG. Transeat per punctum A. Ducta perpendiculari DA et lineis EA, HAO, erit sicut prius probare quod duo anguli ABO, ABE recti, et duo latera AO, AE equalia, et duo anguli HAZ EAZ equales. Et ita H refertur ad E a puncto A. Similiter sumpto quocumque puncto linee TH, erit probare quod refertur ad E ab alio puncto linee AG, quare linea TH refertur a linea longitudinis que est AG.

[5.21] **[PROPOSITIO 18]** Restat probare ymaginem linee TH esse curvam. Palam ex predictis quoniam Q refertur ad E a puncto B, qui est punctus circuli. Sed cum sic refertur a circulo, si ducatur linea a puncto Q ad centrum illius circuli, concurret cum perpendiculari ducta a puncto B, et erit concursus in puncto axis. Ducatur ergo QL concurrens cum ML in puncto axis qui est L, et est centrum circuli FB. Et producatur EB usque concurrat cum QL. Sit concursus in puncto C. Erit C ymago Q, et est C in superficie in qua sunt linee QH, et axis, et linea longitudinis AG.

[5.22] Palam etiam quod T refertur ad E a puncto sectionis columpne, scilicet a puncto G. Est autem a puncto T lineam ducere perpendicularem super lineam contingentem in puncto alio sectio-

171 *post* perpendiculari *add.* super axem *R*/erit equidistans *transp. ER* 172 TGN: ZGN *FP1*/*post* TGN *add.* est *P1*/GOE *alter. ex* GNE *in* NGE *E* 173 et . . . NGE (174) *scr. et del. E*/angulo[2] *om. R* 174 E *om. FP1*/EO *inter. E*/E . . . Z: TGO NGZ *R*/O *om. S*/et . . . E (175): in qua G ergo puncta O *R*/et . . . superficie[2] (175) *om. L3* 175 G[1] *om. O*/*post* E *add.* O *C1*/erunt: erit *O*/*post* superficie[1] *scr. et del.* et in illa *S*; *add.* ET *SE*; *add.* cum ET *O*/ita *inter. O* 176 NG: OG *R*/refertur: reflectitur *R* 177 *post* linea *add.* TG *P1* 178 Q *om. FP1*/punctus: punctum *R*/et *inter. O*/ducta linea *transp. SL3ER*/HO: HD *L3*/transibit . . . HAO (180) *om. FP1* 179 transeat: transiet *SE*/ducta: ducte *S*/*post* ducta *add.* que a puncto A super axem *R*/perpendiculari: perpendicularis *S* 180 DA: DD *S*/lineis: linea *R*; *corr. ex* longitudinis *O*/HAO: HAC *L3E* (*alter. in E*); *om. R* 181 ABO ABE *transp. C1*/AO: AC *FP1*/HAZ: HAR *R* 182 EAZ: EAR *R*/refertur: reflectetur *R* 183 quocumque *corr. ex* quoque *O*/*post* erit *add.* probare quod refertur ad E a puncto A similiter sumpto quocumque puncto linee TH erit *FP1*/refertur: reflectatur *R*/ad E *om. R*/alio: aliquo *FP1C1R* 184 linea[1] *om. C1*/refertur: reflectetur *R* 187 esse: ee *S*/quoniam Q refertur: quod Q reflectitur *R*/a *om. S* 188 qui *corr. ex* que *F*/qui est punctus: quod est punctum *R*/sic: sit *L3E*/refertur: reflectatur *R* 189 a *om. F*/Q: B *L3*/circuli *om. C1* 191 qui: quod *R* 192 usque: quousque *R* 193 *post* C[1] *scr. et del.* erunt *P1* 194 et *inter. O* 195 refertur: reflectitur *R* 196 columpne: columpnaris *R*/lineam: unam *OL3C1ER* 197 contingentem *om. L3*/puncto alio *transp. R*/alio: aliquo *FP1C1ER*/sectionis: sectionem *R*

nis, que quidem concurret cum perpendiculari ducta a puncto G, que est NGZ, sub axe, id est sub puncto Z, qui est concursus perpendicularis NZ et axis, quoniam ducta linea TZ, erit angulus TZN acutus. Ducatur ergo TX concurrens cum NZ in puncto X, et producatur EG donec concurrat cum TX in puncto I. Erit I ymago puncti T.

[5.23] Similiter ducta a puncto H linea, que sit ortogonalis super punctum sectionis a qua refertur, concurret cum perpendiculari DAZ sub puncto D, que est punctus axis. Concurrat in puncto P, et producatur EA donec concurrat cum HP in puncto S. Erit ymago puncti H punctus S. Ducatur autem linea SI.

[5.24] Palam cum linea TI concurrat cum perpendiculari NZ, que est equidistans linee EO, concurret cum linea EO. Similiter linea HS; quoniam concurrit cum perpendiculari DAZ, que est equidistans EO, concurret cum EO. Sed quoniam situs T respectu puncti E idem est cum situ H et eadem longitudo, similiter situs puncti T et puncti H ad punctum O idem, et punctorum I, S respectu O etiam est idem. Erit idem situs linearum TI, HS respectu linee EO.

[5.25] Igitur linee TI, HS concurrent super idem punctum linee EO. Concurrant in puncto U. Erit TUH triangulus, et in superficie huius trianguli erit linea IS. Axis autem non est in hac superficie.

[5.26] Verum TH est in eadem superficie cum axe; ergo superficies illa secat superficiem trianguli super lineam communem que est TH, non super aliam. Cum ergo punctus C sit in superficie linee TH et axis, et non sit in linea TH, non est in superficie trianguli TUH, et duo puncta I, S sunt in superficie illius trianguli, quare linea ICS est curva, et ymago linee TH erit curva, quod est propositum.

[5.27] Sed eius curvitas est modica, quia perpendicularis ducta a puncto C ad superficiem circuli est valde parva, et quanto maior fue-

198 G: igitur *L3E*/que: qui *L3* 199 est[1] *om. L3*/qui: quod *R* 200 NZ: ZNZ *S*/*post* TZ *add.* et *FP1*/TZN: TNZ *E*/*post* acutus *add.* producatur NZ ultra Z in X *R* 201 TX: IX *E*/NZ *corr. ex* NY *a. m. E*/X *corr. ex* I *S* 202 TX: IX *C1*/*post* erit *scr. et del.* Y *F*/*post* I *add.* Y *O* 203 *ante* similiter *add.* et *C1*/a puncto *om. FP1*/*post* super *add.* lineam contingentem in speculum in puncto aliquo *R* 204 punctum *om. R*/qua: quo *R*/*post* qua *add.* sectione *C1*; *add.* H *R*/refertur: reflectitur ad E *R*/cum *inter. O*/DAZ: DAR *R* 205 D: O *L3*/que: qui *SOL3C1E*; quod *R*/punctus: punctum *R* 206 H: B *E* 207 punctus: punctum *R*/*post* autem *scr. et del.* est *C1*/SI: sed *FP1*; ST *R* 208 TI: Q *L3*/NZ *corr. ex* ZN *S*/que: qui *E* 209 concurret *inter. O*/cum linea *mg. C1*/*post* EO[2] *add.* sit concursus in U *R* 210 concurrit: concurret *E*/DAZ: DAR *R* 211 concurret *corr. ex* concurrat *O*/T *inter. O*; est *P1*/est *mg. a. m. C1* 212 situs *corr. ex* secundus *C1*/situs puncti *transp. C1* 213 O[1]: Q *R*/etiam *corr. ex* vel *O*; et *L3E* 214 idem *om. FP1*/TI *inter. E*; N *S*; U *L3* 215 *post* igitur *scr. et del.* linee EO quod *O*/TI: N *S*; TS *O*/HS: HC *P1* 216 concurrant: concurrent *FP1*; *corr. ex* concurrent *C1*/*post* erit *add.* ergo *R*/triangulus *corr. ex* visus *a. m. E*; triangulum *R*/et *om. L3*/in[2] *inter. a. m. C1* 217 huius *inter. O*/trianguli: anguli *L3E*/*post* erit *add.* in *C1*/axis *rep. S*/hac: eadem *R* 218 verum . . . superficie *om. FP1*/ergo *inter. O* 220 *post* super *scr. et del.* line *F*/punctus: punctum *R*/C: S *L3*/*post* superficie *scr. et del.* trianguli *L3*/et axis (221) *inter. O* 221 et[1] *inter. a. m. E*/in[1] *inter. O* 222 IS *corr. ex* IO *a. m. E*/illius *mg. a. m. E*/*post* trianguli *add.* illius *C1*/IES: NS *SE*; US *L3* 223 *post* et *inter.* sic *O*/erit: est *FP1*/erit curva *transp. C1* 224 est *om. SOL3C1E* 225 *post* ad *add.* punctum sectionis linee IS et *R*/superficiem: superficiei *R*/et *rep. E*

rit linea visa equidistans linee longitudinis speculi, tanto ymago eius erit minus curva, quanto minor magis.

[5.28] **[PROPOSITIO 19]** Amplius, si linea TH [FIGURE 6.5.19, p. 314] secet superficiem in qua sunt centrum visus et axis, et sit ortogonalis super eam, visus aut erit in illa superficie linee TH secante ortogonaliter superficiem axis et visus, aut extra.

[5.29] Si fuerit in illa superficie, aut erit supra lineam TH, aut infra. Si supra, cum illa linea sit corporalis, occultabit visui speculum, et ita non reflectetur, sed forsan capita eius apparebunt et reflectentur a circulo columpne qui communis est superficiei linee TH secanti columpnam et columpne. Et erit horum capitum ymago sicut in spericis exterioribus.

[5.30] Similiter, si visus fuerit sub linea TH, occultabitur pars eius propter capud in quo est visus. Pars autem linee visa refertur a circulo eodem penitus modo quo in exterioribus spericis.

[5.31] Si vero visus fuerit extra superficiem linee TH ortogonaliter secantem superficiem visus et axis, sit E visus, et XZG columpna. Refertur H ad E ab aliquo puncto columpne. Sit a B. Sit T eiusdem longitudinis a puncto E. Dico quod T refertur ad E ab alio puncto columpne, et cum puncta H, T sint eiusdem situs et eiusdem longitudinis a puncto E, erunt similiter puncta reflexionum, scilicet B, G, eiusdem longitudinis et eiusdem situs a puncto E. Igitur duo puncta B, G erunt in circulo.

[5.32] Sit circulus BZG, eius centrum D. Ducantur linee HB, BE, TG, GE, et a centro ducantur perpendiculares supra contingentes B, G, scilicet DBO, DGS. Et ducatur linea ED, et producantur HB, TG usque concurrant cum linea ED.

226 longitudinis: US *FP1* 227 *post* curva *add.* et *R*/minor: maior *FP1*/*post* minor *add.* tanto *R*/minor magis *mg. O* 228 si linea *om. FP1* 230 *post* erit *add.* visa *L3*/in illa *mg. F* 231 ortogonaliter: ortogonalis *L3*; ortogonalem *E*/*post* ortogonaliter *add.* super *FP1O* 232 si *inter. O*/illa superficie *transp. L3ER*/erit *om. ER*/infra: intra *S* 233 illa linea *transp. FP1*/sit *om. E*/occultabit: occultabitur *C1* 234 non *om. S*/reflectentur: reflectetur *L3C1E* 235 a: in *FP1*/est *inter. C1E* (*a. m. C1*)/secanti *alter. ex* secant *in* secantis *O* 236 columpne: columpnas *L3*/*post* columpne *add.* erit *S*/et[2] *inter. C1* 237 *ante* spericis *scr. et del.* speculis *E* 239 est *inter. O*/visa: vise *R*/refertur: reflectitur *R* 240 *post* eodem *scr. et del.* puncto *F*/in *inter. a. m. E*/spericis *inter. a. m. E* 241 ortogonaliter: ortogonalis *L3E* 242 visus et[1] *om. L3*/*post* axis *add.* et *L3*/sit E visus: visus sit E *L3*/quod: et *SL3E*/XZG: XZX *OL3C1E*; BGX *R* 243 refertur: reflectetur *R*/aliquo: alio *SL3E*/*post* a *add.* puncto *R*/*post* B *add.* et *R* 244 *post* E[1] *add.* cuius est H *R*/refertur: reflectetur *R*/alio: aliquo *FP1C1R* 245 columpne et *om. C1*/eiusdem: eius *FP1* 246 scilicet *inter. O*/B: L *FP1* 247 eiusdem *corr. ex* eius *O* 248 B: L *FP1* 249 BZG: BRG *S*/eius *om. FP1*/*post* D *add.* et *R*/BE: LE *FP1* 250 ducantur perpendiculares: ducatur perpendicularis *FP1*/supra: super *OL3ER*/*post* contingentes *add.* circulum in punctis *R* 251 scilicet: SI *L3E*/DGS: GDS *FP1*/ducatur: ducantur *L3*/et[2]. . . ED (252) *om. R* 252 usque: usquoque *O*

[5.33] Cum puncta H, T sint eiusdem situs et longitudinis respectu E et respectu D, et similiter puncta B, G eiusdem situs respectu D et respectu E, habebunt linee HB, TG eundem situm respectu linee ED, et ita concurrent in idem punctum illius linee. Sit in puncto L.

[5.34] Fiat linea longitudinis columpne in qua punctus Z, et sit hec linea in superficie visus et axis, que sit AZ, et ducantur LZN, DZC. Q sit punctus linee TH, punctus scilicet qui est in superficie visus et axis, et a puncto Q ducatur equidistans linee DZC. Cadet quidem hec linea super axem, et LZN cadet in hanc lineam supra punctum Q. Cadat in puncto N.

[5.35] Palam ex predictis quod angulus HBO equalis angulo OBE. Sed angulus HBO equalis angulo LBD per contrapositionem, et angulus OBE equalis duobus angulis BED, BDE, quia extrinsecus. Ergo angulus LBD equalis duobus angulis BED, BDE. Fiat ergo angulus MBD equalis angulo BDE. Remanet angulus MBL equalis angulo BEL, quare ductus EM in ML equalis quadrato BM.

[5.36] Ducatur linea MZ. Quoniam angulus BDM maior angulo ZDM, et duo latera ZD, DM equalia duobus lateribus BD, DM, erit MB maior MZ, quare ductus EM in ML maior quadrato MZ. Sit ductus EM in MI equalis quadrato MZ, et ducantur linee IB, IZ. Erit ergo angulus MZI equalis angulo ZEI, quare MZL maior angulo ZED.

[5.37] Sed quoniam angulus MBD positus est equalis angulo BDM, erit linea MD equalis linee MB. Sed MB maior MZ, quare MD maior MZ. Igitur angulus MZD maior angulo MDZ; igitur angulus DZL maior duobus angulis ZDE, ZED. Sed angulus DZL equalis angulo NZC, et angulus CZE equalis duobus angulis ZDE, ZED, quare angulus NZC maior angulo CZE.

[5.38] Secetur ad equalitatem per lineam FZ, que quidem concurret cum linea NQ supra punctum N. Cum ergo angulus FZC equalis

253 T: E *R*/sint *corr. ex* sicut *C1*/longitudinis: lis *FP1* 254 B G *inter. E*/D[2]: E *R* 255 E: D *R* 256 concurrent: concurrant *FP1SC1E*; *corr. ex* concurrant *O*/in[1] *om. S*/idem: eundem *L3*/*post* sit *add.* concursus *R* 257 columpne *corr. ex* ? *O*/*post* qua *inter.* sit *O*/punctus: punctum *R* 258 que . . . axis (260) *om. S*/*post* ducantur *inter.* linee *OR*/DZC *corr. ex* DCZ *C1* 259 punctus[1, 2]: punctum *R*/scilicet: S *L3*/qui: quod *R*/*post* visus *add.* et visus *E*/et *inter. E* 260 et a puncto *rep. P1*/a *om. S*/Q: que *L3*/*post* ducatur *add.* linea *R*/DZC: DZI *S*/quidem: equidistans *C1* 261 et BZN: ZL ZN *L3E*; et LZN *SC1* 263 *post* equalis *inter.* est *OR*/angulo *om. L3ER*/angulo . . . equalis (264) *om. P1* 264 *post* equalis *add.* est *R*/contrapositionem: circa positionem *C1* 265 *post* equalis *add.* est *R*/BDE: HDG *L3*; BDG *E*; *inter. O* 266 LBD: BBD *FP1*/*post* equalis *add.* est *R*/BED: DEB *C1*/BDE *inter. O* 267 remanet: remaneat *FP1O*/equalis . . . BEL (268) *mg. a. m. O* 269 *post* quoniam *add.* igitur *R*/*post* maior *add.* est *R*/angulo *inter. O* 270 ZDM . . . latera *om. P1*/DM[1] *corr. ex* M *P1* 271 *post* maior *add.* est *R*/MZ[2]: ZM *R* 272 EM in MI: in MI EM *S*/in MI *inter. O*/MI: M *L3*; IN *E*/et . . . MZI *mg. a. m. E* 273 MZI: MZL *L3* 274 MBD: MHD *O* 275 MB[1]: MD *R*/quare . . . MZ (276) *om. L3*; *mg. a. m. E* 276 MZD: MZ *FP1*/angulus[2] *om. R* 277 DZL[1] *corr. ex* L/ZED *mg. C1*/*post* maior *add.* est *R* 278 NZC: NZS *L3*/et . . . CZE *mg. a. m. O*/CZE: EZC *R* 279 NZC *corr. ex* NZO *O*; BZT *L3*; BZC *E*/*post* equalis *add.* est *R*/CZE: GZE *E*; EZC *R* 280 per *inter. a. m. E*/concurret: concurrent *C1* 281 supra: super *FP1*/*post* NQ *add.* concurrat *R*/supra: super *R*/N: F *R*/FZC: FCZ *L3*/*post* FZC *add.* est *L3*; *add.* sit *R*/*post* equalis *inter.* est *O*

angulo CZE, refertur F ad E a puncto Z. Q refertur ad E a puncto linee longitudinis que transit per Z, a puncto que est AZ, scilicet ultra Z. Si enim a puncto citra Z, id est propinquiori E, linea ducta a puncto Q ad punctum illud reflexionis secabit lineam FZ, et ita punctus sectionis refertur ad E a duobus punctis, quod est impossibile.

[5.39] Sumatur ergo ultra Z punctus K a quo refertur Q ad E, et ducatur linea EK donec concurrat cum linea NQ in puncto P. Erit P ymago Q. Sed H refertur ad E a puncto sectionis columpne. Si ergo a puncto H ducatur perpendicularis super contingentem sectionem in aliquo puncto, perpendicularis illa concurret cum perpendiculari CZD sub axe. Concurrat in puncto U.

[5.40] Similiter a puncto T est ducere unam perpendicularem super sectionem a cuius puncto refertur ad E. Et quoniam puncta H, T sunt eiusdem situs respectu linee CZD, et puncta sectionis similiter per que transeunt perpendiculares, igitur ille due perpendiculares concurrent in idem punctum linee CZD. Concurrant ergo in puncto U.

[5.41] Linea EB concurret cum linea HU. Sit concursus in puncto R. Similiter EG concurrat cum TU in puncto Y, et ducatur linea RY. Palam quod R est ymago H, Y est ymago T, et habemus triangulum ERY. Extra superficiem huius trianguli est punctum Z, et ita superficies huius trianguli altior est linea EP, et ita P extra. Quare linea RPY erit curva, et illa est ymago linee TH, et est quidem hec ymago curvitatis non modice, quod est propositum.

[5.42] Palam ergo quod in hiis speculis, si linea recta visa equidistans fuerit linee longitudinis columpne, erit ymago eius aut recta aut accedens ad rectitudinem. Si vero linea visa recta equidistans fuerit latitudini columpne, erit ymago eius curva curvitate non modica.

282 refertur[1, 2]: reflectetur *R*/*post* Q *add.* vero *R*/E[2]: ZE *S* 283 a puncto *scr. et del. O*/que[2]: qui *L3*; quod *R*/AZ *corr. ex* AI *O*/AZ scilicet *om. R* 284 citra: circa *L3*/id est *mg. a. m. C1*/propinquiori: propinqiore *R*; *corr. ex* propinquior *O*/*post* a[2] *add.* puncto A *S* 285 punctum illud *transp. C1*/illud: illum *FP1*/FZ: F *FP1*/punctus: punctum *R* 286 refertur: reflectetur *R*/quod est *inter. O* 287 *post* sumatur *add.* punctus *P1*/*post* ultra *add.* punctum *R*/punctus: punctum *R*/a quo refertur *om. FP1*/refertur: reflectatur *R*/Q: AQ *FP1* 289 sed: verum *O*/H *inter. O*/H refertur *transp. O*/refertur: reflectitur *R*/*post* columpne *add.* quo refertur *FP1*/ergo *inter. O* 290 H . . . sub (292) *mg. a. m. O*/*post* super *add.* lineam *R* 291 aliquo: alio *FP1SL3E* (*alter. in E*)/cum *mg. a. m. C1* 292 CZD: ZD *O* 293 T: L *R* 294 refertur: reflectatur *R*; *om. S*/*post* refertur *add.* T *R*/et *inter. O*/T: D *FP1L3E*; *corr. ex* D *O* 295 CZD: ED *R* 296 transeunt: transeant *S*/*post* perpendiculares[1] *add.* ab ipsis ducte *R*/igitur *inter. O*/ille due *transp. C1*/due perpendiculares *om. S* 297 CZD: ED *R* 298 *ante* linea[1] *add.* et quia *R*/linea[1]: linee *FP1SL3E*/EB: EDZ *P1*; *alter. ex* DZ *in* EDZ *F*/concurret: concurrit *R*; concurrent *L3E*; *corr. ex* concurrent *O*/linea[2] *om. R* 299 R: RE *FP1*/TU: TRI *FP1*; FD *C1*; IN *L3E* 300 H Y est ymago *rep. S*/*post* H *add.* et *R* 1 ita superficies: in superficie *R* 2 altior *corr. ex* altius *O*/*post* P *add.* est *R*/*post* extra *add.* P *SO*/quare: quoniam *FP1* 3 curvitatis: curvitas *L3* 4 *post* non *scr. et del.* est *C1* 5 ergo *om. L3*/*post* visa *add.* est *L3* 6 linee longitudinis *transp. E*/*post* eius *scr. et del.* a *S*/aut[1] *om. L3C1E* 7 accedens: accidens *L3*/visa recta *transp. SER*/*post* fuerit *scr. et del.* la *F* 8 latitudini *om. R*/*post* latitudini *scr. et del.* n *O*/non *inter. O*

[5.43] Linee autem inter has duas site, que magis accedunt ad situm linee equidistans longitudini columpne erunt ymagines earum rectitudini magis vicine, et ymagines earum que propinquiores sunt situi equidistantium latitudini erunt magis curve. Et minuetur vel augmentabitur curvitas ymaginum secundum accessum vel elongationem linearum ad alterum horum situum, et hoc est propositum.

[PARS SEXTA
In speculis piramidalibus exterioribus]

[6.1] Amplius in speculis piramidalibus exterioribus idem errores accidunt qui in columpnaribus exterioribus eveniunt, linee enim vise equidistantes longitudinis piramidis aut recte videntur, aut forte equidistantes latitudini curve, et intermedie augmentant vel diminuunt curvitatem secundum propinquitatem harum vel harum remotionem, et hoc quidem probabitur. Quoddam tamen premittendum proponamus et est:

[6.2] **[PROPOSITIO 20]** Si sumatur in superficie piramidis punctus reflexionis et fiat sectio transiens per punctum illud, et in sectione sumatur punctus remotior ab acumine piramidis puncto reflexionis et a puncto sumpto ducatur perpendicularis super contingentem sectionem, hec perpendicularis concurret cum perpendiculari ducta a puncto reflexionis sub axe.

[6.3] Verbi gratia, sit ABGZ [FIGURE 6.6.20, p. 315] piramis erecta super bases suas, A acumen piramidis, BFZ sectio, E punctus reflexionis, Z punctus sectionis remotior a puncto A quam E. Supra punctum Z sit superficies secans piramidem equidistans basi. Secabit quidem

9 inter: intra *O*/duas *om. P1*/site: si de *F*; si DO *P1*/accedunt: accidunt *L3*; *alter. in* accedint *O* 10 equidistans: equidistantis *C1E*/equidistans . . . vicine (11): equidistantis respectu columpne habebunt ymagines suas rectitudini magis vicinas *R*/longitudini: respectu *E*/ymagines . . . et (11) *om. FP1* 11 et *om. O*/que *inter. P1* 14 horum: eorum *C1* 15 *ante* amplius *add.* de erroribus qui accidunt in speculis piramidalibus convexis capitulum sextum *R* 16 accidunt *corr. ex* accedunt *O*/columpnaribus: spericis *FP1SOL3C1ER* 17 equidistantes: equidistant *S*/longitudinis: R *FP1*; scilicet *L3*; respectu *ER*/forte: fere *C1* 18 equidistantes: quidem *S*/latitudini *corr. ex* linea *O* 19 secundum propinquitatem *om. FP1*/harum[1]: earum *R*/harum[2] *om. R*/*post* harum[2] *add.* vel *FP1SOL3C1*/remotionem: removentur *FP1*; *corr. ex* removentur *O* 20 quidem *om. OR*/quoddam: quiddam *FP1R*/tamen: cum *L3*/*post* tamen *add.* proponendum *P1* 22 piramidis: piramidali *L3*/punctus: punctum *R* 23 illud: illum *FP1* 24 punctus remotior: punctum remotius *R*/ab acumine: a vertice *R*/reflexionis: rationis *S* 25 a *corr. ex* in *O*/contingentem: continentem *O* 26 concurret cum perpendiculari *inter. O*/*post* perpendiculari *add.* super contingentem sectionem *R* 28 ABGZ: ABGT *FP1*/piramis: piramidis *O*/erecta *corr. ex* recta *OC1* (*a. m. C1*) 29 bases suas: basim suam *R*/A: et *L3*/acumen: vertex *R*/BFZ: FZ *O*/E: est *S*/punctus: punctum *R* 30 Z: et *S*/punctus: punctum *R*/remotior: remotius *R*/a *om. E*/supra: super *R* 31 Z: E *FP1SL3E*; *corr. ex* E *OC1*/sit: fiat *R*/equidistans: equidistantem *FP1L3C1*; *alter. ex* equidistantem *in* equidistanter *O*; equidistanter *R*/quidem: equidem *FP1*

supra circulum communem. Sit circulus ille GBRZ, et ducantur linee AZ, AE, et producatur AE donec sit equalis AZ. Veniet quidem ad circulum. Cadat ergo in puncto eius O.

[6.4] C sit centrum circuli, et ducatur axis AC, et a puncto E ducatur perpendicularis super superficiem contingentem piramidem. Concurret quidem cum axe circa centrum circuli quod est C. Sit in puncto D, et ducatur linea DZ.

[6.5] Et a puncto O ducatur perpendicularis concurrens cum axe in puncto K, et ducantur linee DZ, KZ. Et supra punctum Z ducatur contingens sectionem, que sit TQ, et alia contingens circulum BGZ, que sit ZY.

[6.6] Et ducatur linea BCZ, et a puncto C ducatur perpendicularis super lineam BCZ, que sit CR. Erit quidem perpendicularis super axem, cum axis sit perpendicularis super superficiem circuli, quare CR est perpendicularis super superficiem ACZ. Et erit equidistans contingenti ZY, quare ZY est perpendicularis super superficiem ACZ, quare TQ non est perpendicularis super eandem superficiem.

[6.7] Verum quoniam K est polus ad circulum BRZ, cum linee KO, KZ sint equales, et axis AK communis, erit angulus AOK equalis angulo AZK, et ita angulus AZK rectus. Cum ergo linea KZ sit perpendicularis super AZ, que est linea longitudinis, erit perpendicularis super superficiem contingentem piramidem super hanc lineam longitudinis. Sed TQ est in superficie contingenti, quia est communis superficiei contingenti et sectioni. Igitur KZ est perpendicularis super TQ.

[6.8] Ducatur autem HZ in superficie sectionis perpendicularis super lineam TQ. Cum autem linea KZ sit extra superficiem sectionis, secabit lineam HZ, nec erit una linea cum illa. Superficies ergo

32 supra: super *R*/GBRZ: GBIZ *FP1E*; BGRZ *C1*/et *om. FP1* 33 et . . . AE[2] *mg. a. m. E*/veniet: venit *FP1* 34 puncto: punctum *R*/*post* puncto *scr. et del.* i *P1*/eius *corr. ex* illo *O*/O: OC *S*; ECO *L3E* 35 *ante* C *add.* et *R*/C: OC *L3E*/circuli *inter. a. m. E* 37 circa: citra *SOL3C1ER* 38 educatur: et ducatur *OE* (*alter. in O*)/DZ: DZI *L3*/*post* DZ *add.* continens angulum acutum cum perpendiculari ED *R* 39 *post* perpendicularis *add.* super lineam AO *R* 40 ducantur linee: ducatur linea *R*/DZ *om. R*/KZ: HZ *FP1*; KT *L3E*; *corr. ex* HC *O*/*post* KZ *add.* HZ *C1*/supra: super *R* 41 contingens[1]: continens *SO*/sectionem: sectioni *P1S*/TQ *corr. ex* TA *O*/contingens[2]: continens *OE* 42 ZY *corr. ex* ZI *O*; *corr. ex* ZQ *L3* 43 BCZ: BOZ *L3*/et[2]. . . BCZ (44) *rep. S*; *scr. et del. E* 44 BCZ: BOZ *E*; *corr. ex* BZ *C1*/que sit CR *mg. a. m. E* 47 contingenti ZY *transp. R*/quare ZY *om. FP1* 48 quare *om. L3*/TQ: DQ *FP1* 49 K: KZ *FP1*; *corr. ex* Q *C1*/est *om. FP1*/ad circulum: circuli *R*/BRZ *corr. ex* BLZ *E*/*post* BRZ *add.* palam *R* 50 equales *corr. ex* quales *O*/AK: AQ *S*/*post* communis *add.* et AO equalis AZ quod *R*/AOK: ACK *FP1SOL3E* 51 KZ: U *FP1*; KI *L3E*/KZ . . . linea (52) *mg. O* 54 est[1]: erit *O*/*post* contingenti *scr. et del.* circuli cum ergo linea KZ sit perpendicularis super AZ que E *O*/quia: quod *S*/*post* communis *scr. et del.* perpendicu *E*; *add.* sectio *R* 55 *post* KZ *add.* est perpendicularis super TK *L3* 57 HZ: BZ *L3*; *corr. ex* KZ *O* 58 KZ *corr. ex* TZ *L3*/sit *corr. ex* sint *O* 59 *post* secabit *scr. et del.* unam *O*/HZ: KZ *L3*/nec: non *FP1*/cum: quare *R*/ergo *om. R*

KZH secat superficiem sectionis super lineam communem HZ, et secat lineam TQ super punctum Z. Et superficies AZK secat superficiem AZH super lineam communem KZ.

[6.9] Verum DZ est in superficie sectionis, et secatur a linea KZ in puncto Z, et punctus T supra superficiem KZH, punctus Q infra. Et ita superficies KZH secat superficiem DZQ super lineam communem, et illa linea communis est perpendicularis super lineam TQ, quia linea illa est in superficie HZK super quam est perpendicularis TQ. Et quoniam superficies HZK secat superficiem DZQ, et declinatio superficiei HZK sit ex parte ZE, erit linea communis sectioni illarum superficierum inter lineas QZ, DZ, et ita concurret cum perpendiculari ED sub axe. Et quod necessario concurrat cum ea probatum est in libro quinto, figura 19, et ita propositum.

[6.10] **[PROPOSITIO 21]** Sit ergo piramis cuius acumen A [FIGURE 6.6.21, p. 316], axis AH, linea longitudinis AZ, et a puncto Z ducatur perpendicularis supra superficiem contingentem piramidem in linea AZ, que necessario concurret cum axe. Sit linea TZH.

[6.11] Ducatur a puncto A linea extra piramidem supra superficiem contingentem piramidem in linea AZ faciens angulum acutum cum axe et cum linea longitudinis AZ, que sit AN. Et in superficie AHN a puncto H ducatur linea cum axe faciens angulum equalem angulo AHZ, que linea necessario concurrat cum linea AN, que sit HO. Et facto supra punctum Z circulo equidistans basi, transibit HO per circulum sicut HZ transit per ipsum.

[6.12] Ducatur autem linea OZ, et producatur usque ad punctum F. Quoniam linea OZ est supra superficiem contingentem piramidem

60 secat *corr. ex* secabit *a. m. E*/*post* sectionis *scr. et del.* sectioni *F*/communem HZ *transp. R*/HZ . . . KZ (62) *mg. O*/et . . . KZ (62) *rep. L3E* (secat[2] [61]: secabit/AZH [62]: AZKB *E*); *rep. et del. C1* (TQ [61]: CO; AZH [62]: AZ KB) 61 secat[1] *corr. ex* secabit *E*/lineam: linea *L3*/TQ: CO *L3E*/Z *om. S*/AZK: AKZ *C1*; HZK *R*/*post* AZK *add.* et *L3E*/*post* secat *add.* super *C1* 62 AZH: AZKO *FP1*; AZBH *S*; AZKL *O*; AZKB *L3*; AZKD *E*; DZK *R*/KZ: HZ *L3E*/*post* KZ *scr. et del.* K est *O* 63 DZ *alter. in* DE *a. m. E* 64 punctus[1, 2]: punctum *R*/*post* T *add.* est *R*/*post* KZH *scr. et del.* secat *S*/Q: quod *FP1* 65 KZH *corr. ex* LTZH *O*/secat: secabit *ER*/superficiem DZQ *rep. S* 66 illa: ita *L3C1E*/super lineam *om. S*/quia . . . TQ (67) *om. S* 69 *post* HZK *add.* a superficie sectionis *R*/ZE: ZC *R*/sectioni *corr. ex* sectionis *S* 70 lineas: lineam *FP1SOL3C1E*/QZ: AZ *FP1* 71 ED *om. R*/cum ea *om. R*/ea: eo *FP1*/est *inter. O* 72 quinto figura 19 *corr. ex* quomodo S in 9[a] *a. m. O*/figura 19 *om. R*/19: 14 *E*/*post* ita *add.* est *R*/propositum: proponimus *L3* 73 acumen: vertex *R* 74 AH *corr. ex* AZ *O*/AZ: AHZ *L3* 75 ducatur *corr. ex* dicucatur *F*/supra: super *FP1R*/piramidem: piramide *L3* 76 necessario *corr. ex* non *O* 77 supra: super *FP1*; ultra *R*/supra superficiem *mg. a. m. O* 78 contingentem: continentem *O* 79 *post* superficie *scr. et del.* a *C1* 80 AHN *corr. ex* ANH *a. m. E*/*post* AHN *scr. et del.* que linea vero concurrat cum linea *C1*/*post* faciens *scr. et del.* lineam *C1*/angulum equalem *transp. C1* 81 necessario: non *FP1*; nec *S*; vero *L3C1E*; *corr. ex* non *O*/concurrat: currat *F*; concurret *R* 82 HO[1]: BO *O*/supra: super *R*/*post* Z *add.* et *P1*; *scr. et del.* et *F*/equidistans: equidistante *OC1R*; equidistanti *E*/HO[2] *corr. ex* O *O* 83 transit: transibit *C1E*; *corr. ex* transibit *O* 84 autem *om. E*/OZ: DZ *FP1O*/usque *om. ER* 85 est supra: secat *R*

in linea AZ, cum linea HZ sit perpendicularis supra illam superficiem, erit angulus OZH maior recto. Igitur angulus FZH acutus.

[6.13] A puncto Z ducatur contingens supra circulum, que sit ZM, et a puncto F ducatur perpendicularis supra AZ cadens in puncto eius E, que producta concurret cum AO, quoniam angulus OAZ est acutus. Concurrat ergo in puncto N, et a puncto E ducatur equidistans linee TH, et sit QE.

[6.14] Et a puncto E ducatur equidistans linee MZ, que sit LE. Palam quoniam MZ est perpendicularis supra AE, quoniam est perpendicularis supra TH et supra dyametrum circuli, cuius est contingens. Igitur LE est perpendicularis super AE.

[6.15] Fiat autem superficies LQD secans piramidem. Erit quidem sectio piramidalis. Cum ergo AE sit perpendicularis super FN, et super QD, et super LE, erit FN in superficie illa secante piramidem. Fiat ergo CF equidistans QE. Erit quidem equidistans TZ.

[6.16] Verum cum angulus OZT sit acutus, angulus TZF sit obtusus. Ducatur a puncto Z linea faciens cum TZ angulum equalem angulo OZT, que quidem linea necessario secabit FC. Secet in puncto C, et ducatur linea EC. Cum ergo CZ, OZ sint in eadem superficie, et angulus OZT equalis angulo TZC, punctus O refertur ad C a puncto Z.

[6.17] Verum quoniam angulus OZT equalis angulo ZFC, et angulus OZT equalis angulo ZCF, erunt latera ZC, ZF equalia, et quia angulus FEZ rectus, quadratum FZ valet quadrata EZ,EF, et quadratum CZ valet quadrata EZ, EC. Igitur CE, FE equalia, et ita anguli ECF, EFC equales, quare anguli NEQ QEC equales. Et cum in eadem superficie sint C, E, N, refertur N ad C a puncto E.

86 *post* AZ *add.* et *C1*/cum . . . HZ *om. P1*/supra: super *FP1ER* 87 *post* recto *add.* quia AZH rectus est *R* 88 contingens: continens *O*/supra *om. R*/ZM: MZ *ER* 89 et *om. C1*/supra: super *R*/puncto[2]: punctum *R* 90 E que: eque *S*/producta *corr. ex* puncta *O*/concurret: concurrent *FP1*; concurrat *R* 92 TH . . . linee (93) *mg. O*/QE: QD *O* 93 linee *om. R*/LE: EL *R* 94 quoniam[1]: quod *R*/supra: super *R*; *om. L3*/quoniam[2]: que *L3*/*post* quoniam[2] *add.* AH *R* 95 supra[1]. . . contingens: super circulum per Z transeuntem et MZ super dyametrum illius circuli quia contingit *R*/contingens: continens *O* 96 *ante* igitur *scr. et del.* ig *S*/super: supra *S*/*post* AE *add.* et producatur QE ultra E hec concurret quidem cum axe concurrat in D *R* 97 LQD: LDQ *OE*; LE DQ *R* 98 piramidalis: piramidis *S*; piramidali *L3* 99 LE *corr. ex* L *a. m. E*/illa *om. S* 100 *post* ergo *add.* in illa superficie *R*/CF: PF *R*/QE: QB *C1*/quidem *om. R* 101 OZT: EZT *FP1L3E*; *corr. ex* EZT *O*; FZH *R*/*post* acutus *add.* erit *R*/sit *om. R* 102 equalem *om. O* 103 necessario: vere *S*; non *L3E*; *corr. ex* non *O*; *om. FP1*/FC: et *L3*; FP *R*/secet *om. L3*/C *inter. E*; P *R* 104 EC: PE *R*/CZ: CT *FP1SOL3C1E*; PZ *R*/OZ: Z *C1*/superficie: superficiei *O*/et . . . TZC (105) *scr. et del. E* 105 equalis: erit *E*/TZC: ZCF *E*; TZP *R*; *corr. ex* TZF *P1*/TZC . . . angulo (106) *om. L3*/punctus *om. R*/punctus . . . ZFC (106) *om. E*/O refertur: reflectetur O *R*/C: P *R*/*post* puncto *add.* speculi *R* 106 verum quoniam: et quia *R*/*post* equalis *add.* est *R*/ZFC: ZFP *R*/et . . . ZCF (107) *mg. C1*; *om. R* 107 OZT: ZT *FP1*; TZC *OC1*/ZC: ZP *R*/ZF: LZF *FP1* 108 rectus *rep. E*/FZ *scr. et del. E*/quadrata: quadratum *L3E*/EF: erit *S*/et *inter. C1* 109 CZ: PZ *R*/quadrata: quadratum *L3E*/EC: et *L3*; EP *R*/CE: PE *R*/anguli *om. R* 110 ECF: EST *S*; EOF *O*; EPF *R*/*post* ECF *scr. et del.* equ *S*/EFC: EFP *R*; EST *S*; *om. P1L3*/*post* EFC *add.* anguli erunt *R*/quare: quarum *L3*/anguli: angulus *L3E*/NEQ *corr. ex* NEC *P1*/QEC: QES *L3*; QEP *R*; *corr. ex* EC *O* 111 *post* sint *add.* que est *R*/C[1, 2]: P *R*/E[1]: O *F*/refertur: reflectetur *R*/C[2] *corr. ex* E *O*

[6.18] Similiter ducatur a puncto F quecumque linea ad aliquod punctum linee ZE, et producatur usque ad ON. Probabitur de puncto linee ON in quam cadit quod refertur ad C a puncto ZE, quoniam secat illa linea. Simili modo et omnium huiusmodi linearum probatio sumet initium a perpendiculari, que est FE, et a parte linee EZ, que erit terminus, et ita quodlibet punctum linee ON refertur ad C ab aliquo puncto linee EZ.

[6.19] **[PROPOSITIO 22]** Hoc ergo declarato, dicamus: cum visus comprehenderit lineas rectas transeuntes per caput speculi piramidalis convexi recti obliquas super axem speculi, in hoc speculo tunc forme earum erunt parum convexe.

[6.20] Sit ergo speculum piramidale erectum ABG [FIGURE 6.6.22, p. 316], cuius caput sit A, et cuius axis sit AD, et extrahamus in superficie eius lineam AZ, quocumque modo sit, in qua signetur punctum Z, quocumque modo sit. Et transeat per Z superficies equidistans basi piramidis, et faciat circulum ZU. Et extrahamus ex Z perpendicularem ZH super AZ. Hec ergo linea concurret cum axe piramidis, et concurrat ergo in H.

[6.21] Et extrahamus ex Z lineam contingentem circulum, et sit ZM, et extrahamus ex A lineam continentem cum utraque linea AZ, AH angulum acutum, et sit extra superficiem contingentem piramidem transeuntem per lineam AZ, et hoc possibile. Sit ergo AO, et extrahamus ex puncto H lineam in superficie in qua sunt AO, AH continentem cum AH angulum equalem angulo ZHA. Hec ergo linea concurret cum AO, nam duo anguli A, H sunt acuti. Concurrant ergo in O.

112 *post* similiter *add.* si *R*/a puncto F *mg. a. m. E*/a . . . linea: quecumque linea a puncto F *R*/aliquod: aliquem *S* 113 linee *om. R*/ZE *corr. ex* EZ *P1*/et . . . linee (114) *mg. a. m. E*/ON *corr. ex* EN *O*/probabitur: probabiliter *P1*; probababiliter *F* 114 *post* linee *scr. et del.* ZE et producatur *S*/ON: OM *S*/quam: quem *OE*; quod *R*/refertur: reflectetur *R*/C: P *R*/*post* puncto *add.* linee *R*/quoniam: quem *SC1*; quod *R* 115 huiusmodi: huius *L3* 116 sumet: sit MET *FP1*/initium: iniqum *L3* 117 terminus: communis *FP1*; communis omnibus illis triangulis *R*/quodlibet: licet *FP1*/*post* ON *scr. et del.* in quam cadit quod *C1*/refertur: reflectetur *R*/C: P *R* 118 aliquo: alio *SOL3E*/linee *inter. O*/*post* EZ *add.* et hoc est quod voluimus *C1*; *add.* que erit communis *E* 119 ergo *corr. ex* modo *E*; *om. R*/*post* dicamus *inter.* quod *O*/cum visus (120) *om. FP1* 120 caput: verticem *R* 121 recti *alter. in* erecti *a. m. C1*/super axem *rep. et del. E*/in hoc speculo *om. R* 123 piramidale: piramidalis *S*/ABG: ABC *R* 124 caput: vertex *R* 125 quocumque: quomodocumque *O*/in^{2}. . . sit (126) *om. P1*/quocumque modo: quomodocumque *FOC1* 128 linea *om. L3* 129 et *om. C1R*; *scr. et del. E* 130 circulum . . . contingentem (131) *rep. L3E* (ex A lineam [131]: lineam ex A *L3*/ZM^{2} [131] *corr. ex* MZ *E*) 131 ZM: MZ *FP1E*/A: HA *FP1* 132 AH: ad *P1*/sit *mg. C1*/contingentem: continentem *O* 133 *post* hoc *add.* est *R*/AO: AN *R*; *corr. ex* BA *S*/*post* AO *inter.* vel O *O*/et *mg. F*/extrahamus: extrahemus *L3* 134 ex: a *SC1*/H: B *E*/AO: AN *R*/continentem: continent *FP1*; contingentem *SL3* 135 angulo *om. C1*/ZHA: ZAH *L3*; AHZ *ER* 136 AO: AC *FP1*; HO *C1*/*post* anguli *add.* ad *R* 137 in . . . ergo (138) *om. S*

[6.22] Linea ergo HO concurret cum circumferentia circuli ZU, nam angulus AHO est equalis angulo AHZ. Concurrat ergo in U, et extrahamus AU recte. Et extrahamus perpendicularem HZ ad T, et continuemus OZ, et extrahatur recte ad F, et extrahatur AZ ad E. Angulus ergo FZH erit acutus, quia linea OZ secat superficiem contingentem piramidem transeuntem per AZ. Linea ergo FZ est sub differentia communi inter superficiem OZH et superficiem contingentem, et hec differentia continet cum linea HZ angulum rectum. Angulus ergo OZH est obtusus; ergo angulus FZH est acutus.

[6.23] Ponatur ergo in ZF punctus F a quo extrahatur perpendicularis FE super AE, et extrahatur recte. Concurret ergo cum linea AO, nam angulus OAE est acutus. Concurrat ergo in N, et extrahatur ex E linea ED equidistans ZH linee. Erit ergo ED perpendicularis super superficiem contingentem piramidem transeuntem per AE.

[6.24] Et extrahatur ex E linea equidistans linee ZM, et sit EL, et extrahatur superficies in qua sunt LE, ED. Secabit ergo superficiem piramidis et faciet sectorem, nam hec superficies est obliqua super axem AD.

[6.25] Sit ergo sector BEG. Et MZ est perpendicularis super superficiem AZH, et hoc declaratum est in predictis. Ergo linea LE est perpendicularis super superficiem AED; ergo angulus AEL est rectus, et angulus AEN est rectus, et similiter angulus AED est rectus. Ergo linee LE, NE, DE sunt in eadem superficie. Ergo linea FEN est in superficie sectoris.

[6.26] Et extrahatur ex F linea equidistans linee DE, et sit FR. Hec ergo linea equidistabit linee HZ. Et extrahatur ex Z in superficie OZH linea continens cum ZT angulum equalem angulo OZT. Hec ergo linea concurret cum FR, quia secabit ZH equidistantem FR, et est in superficie eius, quia ZF est in superficie eius. Concurrat ergo in R.

[6.27] Ergo duo anguli qui sunt apud R, F sunt equales, sunt enim equales duobus angulis qui sunt apud Z. Due ergo linee RZ, FZ sunt

139 nam: linea *L3E* / AHO *corr. ex* HO *O* 140 T et (141) *om. S* 141 et[1] *inter. C1* / extrahatur[1]: extrahamus *R* 143 FZ: AZ *R* 144 OZH: OZA *S* 146 OZH: EZH *L3ER* / est[1] *om. L3ER* / est[2] *om. ER* 147 in ZF: MZF *P1* / punctus: punctum *SOL3C1ER* 148 AE: A *L3* / *post* recte *add.* ON *FP1* / concurret: concurrat *L3C1E* / concurret . . . acutus (149) *om. S* 149 OAE: AOE *FP1* 150 super *mg. a. m. E* 152 *post* extrahatur *scr. et del.* superficies in qua sunt LE ED secabit ergo superficiem *S* / EL: EB *L3E* 153 *post* sunt *add.* linee *R* 154 piramidis: piramidem *S* / faciet *corr. ex* faciat *a. m. E* / sectorem: sectionem *R* 156 sector: sectio *R* / BEG: REG *S*; DEC *R* / est *om. S* 158 AED: ADE *C1* 159 et[1]. . . rectus[1] *om. P1R* / et[1]. . . rectus[2] *mg. a. m. E* / AEN: LEN *SL3*; AEL *E* / est rectus[2] *transp. R* / *ante* ergo *add.* et AEN similiter rectus *R* 161 sectoris: sectionis *R* 162 FR: FFE *S* 163 equidistabit: equidistat *R* / OZH: EZH *FP1* 164 hec ergo *transp. C1* 165 quia *alter. in* et *a. m. E* / secabit: secat *R* / equidistantem: equidistanter *E* / FR: FRA *FP1*; FA *S*; FK *L3* 166 quia . . . eius[2] *om. P1*; *mg. C1* / ZF: FZ *C1* / R: FE *S*; K *L3* 167 *post* sunt[1] *scr. et del.* ZF *C1* / R F: SEF *S* / sunt[2]. . . equales (169) *mg. O* 168 *post* sunt[1] *scr. et del.* i *P1* / RZ: KZ *E*; *corr. ex* RI *L3*

equales. Et declaratum est quod linea FEN est in superficie sectoris, et linea FR est equidistans linee ED. Est ergo in superficie sectoris.

[6.28] Et continuemus RE. Erit ergo in superficie sectoris. Et extrahatur DE ad K, et declaratum est quod EA est perpendicularis super superficiem sectoris. Uterque ergo angulus AER, AEF est rectus, et due linee FZ, RZ sunt equales. Ergo due linee RE, FE sunt equales; ergo duo anguli ERF, EFR sunt equales.

[6.29] Ergo forma N convertetur ad R ex E, et forma O convertetur ad R ex Z. Et omnis linea extracta ex F ad aliquod punctum linee AN secabit AE. Et patet quod illa linea erit equalis linee extracte ex R, nam AE est perpendicularis super superficiem in qua sunt linee RE, FE, nam hec superficies est superficies sectoris, et due linee RE, FE sunt equales. Ergo omnes due linee extracte ex R, F ad aliquod punctum linee AE sunt equales.

[6.30] Patet ergo quod forma puncti quod est in AN convertetur ad R ex illo puncto quod est in AE. Et similiter de omni puncto posito in AN ultra N, si copulatum fuerit cum F, et per lineam rectam, illa linea secabit AE ultra E. Et patet quod forma puncti quod est in AN convertetur ad R ex puncto in AE. Patet ergo ex hoc quod forma linee AN, et quicquid continuatur cum ipsa, convertetur ad R a superficie piramidis ABG ex linea recta, et similiter omnis linea extracta ex A oblique super axem piramidis.

[6.31] Et continuemus ND. Secabit ergo circumferentiam sectoris, nam duo puncta N, D sunt in superficie sectoris, et N est extra sectorem, et D est intra sectorem. Secet igitur circumferentiam sectoris in C, et quia triangulus AOH est in eadem superficie, erit ND in superficie trianguli AOH.

169 quod: quia *C1*/FEN: FN *L3*/sectoris: sectionis *R* 170 linea: linee *S*/linee *om. R*/sectoris: sectionis *R* 171 et[1] *om. FP1*/et[1]. . . sectoris *om. L3*/continuemus: continueremus *FP1*/RE: SE *S*; KE *E*/sectoris: sectionis *R* 172 quod: quia *C1*/EA *om. FP1* 173 sectoris: sectionis *R*/uterque: utrique *L3E*/angulus: angulo *L3E*; angulorum *R*/*post* AEF *inter.* vel D *O*/est rectus *transp. R* 174 FZ RZ *transp. O*/RZ: IZ *FP1*; RBZ *S*/ergo . . . equales[2] *om. FP1*; *mg. a. m. E* 175 ergo . . . equales *om. P1*/EFR: EFK *E* 176 N: M *E*/convertetur[1, 2]: reflectetur *R*/O: D *O* 177 R: FI *FP1*/ex[2]: a *L3*/ad[2] *inter. O*/aliquod: O ad *FP1*; *mg. a. m. E* 178 AN: ON *R*/quod: quia *C1*/illa linea *transp. L3ER* 179 R *corr. ex* ER *FP1*; F *O*; K *L3E*/*post* R *add.* ad idem punctum scilicet AE ad quod extrahitur linea a puncto F *C1*; *add.* ad idem punctum *R*/nam: namque *E*/super *inter. a. m. E* 180 *post* FE *scr. et del.* sunt equales *C1*/est superficies *mg. F*/sectoris: sectionis *R*/RE[2]. . . linee (181) *mg. O* 181 linee: ? *O*/*post* ad *add.* unum *R* 183 quod[1]: quia *C1*/AN: ON *R*/convertetur: reflectetur *R* 184 R: N *L3*; K *E*/est in AE: secatur in ZE *R*/posito *om. P1* 185 N: QN *FP1*; quod *O*; TI *L3*/et *om. R* 186 et . . . AE (187) *om. R* 188 AN *om. L3*/ipsa *inter. E*/convertetur: reflectetur *R*/superficie *corr. ex* superficicie *F* 189 ABG: AB *FP1*; ABH *OL3E* (*deinde inter.* vel G in arabico *O*); ABHT *S*/*post* ex[2] *scr. et del.* ea *F*/A *corr. ex* ea *P1* 190 oblique: obliqua *R*/piramidis *om. L3ER* 191 et: si *FP1*/continuemus: continuemuemus *S*/sectoris: sectionis *R* 192 N D: Z E *FP1SL3C1E*; Z D *O*; *transp. R*/sectoris: sectionis *R*/et: Z *S*/N: enim *P1*/*post* extra *add.* circumferentiam *R*/sectorem: sectionis *R* 193 et: SZ *FP1*/sectorem: sectionem *R*/sectoris: sectionis *R* 194 triangulus: triangulum *R* 195 AOH: ACH *L3*/*post* AOH *scr. et del.* et duo puncta A U sunt *S*

[6.32] C ergo est in superficie trianguli AOH, et duo puncta A, N sunt in superficie huius trianguli. Ergo puncta A, N, C sunt in superficie trianguli AOH. Sed puncta A, U, C sunt in superficie piramidis. Ergo puncta A, U, C sunt in differentia communi superficiei piramidis et superficiei AND. Sed hec differentia est linea recta. Ergo puncta A, U, C sunt in linea recta.

[6.33] Extrahatur ergo AU recte ad C, et extrahatur RZ recte. Secabit ergo OH. Secet ergo in P. P ergo est in superficie trianguli AOH. Continuetur ergo AP, et transeat recte. Secabit ergo ND in G, et quia F est sub superficie contingente piramidem transeunte per lineam AZE, erit angulus FED acutus, et angulus DEN est obtusus. Ergo angulus ENC est acutus.

[6.34] Et sit linea CZ contingens sectorem. Patet ergo, ut in figura predicta, quod angulus DCZ est obtusus et quod perpendicularis extracta ex C super CZ secat angulum DCZ, et concurret cum ED sub D. Hec ergo perpendicularis secabit ED in S.

[6.35] Perpendicularis ergo extracta ex N super lineam contingentem sectorem secabit sectorem ultra C, scilicet, remotius ex E quam C, nam iste perpendiculares concurrent ultra circumferentiam sectoris. Perpendicularis ergo extracta ex N super lineam contingentem sectorem non secabit angulum DCZ. Erit ergo remotior ex NE quam CD, et hec perpendicularis secat ED sub D.

[6.36] Sit ergo perpendicularis extracta ex N super lineam contingentem sectorem linea NQ. Et RE secat EN, et secat circumferentiam sectoris, et est in superficie eius, et NQ est in superficie sectoris. Si ergo RE extrahatur recte, secabit NQ. Secet ergo in Y.

196 C: S *FP1L3*/AOH: ACH *FP1*/N: O *O*; U *R* 197 huius trianguli *transp. L3ER*/*post* trianguli *add.* AOH *R*/ergo ... superficie[2] *om. E*/ergo ... AOH (198) *om. R* 198 AOH: ACH *FP1S*/sed ... C *mg. a. m. E* 199 ergo ... piramidis *mg. a. m. E* 200 AND: AUD *ER* (*alter. in E*)/*post* hec *scr. et del.* est *E* 201 U C *corr. ex* N T *a. m. E* 202 *post* extrahatur *scr. et del.* ergo AN recte *S*/RZ: X *FP1*; RR *deinde inter.* vel Z in arabico *O*; TZ *L3*; KZ *E* 203 OH: H *L3E*/ergo[2] *om. O*/*post* in[1] *add.* puncto *ER*/P[2] *inter. O*; *om. L3E*/P ergo est: est ergo P *R*/AOH *om. S*; ACH *L3E* 204 continuetur *corr. ex* continetur *C1*/ND: AND *O*/G: H *OL3E* 205 est sub: non est in *R*/sub: in *L3*/*post* superficie *scr. et del.* trianguli *P1*/contingente: continente *P1*/contingente piramidem *transp. R*/transeunte: transeuntem *E* 206 AZE: AZ *R*; *corr. ex* AE *C1* 207 ENC: EQC *O*; END *R* 208 CZ: Z *FP1*; CX *R*/sectorem: sectionem in puncto C *R*/in *inter. C1*/figura predicta (209) *transp. R* 209 DCZ: DCX *R*/est obtusus *mg. C1*/quod *om. FP1* 210 CZ: CX *R*; *corr. ex* Z *O*/secat: secabit *ER*; *corr. ex* secet *C1*/DCZ: DCX *R*/ED: AD *FP1SL3C1E*/*post* ED *add.* et *L3*/D *corr. ex* DH *E* 211 hec *om. FP1*; *inter. a. m. E*/hec ergo *transp. ER*/secabit: secet *R*/ED: AD *FP1SOL3E*; *corr. ex* AD *C1*/S: Z *FP1*; C *O* 212 N *corr. ex* A *S*/contingentem: continentem *O* 213 sectorem[1, 2]: sectionem *R*/secabit *rep. FP1*/sectorem[2] *mg. F*/C: S *FP1SL3C1ER*/scilicet *om. S*; sed *ER*/ex E: a D *R*/E: eo *S* 214 C: S *FP1SL3C1ER*/sectoris: sectionis *R* 215 perpendicularis: perpendiculares *L3*/*post* ex *add.* puncto *R*/contingentem: continentem *OE* 216 sectorem: sectionem *R*/DCZ: CZG *FP1O*; CZH *SL3C1E*; DCX *R*/ex: ab *R*/*post* quam *add.* sit *R* 217 CD: ND *R*/et: ergo *R*/ED: AD *FP1SL3ER*; *corr. ex* AD *C1*/sub: supra *R* 218 *post* super *scr. et del.* perpendicularem *C1*/contingentem: continentem *O* 219 sectorem: sectionem *R*/RE: KE *E*/secat[1] *alter. in* secet *E*/EN: EQ *O* 220 sectoris[1, 2]: sectionis *R*/eius *inter. O*/et[2] *inter. a. m. E* 221 RE *corr. ex* EY *a. m. E*/ergo *om. S*

[6.37] Et superficies AND secat superficiem sectoris. Quia punctum E est extra superficiem AND, nam superficies AND non est superficies sectoris, A enim est extra superficiem sectoris, quia AE est perpendicularis super superficiem sectoris, et E est in circumferentia illius, ergo ND est differentia communis superficiei AND et superficiei sectoris, et NQ concurret cum sectore ultra C. Ergo NQ est ultra superficiem AND. Y ergo est ultra lineam APG.

[6.38] Si ergo visus fuerit in R, et linea AON fuerit in aliquo visibili, tunc P erit ymago O, et Y erit ymago N, et A videbitur in suo loco, quia est in capite piramidis. Et erit ymago linee AON linea transiens per puncta A, P, Y, sed hec linea est convexa, quia est ultra APG.

[6.39] Sit ergo illa linea APY, et patuit iam quod forme omnium punctorum que sunt in AN convertuntur ad R ex AE. Linee ergo radiales per quas convertuntur ille forme sunt in superficie trianguli RZE; omnes ergo ymagines linee AN sunt in hac superficie.

[6.40] Ergo linea APY convexa est in hac superficie, et P est propinquius ad R quam Y, et erit convexitas huius ymaginis ex parte visus, et erit convexitas parva. Et dyameter huius ymaginis erit minor ipsa linea modica quantitate. Ymagines ergo linearum rectarum que extrahuntur ex capite piramidali oblique super axem comprehenduntur a visu in tali speculo convexe, et forme harum linearum convertuntur a lineis rectis ex lineis extensis in longitudine piramidis, et hoc est quod voluimus declarare.

[6.41] Forme vero linearum equidistantium latitudini speculi piramidalis convexi convertuntur a lineis convexis in superficie speculi, et convexitas harum linearum patet ut in columpnali speculo con-

222 et: in *S*/superficies *corr. ex* superficiess *F*/secat: secabit *ER*; *corr. ex* secet *C1*/sectoris: sectionis *R*/*ante* quia *add.* item *R* 223 est[1] *om. S*/*post* extra *scr. et del.* civitatem *S*/AND[1] *corr. ex* AMD *P1* 224 sectoris[1, 2]: sectionis *R*/*ante* A *add.* quia punctum *R*/A . . . sectoris[2] *inter. a. m. E*/enim *om. R*/est[1] *om. P1E*/*post* superficiem *add.* corporis *P1*/*ante* quia *add.* et *R* 225 sectoris: sectionis *R*/et: Z *L3E*/in *inter. C1* 226 ND: NCD *R* 227 sectoris: sectionis *R*/NQ: NRQ *FP1*; LQ *L3E*; *corr. ex* Q *a. m. C1*/concurret: concurrit *R*/sectore: sectione *R* 228 Y *om. P1*/*post* ultra *add.* C ergo NQ *F*/APG: APH *FP1SOL3E* 229 linea . . . visibili: forma alicuius visibilis reflectatur a linea longitudinis *R*/AON: longitudinis *FP1L3C1E*; TO *S*; LON *O* 230 O *om. FP1SOL3E*; *inter. a. m. C1*/A *om. FP1SOL3E*; *inter. a. m. C1*/suo loco *transp. C1*/*post* loco *scr. et del.* lo *C1* 231 in *inter. O*/capite: vertice *R* 232 *post* ultra *add.* lineam *R*/APG: APH *FP1SOL3C1E* 233 illa linea *transp. C1*/iam quod *transp. P1* 234 convertuntur: concurruntur *O*; reflectantur *R*/R: K *L3E* 235 convertuntur: reflectuntur *R* 236 RZE: KZE *L3E*; RAE *R*/ergo *inter. O* 237 ergo . . . superficie *om. S*/et: Z *FP1*/propinquius: propinqus *P1* 238 ad *om. R*/R: K *L3E*/ex . . . ymaginis (239) *rep. F*/huius ymaginis *transp. R* 239 *post* ymaginis *add.* ex parte visus *P1S* 241 capite piramidali: parte piramidis *O*; vertice piramidis *R* 242 convexe *inter. a. m. E*/convertuntur: reflectuntur *R* 243 a lineis: alienenis *S*/rectis *inter. a. m. E*/ex lineis *om. R*/piramidis: piramidalis *O* 244 voluimus: volumus *P1L3C1*/declarare: dicere *FP1* 245 equidistantium *om. FP1*/latitudini: iam *L3* 246 convexi *corr. ex* convexitas *S*/convertuntur: reflectuntur *R*/a lineis: alienis *SE* 247 *post* convexitas *add.* formarum *FP1*/ut *om. S*/columpnali: columpnari *C1R*/columpnali speculo *transp. R*/convexo *om. P1*

vexo, et per illam eandem viam, et patebit similiter quod ymagines harum linearum erunt nimium convexe et manifeste sensui. Et erit centrum visus extra superficies in quibus est convexitas formarum harum linearum, et erunt dyametri ymaginum harum linearum multum minores ipsis lineis.

[6.42] De lineis vero obliquis existentibus inter hos duos modos, que appropinquant in suo situ lineis extensis in longitudine piramidis habent formas parum convexas, que vero appropinquant lineis equidistantibus latitudini piramidis habent formas manifeste convexas.

[6.43] Sed tamen linee tortuose que appropinquant capiti piramidis habent formas minores, et strictiores, et convexiores, que vero appropinquant basi piramidis habent formas ampliores propter illud quod declaratum fuit in speculis spericis convexis—scilicet quod quanto minus fuerit speculum tanto minores erunt circuli qui cadunt in superficie eius—et sic ymagines erunt propinquiores centro, ideo ergo erunt minores.

[6.44] Et similiter sectores qui cadunt in speculo piramidali qui sunt ex parte capitis piramidis sunt strictiores et minores, et sic ymago erit propinquior puncto in quo concurrunt perpendiculares exeuntes a linea visibili perpendiculariter super lineas contingentes sectores que sunt differentie communes, et ideo iste ymagines erunt minores.

[6.45] Sectores vero qui sunt ex parte basis piramidis econverso, unde accidit quod forma comprehensa in speculo piramidali convexo erit piramidata, quod scilicet fuerit ex parte capitis speculi erit strictius, et quod ex parte basis erit amplius, et convexitas latitudinis forme erit manifesta.

[6.46] Et accidit etiam in hiis speculis quod quanto magis res visa appropinquaverit speculo videbitur maior, et quanto magis erit remota videbitur minor.

248 illam eandem *transp. P1*/viam *om. P1*/et: etiam *L3ER*/patebit *om. E*/patebit similiter *transp. R* 249 nimium *corr. ex* in unum *E* 250 formarum harum (251) *transp. O* 251 et . . . linearum[2] *mg. a. m. E*/multum: multo *R* 253 lineis: linei *L3*/lineis vero obliquis: obliquis vero lineis *C1*/*post* existentibus *add.* et *O* 254 suo situ *transp. L3*/situ: motu *R*; *corr. ex* motu *a. m. E* 255 habent: habens *FP1*/habens . . . piramidis (256) *om. L3* 256 *post* equidistantibus *scr. et del.* inter hos duos modos *C1*/habent: habens *P1S* 258 capiti: vertici *R* 259 vero *om. FP1* 261 quod[2] *om. L3*/speculis: speculus *R* 263 superficie: superficiem *R*/*post* propinquiores *scr. et del.* circulo *C1*/ideo: idcirco *R* 264 ergo *om. R*/ergo erunt *transp. C1*/erunt *inter. a. m. E* 265 sectores: sectiones *R*/qui[1, 2]: que *R*/speculo piramidali: speculum piramidale *R* 266 capitis: verticis *R*/piramidis: piramidalis *L3* 268 exeuntes: exeunte *L3* 269 sectores: sectiones *R*/sectores . . . minores (270) *rep. et del. F* 271 sectores: sectiones *R*/qui: que *R*/piramidis: piramidalis *L3*/*post* piramidis *inter.* erunt *O*/econverso: econtrario *R* 272 unde: non *E*/quod: ut *R*/piramidali: piramidalis *SL3* 273 erit[1]: sit *R*/capitis: verticis *R*/erit[2] *inter. a. m. E* 274 strictius *corr. ex* strictus *C1* 276 etiam *om. O*/quanto: quando *C1* 277 *post* speculo *add.* tanto *R*/et . . . minor (278) *rep. et del. E*/*post* magis *scr. et del.* res visa *S* 278 *ante* videbitur *add.* tanto *R*

[6.47] Fallacie ergo que accidunt in huiusmodi speculis sunt similes in omnibus dispositionibus illis que accidunt in speculis columpnalibus convexis preter quam in piramidatione forme. Et omnino forma rei vise que comprehenditur per conversionem semper assimulabitur forme superficiei speculi a qua convertitur forma, et huius causa est quod semper locus ymaginis constituitur ex forma superficiei speculi et ex loco concursus perpendicularium, ideo semper superficies speculi habet aliquam dignitatem in forma rei vise que comprehenditur in speculo. Fallacie vero composite in hoc speculo similes sunt fallaciis in speculis predictis.

CAPITULUM SEPTIMUM
De fallaciis que accidunt in speculis spericis concavis

[7.1] In hiis vero plures accidunt quam in omnibus speculis convexis et superficialibus, accidit enim in eis que in illis accidunt—scilicet debilitas lucis et coloris et diversitas situs et remotionis—nam causa huius est tantum conversio, non forma speculi. Accidit etiam in hiis speculis ex diversitate quantitatis plus quam in speculis convexis, nam in convexis, in maiori parte, res comprehendetur minor, in concavis vero quandoque comprehendetur maior, quandoque minor, quandoque secundum quod est, et hoc secundum diversitatem positionum eius ex speculo et ex visu, prout nos declarabimus in hoc capitulo.

[7.2] Accidit etiam in hiis speculis quod unum visibile videatur duo, et tria, et quattuor, et non est ita in speculis superficialibus et convexis, unum enim visibile non comprehenditur in illis nisi unum, in concavis vero non.

279 huiusmodi: hiis *L3ER* 280 columpnalibus: columpnaribus *L3C1ER* 281 convexis: convexa *L3*/preter: per *S* 282 comprehenditur: comprehenderunt *P1*/conversionem: reflexionem *R*/assimulabitur: assimilabitur *SER* 283 convertitur: reflectitur *R* 284 constituitur: est *R* 285 concursus: conversus *L3E*/perpendicularium *corr. ex* perpendiculariter *F* 286 speculi *om. C1* 288 in . . . predictis: predictis in speculis *L3E*/speculis predictis *transp. SR* 289 capitulum . . . concavis (290) *om. FP1S*; *mg. a. m. E*; de erroribus qui accidunt in speculis spericis concavis capitulum septimum *R* 291 *post* vero *add.* lineis *L3*/*post* plures *add.* errores *R* 292 superficialibus: superficiebus *L3E*/enim *om. L3E*/que: qui *O*/in^{2} *om. O*/illis: illum *O* 293 remotionis: remotius *SL3* 294 conversio: reflexio *R*/etiam *inter. O* 295 *post* plus *add.* erroris *R* 296 nam in convexis *om. L3*/convexis *corr. ex* vexis *O*; *corr. ex* concavis *C1*/maiori: maiore *R*/parte *corr. ex* partes *C1*/comprehendetur: comprehendere *FP1*; comprehenderetur *S*; comprehenditur *R* 297 *post* concavis *add.* q *P1*/quandoque: quando *L3*/comprehendetur: comprehenditur *R*/*post* comprehendetur *scr. et del.* ma *F* 298 quod *rep. et del. F*/*post* est *scr. et del`.* secundita *S*; *add.* quandoque *FP1*/et hoc *om. SO* 299 eius *om. R*/nos *om. FP1*/declarabimus: declaravimus *O* 1 videatur: videtur *ER* 2 est *om. FP1*/et^{4} *om. L3* 3 comprehenditur: comprehendetur *E*

[7.3] Item ordinatio partium rei vise comprehenditur in speculis convexis et superficialibus secundum quod est; in spericis vero concavis in pluribus sitibus alio modo, et hec duo: scilicet comprehensio unius ut unius et comprehensio ordinationis partium secundum quod est, non habent aliquam deceptionem in speculis convexis spericis, et cum in hiis accidit deceptio in speculis spericis concavis, patet quod nichil comprehenditur in huiusmodi speculis nisi cum fallacia, aut semper, aut in aliqua hora secundum diversitatem positionis.

[7.4] Debilitas vero lucis et coloris et diversitas positionis et distantia accidunt in hiis speculis sicut in aliis semper, et in omni positione. Quantitas vero, et forma, et numerus habent deceptiones in hiis speculis in aliquibus sitibus, prout declarabimus.

[7.5] De numero vero declaratum est in capitulo de ymagine quod unum visum in speculis spericis concavis habet unam ymaginem, et duas, et tres, et quattuor, et quod forma rei vise semper comprehenditur in loco ymaginis. Verum unum visum comprehensum in speculis spericis et concavis forte comprehendetur unum, et forte duo, et forte tria, et forte quattuor, quod non accidit in speculis convexis et superficialibus.

[7.6] De ordinatione vero partium rei vise dictum est etiam in capitulo de ymagine quod forma unius puncti convertitur ex circumferentia circuli, et quod visibilia quorum ymagines sunt retro vel post visum, et ante, et in centro visus apparent dubia, non certificata, et quod est huiusmodi non habet ordinationem partium sicut ipsa res visa habet. Et hoc etiam est in hiis speculis aliter quam sit in speculis convexis et superficialibus. Cause autem huius rei declarate sunt in capitulo de ymagine.

[7.7] Restat ergo declarare quod illud quod comprehenditur in hiis speculis forte comprehendetur maius, et forte minus, et forte equale,

5 *post* speculis *scr. et del.* spericis *S* 6 *post* convexis *scr. et del.* i *F* / *post* et *add.* in *FP1* / superficialibus *corr. ex* superficiebus *E* / spericis: speculis *L3ER* / vero *om. C1* 7 scilicet *inter. E* 8 ut *corr. ex* non *S* / unius[2]: unis *L3*; unum *ER* 9 habent: habet *ER* / convexis spericis *transp. R* 10 in[1] *inter. E* / accidit . . . concavis: speculis spericis concavis accidit deceptio *R* / patet . . . speculis (11) *rep. P1* 11 huiusmodi: huius *L3* / *post* speculis *add.* concavis *P1* 12 semper: super *S* / in *om. L3ER* / diversitatem *om. FP1* / positionis: reflexionis *F*; reflexionem *P1* 13 lucis *corr. ex* luci *C1* 15 et[2] *om. L3* / deceptiones: deceptionem *L3ER* 16 sitibus *om. L3* 17 in *om. P1* 18 spericis *om. ER* / habet: habent *S* 19 quod *inter. O* 20 *post* verum *inter.* vel unde *O* / comprehensum . . . concavis (21) *rep. P1* 21 et[1] *om. OR* / *post* concavis *add.* et *E*; *add.* etiam *R* / comprehendetur: comprehenditur *R* / unum *inter. O* 22 *post* forte *scr. et del.* accidit *C1* / *post* speculis *add.* spericis *R* / et[2] *mg. a. m. E* 24 vero *om. L3* / *post* vise *add.* ut *L3E*; *scr. et del.* ut *C1* / etiam *om. FP1OR* 25 quod: et *S* / puncti *rep. et del. F* / convertitur: reflectitur *R* 26 *ante* circuli *add.* unius *R* / *post* quorum *scr. et del.* or *C1* / *post* retro *inter.* vel *O* / post *inter. O* 28 huiusmodi: huius *FP1L3* 29 aliter . . . speculis *mg. a. m. E* / *post* speculis *scr. et del.* concavis *F* 30 huius: huiusmodi *C1* 32 declarare: determinare *P1* 33 comprehendetur: comprehenditur *ER* / maius *corr. ex* maior *P1*

et quod in quibusdam positionibus comprehendetur conversum, et in quibusdam erectum, et quod rectum in huiusmodi speculis comprehenditur concavum et convexum et rectum, et quod convexum et concavum comprehenduntur etiam aliter quam sint. Et hec etiam sunt ex diversitate ordinationis partium rei vise, et nos declarabimus hoc hoc modo.

[7.8] **[PROPOSITIO 23]** Sit ergo speculum spericum concavum in centro A [FIGURE 6.7.23, p. 317], et secetur superficie equali transeunte per centrum, et faciat circulum BG, et extrahatur in ipsa linea, quocumque modo sit, et dividatur in duo equalia in O.

[7.9] Et ponatur A centrum, et in distantia AO faciamus circulum, et sit EZ. Et ponatur in linea OU punctum T casualiter, quocumque modo sit, et ex T extrahantur linee TN, TM recte super lineam AU. Et extrahantur ex T linee TE, TZ tangentes circulum EZ, et continuemus AE, AZ, et transeant ad B, G. Et continuemus TB, TG, et extrahamus BM equidistantem ad AT, et GN etiam equidistantem AT, et continuemus AM, AN et extrahantur recte.

[7.10] Quia ergo AO est sicut OU, erit AE sicut EB, et AZ sicut ZG, et quia TE tangit circulum EZ, erit TE perpendicularis super BA, et similiter TZ perpendicularis super AG. Linea ergo BT est sicut TA, et TG sicut TA, et angulus TBA sicut angulus TAB, et angulus TGA sicut angulus TAG. Et quia BM est equidistans AT, erit angulus MBA sicut angulus BAT. Ergo angulus MBA est sicut angulus ABT, et similiter angulus TGA est sicut angulus AGN.

[7.11] Cum ergo visus fuerit in T, et M, N fuerit in aliquo visibili, tunc forma M extendetur per lineam MB et convertetur per BT, et

34 comprehendetur: comprehendentur *OL3*; *corr. ex* comprehenderetur *C1* 35 rectum: erectum *ER*/huiusmodi: huius *OL3*/comprehenditur: comprehendetur *ER* 36 convexum[1]: conversum *L3*/et rectum *om. P1*/et . . . convexum[2] *mg. O* 37 *post* et[1] *add.* quod *FP1*/aliter: alter *P1* 38 sunt *corr. ex* sint *S* 39 hoc[1]: hec *C1R* 40 ergo: vero *C1*; *om. R*/concavum *inter. O* 41 in centro: cuius centrum *R*/equali: plana *R*/*post* equali *add.* et *P1* 42 faciat: fiat *P1*/et[2] *om. R*/in ipsa: ab ipsius centro *R* 43 quocumque modo: quomodocumque *O*/modo sit *om. S*/in[1]: inter *P1*/in O *om. R* 44 in *om. FP1L3*/AO: AD *L3* 45 EZ: EX *FP1*/OU: TU *FP1S*; TN *L3C1E* (*inter. E*)/quocumque: quandoque *FP1*; quandocumque *S*; quomodocumque *O* 46 modo *om. FP1SOC1*/*post* ex *scr. et del.* TE *S*/TN: IN *S*/TN . . . linee (47) *inter. O*/TM: tantum *FP1*; IM *S* 47 *post* ex *scr. et del.* TE *S*/TE *corr. ex* T *F*; *om. L3*/TZ: ZZ *FP1*; *corr. ex* DZ *E*/tangentes: tanges *S*/*post* tangentes *scr. et del.* lineam *C1* 48 *post* continuemus *scr. et del.* in figura cave G et H sunt similia in arabico [sunt *inter.*] *O*/TG: BG *R*/extrahamus: protrahamus *R* 49 equidistantem[1]. . . etiam *om. S*/AT[1, 2]: AU *R*/etiam: et *FP1C1*/equidistantem[2]: equidistans *S* 50 AM AN *transp. R* 51 ergo *om. FP1*/AO: OA *FP1*/OU: OF *O*; *corr. ex* OB *E*/AE: AB *S*/sicut[3]: sunt *L3*; *rep. et del. E* 52 BA: AB *R* 53 est *om. P1*/et TG (54) *rep. P1* 54 TA: TQ *O*/TGA *corr. ex* TAG *E* 55 angulus[1] *om. FP1*/TAG: TAH *deinde inter.* G in arabico *O*/AT: AU *R*/angulus[2] *om. R* 57 angulus[1] *inter. a. m. E*/TGA: TGAG *F*; *alter. ex* TGIAG *in* TGAG *P1*; THA *deinde inter.* G in arabico *O*; *corr. ex* TAG *E*/est *om. R*/AGN: AGNG *FP1*; AHN *deinde inter.* G in arabico *O* 58 N: B *FP1SOL3ER*/fuerit . . . MB (59) *om. S*/in . . . visibili: aliquod visibile *R* 59 extendetur *corr. ex* extenditur *a. m. E*/convertetur: reflectetur ad visum *R*/*post* per *add.* lineam *R*/et[2]. . . GT (60) *scr. et del. E*

forma N extendetur per NG et convertetur per GT. Visus ergo T comprehendet puncta M, N ex punctis B, G, et lineam MN ex arcu BG.

[7.12] Et quia TE est perpendicularis super AB, erit angulus ABT acutus. Sed angulus MBA est sicut angulus ABT. Ergo TB est maior BM; ergo AT est maior BM, et sunt equidistantes. Ergo TB concurret cum AM. Concurrant ergo in F. F ergo est ymago M, et sic declarabitur quod TG concurret cum AN. Concurrat ergo in Q. Q ergo erit ymago N.

[7.13] Et continuemus FQ, que est dyameter ymaginis MN, et quia TE, TZ sunt equales, erunt anguli TAB, TAZ equales, et erunt linee TB, TG equales, et linee BM, GN equales, et linee AM, AN equales. Et proportio AF ad FM sicut proportio AT ad BM, et proportio AF ad FM est sicut proportio AT ad GN, et proportio AT ad BM; ergo proportio AF ad FM est sicut proportio AQ ad QN, et AM est sicut AN. Ergo AF est sicut AQ; ergo FQ equidistat MN. Ergo FQ est maior MN. Sed FQ est dyameter ymaginis MN. Ergo si visus fuerit in T et MN fuerit in aliquo visibili, tunc visus comprehendet formam maiorem quam sit.

[7.14] **[PROPOSITIO 24]** Item iteremus circulum BG [FIGURE 6.7.24, p. 317], et lineam AT, et lineas AB, AG, TB. Et super punctum T sit perpendicularis super superficiem circuli BG, et sit TK, et continuemus KA, KB, KG. Superficies ergo KBA, KGA sunt secantes speram super suum centrum perpendiculariter super superficies tangentes ipsam. Ex ipsis ergo convertitur forma, et due differentie communes inter has duas superficies et speram sunt circuli magni a quorum circumferentia convertuntur forme.

60 N: B *FP1SOL3E*; *corr. ex* B *C1*/*post* per[1] *add.* lineam *R*/NG: ZH *deinde inter.* G in arabico *O*/ convertetur: reflectetur *R*/GT: HT *deinde inter.* G in arabico *O* 61 *post* N *add.* et *L3*/*post* B *add.* H *FP1S*/G *corr. ex* H *O*/BG: BHG *S*; *corr. ex* BH *O* 62 TE: TG *S*; MB *O*; MT *R*/AB: AT *OR*/ABT: MTB *R* 63 *post* acutus *add.* ergo angulus BLT est obtusus *O*/sed: et quia *R*/MBA *corr. ex* MAB *E*; BMT *R*/ABT: MTU *R* 64 *post* BM[1] *add.* et linea TB est equalis linee AT *R*/ ergo[1]. . . BM[2] *om. P1E*/*post* ergo[1] *add.* linea *R*/*post* maior *add.* linea *R*/ergo[2]: OG *S* 65 concurrant: concurrent *FP1*; concurram *L3*/F[2] *inter. O* 66 quod *om. L3*/TG: THG *FP1S*; *corr. ex* TH *O*; *corr. ex* GT *E*/concurret: concurrat *L3*/AN: AZN *FP1*; AZH *S*; AZ *O*/ergo[1] *om. E*/*post* in *add.* puncto *P1*/*post* Q[1] *add.* L *O*/Q[2] *om. L3E* 67 ymago *rep. et del. P1* 68 continuemus *corr. ex* continemus *C1*/MN: MB *R* 69 TZ: EZ *FP1*; TS *L3E*/TAB: TAE *R* 70 TG: TH *FP1*/et[1]. . . equales[2] *om. FP1*/*post* GN *inter.* H in arabico *O* 71 proportio[1]. . . sicut *om. S*/AT . . . proportio[2] (72) *om. L3*/BM: MB *R*/AF[2]: AQ *R*/FM[2]: QN *R* 72 GN: NG *R*/*post* GN *inter.* H in arabico *O*/et . . . BM *om. ER*/BM: BN *FP1SOL3* 73 FM: FQ *L3C1E*; *alter. ex* FQ *in* MQ *O*/proportio *om. L3*/est[2] *om. P1* 74 ergo[2]. . . MN[2] *mg. O*/FQ[2]: FA *E*/*post* MN[2] *add.* quia AF ad AM sicut FQ ad MN sed AF maior est AM *R* 75 *post* et *add.* linea *R* 76 tunc: nunc *L3* 78 AT: AU *R*/TB: FB *O*/*post* TB *add.* TG *R* 79 T *om. L3E*; *inter. O*/et[2] *om. R* 80 KA: KL *O*/KG *mg. F*; KH *deinde inter.* G in arabico *O*/superficies *om. P1*/KBA: KB *O*/*post* KBA *add.* AK *FP1SO*/sunt secantes: secant *R* 81 suum centrum *transp. ER*/super[2]: et *R* 82 tangentes *corr. ex* contingentes *P1*/*post* tangentes *scr. et del.* eam *F*/convertitur: reflectitur *R* 83 inter: in *E*/*ante* has *add.* et *E* 84 convertuntur: reflectuntur *R*

[7.15] Et extrahamus BM in superficie BKA equidistantem AK, et sit minor quam AK. Et continuemus AM, et extrahatur recte, et extrahatur KB donec concurrant in F. Et extrahatur NG in superficie KGA, et sit equidistans AK, et ponatur equalis BM. Et continuemus AN, extrahatur recte, et extrahatur KG recte donec concurrant in Q. Et continuemus MN, FQ.

[7.16] Quia ergo BT est sicut TA, erit BK sicut KA, et GK sicut KA. Ergo BK est sicut GK, et angulus KBA est sicut angulus KGA, et angulus KAB est sicut angulus KBA. Et similiter angulus KGA est sicut angulus KAG; ergo angulus ABM est sicut angulus ABK, et angulus AGN est sicut angulus AGK, et erit angulus ABM sicut angulus AGN. Et linea BM erit sicut linea GN. Tunc linea AM erit sicut linea AN; tunc AF erit sicut linea AQ. Tunc due linee FQ, MN erunt equidistantes; tunc FQ erit maior linea MN.

[7.17] Tunc quando fuerit visus super punctum K, et fuerit linea MN in aliquo visibili, tunc forma M extendetur super lineam MB et convertetur per lineam BK in superficie circuli transeuntis per puncta B, A, K, et forma puncti N extendetur super lineam NG et convertetur super lineam GK in superficie circuli transeuntis per puncta G, A, K.

[7.18] Et erit punctum F ymago puncti M, et punctum Q erit ymago puncti N, et erit linea FQ dyameter ymaginis MN. Et iam declaravimus quod linea FQ est maior linea MN; tunc quando fuerit visus super punctum K, et fuerit linea MN in aliquo visibili, tunc visus apprehendet formam linee MN super lineam FQ. Tunc comprehendet formam maiorem re visa.

85 BM: BN *L3*/BKA: BA *FP1*/*post* BKA *scr. et del.* et continuemus *E*/equidistantem: equidistans *S*; equidistanter *L3C1* 86 AK: AB *FP1E* 87 concurrant: concurrat *OR*/*post* concurrant *add.* cum AM *R*/et: sed *FP1*/NG: MG *FP1*; HZ *deinde inter.* ZG in arabico *O* 88 KGA *corr. ex* KHA *O*/AK: TK *O* 89 *post* AN *add.* et *OC1E*/extrahatur recte *om. SL3*/et extrahatur *om. C1*/et . . . recte[2] *om. R*/KG: BG *E*; KH *deinde inter.* G in arabico *O*/KG recte *transp. L3*/ concurrant: concurrat *R*/Q: que *P1* 91 BK: BQ *FP1*/KA: K *S*/et . . . KA (92) *mg. F*/GK *corr. ex* HK *O* 92 GK *corr. ex* HK *O*/et . . . KGA *om. R*/KGA *corr. ex* KHA *O*/KGA et angulus (93) *mg. F* 93 KAB . . . angulus[2] *om. P1*/est[1]. . . KGA *om. L3*/KGA *corr. ex* KHA *O* 94 KAG: KGA *FP1O*/ABM: AMB *FP1L3*/et . . . AGN (96) *mg. a. m. E* 95 *post* AGN *inter.* HZ in arabico *O*/AGK: ABK *FP1*; KBA *O*/*post* AGK *add.* ergo *R*/erit *om. O*/angulus[2] *om. SC1*/angulus ABM sicut *om. P1*/*post* ABM *scr. et del.* est *S*; *add.* erit *O* 96 AGN: ANG *L3E*; AGZ *deinde inter.* N in arabico *O*/linea[2] *inter. E*/GN: GM *FP1*/*post* GN *inter.* Z in arabico *O*/AM *om. S*/erit[2]: est *L3* 97 AN: AZN *FO* (*alter. in O*); GAZN *P1*; ANZ *S*; AQ *E*/tunc[1] . . . AQ *scr. et del. E*; *om. R* 99 fuerit visus *transp. R* 100 *post* visibili *add.* inferiore *R*/extendetur *corr. ex* existetur *P1* 101 convertetur: reflectetur *R*/per[1] *inter. O*/BK: KK *P1*/*post* circuli *add.* qui transit per puncta B A K et forma puncti N extendetur super lineam NG et convertetur super lineam GK in superficiei circuli *mg. O* 102 B *corr. ex* G *O*/A *inter. L3*/et[1]. . . K (103) *om. O*/convertetur: reflectetur *R* 103 GK *corr. ex* NGK *F*/*post* per *add.* tria *R* 104 punctum F ymago: ymago puncti F *FP1SL3C1ER* (F: M *C1*)/puncti: punctum *FP1L3C1ER*; puncto *S*/M: N *L3*; F *C1* 105 *post* puncti *scr. et del.* F *F* 106 fuerit visus *transp. R* 107 apprehendet: comprehendet *L3* 108 linee . . . formam (109) *om. R*

[7.19] Et sic, si revolverimus item totam figuram in circuitu linee AU, ipsa immobili, tunc punctus K faciet circulum perpendicularem super lineam AU, et sic omne punctum ultra illum punctum illius circuli habebit situm respectu linee comparis linee MN sicut est situs K respectu MN.

[7.20] Si ergo visus fuerit in aliquo puncto circumferentie huius circuli et linea compar linee MN fuerit in superficie alicuius rei vise, tunc visus comprehendet formam illius linee maiorem. Et similiter si extraxerimus TK recte et posuerimus in ipsa aliquod punctum preter K, et extraxerimus semper ab illo puncto, quod est quasi punctum K, erit modus eius sicut modus puncti K.

[7.21] Ex hiis ergo duabus figuris patet quod in spericis speculis concavis multa et ex multis sitibus comprehenduntur maiora.

[7.22] **[PROPOSITIO 25]** Item sit speculum spericum concavum AB circa centrum E [FIGURE 6.7.25, p. 318], et extrahamus superficiem transeuntem per E, et faciat circulum AB. Et extrahamus ex E lineam EZ, quocumque modo fuerit, usque ad G, et ex G extrahamus GD perpendicularem super superficiem circuli AB, et in ipsa signemus punctum D, quocumque modo fuerit. Et continuemus DE, et extrahamus ipsam usque ad O, et extrahamus EB ita quod contineat cum ED angulum obtusum, et extrahamus EA ita quod contineat cum ED angulum equalem angulo DEB. Et continuemus DA, DB. Sic ergo superficies duorum triangulorum DAE, DBE secant se super lineam DE, et duo anguli acuti DBE, DAE erunt equales.

[7.23] Et extrahamus ex B lineam in superficie trianguli DBE continentem cum EB angulum equalem angulo DBE. Hec ergo linea con-

110 si revolverimus *corr. ex* scire voluerimus *a. m. E*/item *om. OC1R* 111 punctus: punctum *R*/perpendicularem *om. O* 112 AU: AY *FP1*/*post* punctum[1] *inter.* quod est *O*/ultra illum punctum *om. R*; *inter. O*/punctum[2]: circulum *FP1SOL3C1E* 113 comparis: operis *S*; corporis *L3*/linee[2] *om. R*/MN: MH *FP1*; MZ *O*; MQ *L3E*; unius *C1*/sicut . . . MN (114) *om. P1* 114 MN: MK *F*; MZ *O*; MQ *L3E* 115 ergo: vero *O*/fuerit in aliquo: in aliquo fuerit *FP1* 116 et *mg. a. m. E*/MN: MK *FP1*; MZ *O*; MQ *L3E* 117 comprehendet: comprehendit *C1*/*post* maiorem *scr. et del.* et quando extraxerimus semper ab illo puncto quod est Q punctum R erit modus eius sicut modus puncti R *O*/et *om. FP1* 118 extraxerimus: extra erimus *S*; extrahamus *L3ER*; extraherimus *C1*/TK: KT *FP1* 119 et . . . K (120) *om. O*/extraxerimus: extra erimus *S*; extraherimus *L3C1*; *corr. ex* extraherimus *E*/*post* extraxerimus *add.* lineas *R*/quasi: Q *SL3E*/K[2] *om. S* 120 sicut: si *FP1* 122 *post* concavis *add.* et *P1R* 123 sit: si *L3*/concavum *om. L3C1ER* 124 circa: citra *FP1*/extrahamus: extramus *S*/superficiem . . . extrahamus (125) *om. S* 125 transeuntem *mg. F*/*post* per *scr. et del.* O *F*/faciat: faciet *FP1*/ex *om. L3* 126 *post* G[2] *add.* et *L3E* 127 GD: ergo *O*/perpendicularem: perpendiculariter *O*/signemus: signavus *S* 128 quocumque: quomodocumque *O*/DE: D *O*; TE *L3E*/et[2] *om. S* 129 O *corr. ex* A *P1*/EB: EA *S*/quod: ut *R*/contineat: concurrat *L3* 130 angulum . . . ED (131) *om. S*/quod: ut *R* 131 angulo: modo *L3*/et *rep. C1* 132 secant: secent *FP1* 133 acuti *om. P1* 134 et *om. L3ER*/*post* extrahamus *add.* ergo *L3ER*/DBE: DEB *ER*/continentem: contingentem *FSL3C1*; contingente *P1* 135 *post* cum *add.* FP *FP1*/EB: ED *L3*/DBE *corr. ex* DEB *a. m. E*/concurrit: concurret *R*

currit cum linea DE, quia angulus BEO est acutus, et angulus qui est apud B est acutus. Concurrat ergo in O.

[7.24] Et extrahamus etiam ex A lineam in superficie trianguli DAE continentem cum AE angulum equalem angulo DAE. Concurrat ergo cum DE in O, quia duo anguli AEO, BEO sunt equales, et anguli qui sunt apud duo puncta A, B sunt equales.

[7.25] Et extrahamus ET ita quod contineat cum EB angulum rectum, et extrahamus TE in parte E, et BO in parte O, et concurrant in H, et erit TE equalis EH. Et similiter extrahamus EK ita quod contineat cum EA angulum rectum, et extrahamus illam in parte E, et extrahamus AO, et concurrant in L. Sic ergo KE erit equalis EL.

[7.26] Et continuemus TK, LH. Erunt ergo equales. Si ergo visus fuerit in D et LH fuerit in aliquo visibili, tunc D comprehendet LH in speculo AB, et erit T ymago H, et K ymago L, et sic TK erit dyameter ymaginis LH, et est ei equalis.

[7.27] Si ergo revolverimus totam figuram, HL immobili, tunc D faciet circulum, et si visus fuerit in aliquo puncto circumferentie illius, poterit comprehendere aliquod visibile compar linee LH, et erit ymago eius equalis ei. Et similiter si visus fuerit in O, et res visa fuerit TK, erit ymago equalis rei vise.

[7.28] Sed tamen cum res visa fuerit LH, et visus fuerit D, et fuerit ymago TK, ymago erit conversa; si H fuerit in dextro, erit T in sinistro, et si H fuerit in sinistro, T erit in dextro, et si H fuerit supra lineam, erit T infra lineam, et similiter L.

[7.29] Et si res visa fuerit TK, et visus fuerit O, et ymago fuerit LH, forma erit recta, nam ymago LH erit retro post visum, et comprehen-

136 *post* DE *scr. et del.* qua *S*/BEO: BED *L3R*; *alter. ex* DEB *in* BED *E*/acutus: obtusus *R*/et . . . acutus (137) *om. P1* 137 B *corr. ex* BF *O*/concurrat: currat *FP1*; concurrant *R*/ergo *om. R*/O *corr. ex* E *S* 138 *post* ex *inter.* A *O* 139 continentem: contingentem *SL3C1*; *corr. ex* contingentem *E*/concurrat: concurret *R* 140 in O *om. L3*/*post* O *add.* quod *FP1S*/quia *om. S*/AEO: AED *FP1* 141 duo puncta *om. R* 142 et *om. FP1*/ET *inter. a. m. E*/*post* ET *scr. et del.* et *C1*/quod: ut *R*/contineat *corr. ex* continuet *P1*; concurrat *S*/*post* angulum *scr. et del.* acutum *P1*/rectum *inter. a. m. E* 143 et . . . H (144) *rep.* [extrahamus: erit] *FP1*/TE: DE *FP1O*/in[1, 2]: ex *R*/BO: HO *E*/concurrant: concurrat *FP1O* 144 TE: ET *R*/*post* EH *add.* et BT equalis BH *R*/extrahamus . . . et[1] (145) *om. S*/*post* EK *add.* et *P1*/quod: ut *R* 145 angulum: angulus *F*/et[1]. . . E *rep. S*/in: ex *R* 146 AO: AD *SL3*/EL: LE *O*; KL *FP1SL3E*/*post* KL *add.* et KA equalis AL et TE equalis EH *R* 148 comprehendet: comprehendit *S*/*post* in *add.* hoc *L3E* 149 *post* K *add.* etiam *E*/et[3] *om. ER*/*post* sic *add.* igitur *R*/dyameter: dyametrum *FP1* 150 ymaginis *om. FP1*/equalis *corr. ex* equales *L3* 151 revolverimus: volverimus *L3*; *corr. ex* volverimus *a. m. E*/HL: LH *R* 152 faciet *corr. ex* fiet *O*/circumferentie *corr. ex* cirferentie *F*/circumferentie illius *transp. R* 153 LH: LB *O* 154 fuerit: fuit *O* /in . . . fuerit (156) *mg. F*/O: eo *FP1* 155 TK *corr. ex* TH *a. m. E* 156 visa *om. FP1S*/LH *inter. E*/*post* et[1] *scr. et del.* si *F*/et fuerit: fueritque *ER* 157 ymago erit *transp. ER*/*post* fuerit *add.* visa *L3*/dextro: dextra *R*/erit[2]: et si *O*/T: et *FP1*; *corr. ex* et *C1*; L *O*/*ante* in[2] *add.* fuerit *O*/sinistro: sinistra *R* 158 *post* fuerit[1] *add.* visa *L3*/sinistro: sinistra *R*/T erit *transp. L3ER*/dextro: dextra *R*/H[2] *corr. ex* B *a. m. E*/supra: extra *L3* 159 *post* lineam *add.* erit T N *P1*/L: B *E* 160 LH: LB *O* 161 forma . . . LH *mg. a. m. E*/erit[1]: est *R*/post *om. R*

detur ante rem visam, sicut declaravimus in capitulo ymaginis quinti tractatus, et visus comprehendet H, quod est ymago T, in linea BO, et L quod est ymago K, in LO.

[7.30] Patet ergo quod in speculis concavis comprehenditur res visa quandoque equalis sibi.

[7.31] **[PROPOSITIO 26]** Item extrahamus BH recte, et in ipsa signemus R, et continuemus RE. Sic ergo angulus REB erit obtusus.

[7.32] Et extrahamus RE ad N. Sic ergo RB erit maior quam BN. Et proportio RB ad BN est sicut proportio RE ad EN; ergo linea RE est maior quam EN.

[7.33] Et extrahamus AL recte, et sit AM equalis BR. Et continuemus ME, et transeat usque ad U. Erit ergo ME maior quam EU. Et continuemus MR, NU. Erit ergo MR maior quam NU.

[7.34] Si ergo MR fuerit in aliquo visibili, et visus fuerit in D, erit NU dyameter ymaginis MR, et NU est minor quam MR. Et si visus fuerit in O, et NU fuerit in aliquo visibili, erit MR ymago NU, et est maior quam NU.

[7.35] Sed cum MR fuerit visibile, et fuerit ymago NU, tunc ymago erit conversa, et si NU fuerit visibile et MR fuerit ymago, ymago erit recta, nam ymago, si fuerit ultra visum, videbitur ante, et omne punctum ymaginis videbitur in linea in qua est de lineis radialibus.

[7.36] **[PROPOSITIO 27]** Item signemus in linea OH punctum Q. Et continuemus QE, et transeat ad C. Et sit OF equalis OQ, et continuemus EF, et transeat ad I. Erunt ergo due linee CE, EI maiores duabus lineis EF, QE, et erit linea CI maior quam linea FQ.

[7.37] Si ergo visus fuerit in O, et CI in aliquo visibili, erit FQ ymago CI, et FQ est minor quam CI. Et FQ videbitur super duas lineas

163 tractatus: tractus *F*/H: B *O*/quod *inter. a. m. E*; Q *P1*/BO: HO *R*   164 quod: et *FP1*/*post* in *add.* linea *C1*/LO *corr. ex* BO *C1*   165 comprehenditur: comprehendetur *E*; comprehendatur *R*   167 BH: LH *P1*/recte *corr. ex* recta *a. m. E*   168 signemus *corr. ex* assignemus *O*/R: K *L3*/continuemus: continemus *R*/RE: BE *L3*   169 *post* ergo *add.* TB erit maior BN ergo linea *R*/RB: TB *L3E*/erit: est *R*/BN . . . quam (171) *rep. E*; *rep. et del. F*/quam *om. R*   170 et . . . EN (171) *rep. et del. C1*/RB: TB *L3*; *corr. ex* TN *a. m. E*/ad BN *rep. P1*/BN: DN *S*/EN *corr. ex* N *S*/ergo: quare *R*   171 *post* quam *add.* linea *R*/EN: BN *S*; *corr. ex* BEN *L3*   172 *post* recte *add.* in M *R*/sit: sicut *FP1*/BR: BZ *L3*   173 transeat: transeamus *C1*   174 maior . . . MR (175) *om. S*/NU: MU *C1*   175 MR: ME *FP1*   176 NU: MR *C1*/minor: maior *FP1SL3C1E*/MR: NU *C1*/visus fuerit (177) *transp. FP1C1*   177 fuerit *om. S*/*post* visibili *add.* et *L3*/NU[2]: NB *FP1*/et . . . NU (180) *mg. O*   178 NU: QM *FP1*; MR *O*   179 et . . . visibile (180) *om. FP1SL3E*; *mg. C1*/fuerit . . . NU: NU fuerit ymago et D visus *R*/tunc *om. R*/ymago erit conversa (180): erit ymago conversa *R*   180 *post* si *add.* res visa *R*/NU fuerit *transp. R*/visibile et: et visus O *R*/MR fuerit ymago: ymago MR *R*/ymago[2] *scr. et del. F*; *om. P1R*   181 recta: erecta *L3*/si *om. S*   182 in[1] *om. L3*   184 C: P *R*/et[3] . . . linee (185) *mg. a. m. E*/OQ: EQ *S*   185 I: L *E*/CE *om. FP1*; QE *SL3E*; TI *O*; PE *R*; *corr. ex* QE *C1*/maiores *corr. ex* maior *C1*   186 QE: EQ *R*/CI: Q *L3E*; PI *R*/linea[2] *om. C1*   187 visus: visusus *F*/visus fuerit *transp. L3*/CI: Q *L3E*; PI *R*/visibili: visibile *FP1SO*/FQ: QF *C1*   188 CI[1]: Q *L3E*; PI *R*/minor: maior *E*/CI[2]: PI *R*/*post* FQ[2] *scr. et del.* est minor quam TI *P1*/*post* videbitur *add.* semper *L3*

AO, OB. Erit ergo forma ante visum et minor quam res visa, et erit recta.

[7.38] Et si visus fuerit in D, et FQ fuerit in aliquo visibili, erit CI ymago FQ. Et est maior quam FQ, et erit forma ante visum conversa.

[7.39] Patet ergo quod in speculis concavis comprehenditur forma rei vise minor, et maior, et equalis.

[7.40] **[PROPOSITIO 28]** Item sit speculum concavum AB [FIGURE 6.7.28, p. 319], et centrum G, et habeat superficiem equalem transeuntem per centrum, et faciat circulum AB. Et extrahamus lineam GD, quomodocumque sit, et transeat in parte G ad E, et sit visus in E, et sit T in superficie visus.

[7.41] Et extrahamus TH perpendiculariter super lineam ED, et sit ZT equalis TH, et comprehendat E punctum H ex A. Sic ergo erunt duo puncta A, H a duobus lateribus puncti G, nam si in eodem essent, tunc linea que exierit a speculo ad A non divideret angulum quem continent due linee radiales.

[7.42] Et extrahamus lineas EA, AH, GA, GH, et transeat GH recte ad K. Duo ergo anguli qui sunt apud A erunt equales, et erit K ymago H.

[7.43] Et sit arcus BD equalis arcui DA, et continuemus lineas EB, BZ, BG, et extrahamus ZG ad L. Erunt ergo duo anguli apud B equales, et comprehendetur Z a visu ex B, et erit L ymago Z.

[7.44] Et continuemus KL. Erit ergo KL dyameter ymaginis ZH, et quia ZTH est perpendicularis super DE, et ZT est equalis TH, erunt due linee EA, AH equales duabus lineis EB, BZ, et duo anguli apud A sunt equales duobus angulis apud B, et linea GH est equalis linee ZG.

189 OB: BO *R*/ante: retro *R* 190 recta: erecta *L3* 191 visus *inter. a. m. E*/fuerit[1] *rep. P1*/FQ: QF *C1*/CI: PI *R*; *corr. ex* Q *E* 192 *post* et[1] *add.* T *L3*; *add.* R *E* 193 forma rei vise (194): rei vise forma *C1* 194 *ante* minor *scr. et del.* mor *C1* 195 sit: si *L3*; *corr. ex* si *O* 196 *post* habeat *scr. et del.* f *F*/equalem: planam *R* 197 AB: OB *L3*/lineam *om. C1* 198 quomodocumque: quocumque modo *OC1ER*/*post* quomodocumque *add.* quisque *L3*/transeat: transeant *E*/in: ex *R* 199 T *om. S*/*post* visus *inter.* vel rei vise *O* 200 perpendiculariter *corr. ex* perpendicularium *P1* 201 ZT: ZD *L3E*/*post* A *add.* et GH producta in P comprehendat arcum AP maiorem quarta circuli *R* 202 G *corr. ex* H *L3*/si *mg. F* 203 exierit: exiret *R*/ad A *rep. P1*/divideret: dividet *L3* 204 quem *corr. ex* quoniam *E*/*post* continent *scr. et del.* duo anguli *L3*/*post* radiales *add.* per equalia *R* 205 EA: EH *L3*; *corr. ex* TA *a. m. C1*/AH: HAH *P1*; *corr. ex* HAH *F*; *inter. E*/transeat *corr. ex* transeant *S*/GH[2]: EH *P1* 206 qui sunt *om. R*/erunt: essent *FP1* 208 BD: HD *S*/*post* equalis *scr. et del.* anguli *S* 209 *post* BZ *add.* GZ *SO*/BG: BZ *F*; *om. P1*; *corr. ex* BZ *E*/*post* BG *add.* GZ *C1*/L: I *E*/*post* L *add.* et secet ZB dyametrum DG in F *R*/duo *om. P1* 210 equales *om. P1*/Z[1] *om. FP1S*; *inter. E*/B: A *FP1*/*post* erit *add.* punctum *R*/L: B *FP1SL3C1E* 211 dyameter: dyametri *S*/ymaginis: ymagines *FP1* 212 ZTH: ZH *L3*; TH *R*/DE: D *S* 214 duobus *rep. et del. E*/*post* B *add.* erit HE equalis ZE *R*/est *om. C1* 215 ZG: ZH *FP1SOL3ER*

[7.45] Ergo due linee AG, GH sunt equales duabus lineis BG, GZ, et basis AH est equalis basi BZ. Ergo angulus AHG est equalis angulo BZL, et angulus HAK est equalis ZBL. Ergo HK est equalis ZL, et linea HG est equalis ZG; ergo GK est equalis GL. Ergo KL est equidistans ZH.

[7.46] Item angulus HGA est obtusus, et duo anguli apud A sunt equales; ergo linea GH est maior linea GK, et similiter ZG est maior quam GL. Linea ergo KL est minor quam ZH. Sed KL est dyameter ymaginis ZH. Linea ergo ZH videbitur minor quam sit secundum veritatem. Et linea ZH est superficies faciei aspicientis.

[7.47] Si ergo revolverimus circulum ad B, EG immobili, in circuitu ED, fiet circulus, et fiet ex duobus punctis A, B circulus in superficie speculi. Et erit situs visus E respectu cuiuslibet comparis linee ZH ex illo circulo quam signat ZH, et ex omni arcu compari arcui AB ex portione quam dividit circulus quem signant duo puncta A, B sicut est situs quem visus E habet ex linea ZH et ex arcu AB. Et similiter declarabitur si posuerimus lineam maiorem quam ZH aut minorem.

[7.48] Patet ergo ex hiis omnibus quod dyameter superficiei faciei aspicientis comprehenditur in speculo concavo minor quam sit. Sequitur ergo quod si visus fuerit in E, tunc aspiciens comprehendet suam formam in tali speculo minorem quam sit, et quia K est ymago H, et L est ymago Z, erit ymago conversa.

[7.49] Et sic visus E comprehendet suam formam, scilicet quod est in dextro comprehendet etiam in sinistro et sursum deorsum, et econverso. Et similiter si visus fuerit in quolibet puncto inter quod et superficiem speculi fuerit centrum speculi, comprehendet suam formam conversam, et hoc est quod voluimus.

216 AG: EAG *FP1*/GZ: GH *FP1* 217 AH *corr. ex* AG *O*/*post* basi *scr. et del.* L *F*/AHG: ABG *S*; AHK *R* 218 ergo HK *rep. C1* 219 *post* linea *scr. et del.* HK *P1*/ZG: ZH *FP1SOL3ER*/GK: HK *FP1O*; KH *L3E*; KG *C1*/GL: HL *O*/GL ergo *om. P1*/KL: ZL *P1*; *corr. ex* LK *a. m. E* 221 item *om. P1*/HGA: LGA *S*/apud *om. O* 222 linea[2] *inter. P1*/GK *corr. ex* KG *S*/ZG: ZK *P1* 223 GL: GK *FP1*; *corr. ex* AGL *S*/sed . . . ZH[1] (224) *mg. a. m. E* 224 linea *om. R*/videbitur: videtur *R* 226 EG: ED *R*/in *om. P1*/in . . . et (227) *om. R* 227 *post* ED *scr. et del.* fa *O*/fiet[1] *corr. ex* fiat *a. m. E*/fiet[2] *corr. ex* fient *O*; *corr. ex* fiat *a. m. E* 228 *post* ZH *scr. et del.* exit *F*; *add.* et *L3* 229 quam: quem *SL3ER*/signat: signant *R*/ZH: puncta Z H *R*/compari: comparari *S*/*post* arcui *scr. et del.* ex *C1* 230 *post* portione *add.* speculi *R* 231 quem: quoniam *FP1S*/E: est *FP1* (*scr. et del. F*) 232 *post* lineam *add.* ZH *R*/quam ZH *om. R* 234 sequitur: sciendum *FP1R* 235 si visus: similis *L3E*/visus fuerit *transp. R*/fuerit *om. L3E* 236 suam formam *transp. C1ER*/in tali speculo *om. R*/*post* et *scr. et del.* linea *P1*/K *om. L3* 238 *post* comprehendet *scr. et del.* in sinistro *S*/*post* formam *inter.* conversam *O*/scilicet: secundum *P1L3ER* 239 in dextro: dextrum *R*/comprehendet etiam *om. R*/etiam *om. SO*/sursum *corr. ex* sur *O*/*post* sursum *add.* et *FP1* 240 econverso: econtrario *R*/et[1] *om. ER*/inter quod et: uterque *L3* 241 speculi[1] *corr. ex* speculum *S*/comprehendet: comprehendit *C1*/suam formam *transp. ER* 242 est *inter. O*/voluimus: volumus *C1E*

[7.50] Patet ergo ex hiis quattuor figuris quod in speculo concavo quandoque comprehenditur maior, quandoque minor, quandoque equalis, et nunc recta, et nunc conversa.

[7.51] Et in capitulo de ymagine diximus quod in speculo concavo quandoque ymago erit una, quandoque due, et quandoque tres, et quandoque quattuor, et hoc idem accidit in hiis predictis.

[7.52] Illud ergo quod habet ymaginem se maiorem forte habebit alias minores et equales, et quod habet minorem ymaginem forte habebit alias maiores et equales, et quod habet equalem forte maiorem et minorem, et quod rectum videtur forte videbitur sub alia ymagine conversum, et econverso. Restat ergo declarare formas eorum que comprehenduntur in huiusmodi speculis.

[7.53] **[PROPOSITIO 29]** Sit ergo speculum spericum AB [FIGURE 6.7.29, p. 320], et extrahamus in ipso speculo superficiem equalem transeuntem per centrum, et faciat circulum AB circa centrum E. Et extrahamus in hoc circulo duos dyametros se secantes AEO, BED, et speculum non excedat arcum BADO. Et ponamus in BE punctum Z, quomodocumque sit, et ponamus in linea AE punctum K, et sit AK maior quam KE. Et continuemus ZK, et transeat ad F. Et continuemus EF, et sit angulus EFG equalis angulo EFZ.

[7.54] Quia ergo FK est maior quam KA, et KA est maior quam KE, erit FK maior quam KE. Angulus ergo FEK est maior angulo EFK; est ergo maior angulo EFG. Linea ergo FG concurret cum linea KE. Concurrant ergo in G. Due ergo linee ZF, FG convertuntur per angulos equales; K ergo est ymago G, si visus fuerit in Z.

243 *post* concavo *add.* ymago *R* 245 nunc[2]: ? *S* 246 *post* concavo *add.* ymago *R* 247 *post* una *add.* et *FP1*/et[1] *om. SE* 248 et *om. S*/idem: quidem *FP1*/*post* hiis *add.* speculis *L3* 249 quod *corr. ex* quo *L3*/*post* maiorem *add.* et quod habet equalem *SL3C1E* (habet: habebit *E*)/habebit *corr. ex* habet *F* 250 minores *corr. ex* maiores *C1*/et[1] *inter. E*/habet . . . ymaginem: ymaginem habet minorem *R*/minorem . . . habet (251) *mg. a. m. E*/*post* ymaginem *add.* ne *L3* 251 et[1]. . . maiorem *om. R*/forte . . . videtur (252) *om. L3* 252 minorem: minores *R*/rectum: recte *P1E* (*alter. in a. m. E*)/videtur: videbitur *R*/videbitur *corr. ex* videtur *a. m. E* 253 conversum *om. E*/econverso: econtra *SL3*; *alter. ex* econtra *in* conversum *a. m. E*; econtrario *R*/declarare: declare *F* 254 huiusmodi: hiis *R* 255 sit: si *SL3*; *corr. ex* si *E*/*post* spericum *add.* concavum *R* 256 speculo *om. O* 257 equalem: planam *R*/faciat: fiat *P1*/circa: citra *FP1*; *alter. in* citra *E* 258 *post* hoc *scr. et del.* speculo *E*/duos: duas *R* 259 excedat: ecedat *F*; excedet *S*/arcum *om. L3*/et[2] *inter. O* 260 quomodocumque: quocumque *FP1*; quocumque modo *C1ER*; *corr. ex* quocumque *O* 261 AK *corr. ex* KA *C1* 262 sit: si *O*/EFG: EFH *FP1SOL3*; EHF *E*; GFE *R*/EFZ: ZFE *R* 263 quam[1] *om. R* 264 erit . . . KE[2] *mg. a. m. E*/est maior *transp. R* 265 EFK *corr. ex* FK *O*/est *inter. O*/est ergo *transp. C1R*; *transp. deinde corr. E*/EFG: EFH *FP1SOL3E*/FG: FH *FP1O*; FR *L3E* 266 G *corr. ex* H *L3*/ergo[2] *om. L3*/ergo linee *transp. C1*/FG: FH *FP1O*; FR *L3E*/convertuntur per: reflectuntur propter *R* 267 *post* angulos *scr. et del.* ano *P1*/*post* equales *add.* ZFE GFE *R*/G: HG *O*/fuerit *om. P1*/*post* Z *add.* et R *E*; *scr. et del.* et R *C1*

[7.55] Et extrahamus lineam ZLH, quomodocumque sit, et continuemus EH, HG, ZG, et extrahamus FE usque ad M. Proportio ergo ZM ad MG est sicut proportio ZF ad FG. Et ZH est maior quam ZF, et GH est minor quam GF. Ergo proportio ZH ad GH est maior quam proportio ZF ad FG; est ergo maior quam proportio ZM ad MG. Linea ergo que dividit angulum ZHG in duo equalia secat lineam MG; secat ergo lineam EG. Angulus igitur GHE est maior angulo EHZ.

[7.56] Et ponamus angulum EHR equalem angulo EHZ. Linea ergo HR secat lineam GF, et secat lineam EG; secet ergo lineam EG in R. Ergo due linee ZH, HR convertuntur per angulos equales, et erit L ymago R. Dico ergo quod forma cuiuslibet puncti linee GR convertitur ad visum Z ex puncto arcus FH, et non ex alio.

[7.57] Huius rei demonstratio est quoniam in capitulo de ymagine quinto tractatu, in duabus figuris viginti septem et viginti octo, dictum est quod duo arcus AB, DO non possunt esse tales quod ex illis convertetur aliquid de linea EO ad Z, et arcus BO non est de speculo. Non ergo remanet nisi arcus AD.

[7.58] Sed in tricesima quinta figura dictum est quod forma cuiuslibet puncti dyametri EO convertitur ad aliquod punctum arcus AD, et in tricesima sexta capitulo de ymagine patuit quod numquam convertitur forma puncti R ad Z ex arcu AD nisi uno solo puncto. Forma ergo cuiuslibet puncti linee GR convertitur ad Z ex uno solo puncto arcus AD.

[7.59] Et ponamus in linea GR punctum C. Forma ergo C convertitur ad Z ex uno puncto arcus AD. Dico ergo quod illud punctum

268 et[1] *om. E*/ZLH: ZLLY *P1*; *alter. in* ZLLY *mg. F*/quomodocumque; quocumque modo *ER* 269 ZG: ZH *FP1O*; *om. S*/FE: FS *L3*/ergo *rep. S* 270 et *corr. ex* Z *P1*/ZH *inter. a. m. C1*; ZG *FP1SL3E*/*post* ZH *inter.* G in arabico *O*/ZF[2]. . . quam[1] (271) *om. S* 271 GH[1] *corr. ex* HGH *F*/ZH: ZG *FP1SL3E*/GH[2]: DG *O* 272 linea ergo (273) *transp. R* 273 *post* equalia *add.* et *L3*/MG *corr. ex* M *P1* 274 secat *om. L3*/ergo *inter. a. m. E*/*post* EG *add.* secet ergo lineam EG in R ergo *R*/igitur *om. R*/est maior *transp. R*/EHZ: ZHE *R*/*post* EHZ *add.* et HZ secet AE in L *R* 275 et . . . R (277) *om. R*/EHR: EHI *S*; EAHR *O*/EHZ: EH *FP1*; EHG *S* 276 GF . . . lineam[2] *om. FP1*/GF: HQ *O*/EG: EH *FP1O* 277 ZH: ZB *FP1*/convertuntur per: reflectuntur propter *R*/et *inter. E* 278 convertitur: reflectitur *R* 279 *post* puncto *add.* aliquo *R*/FH: FGH *FP1*; *alter. ex* FDH *in* FDG *O* 280 de *inter. C1* 281 tractatu: tractu *O*/viginti[1]. . . octo *om. R*/viginti septem: ZA *FP1SL3E*; viginti sex *O*/et *om. SOL3C1E*/viginti octo: ZG *FP1SL3E*; viginti septem *O*/*post* viginti octo *add.* ZA ZG *FP1*/dictum est (282) *om. O* 282 non: item *FP1*; etiam *SL3E* 283 convertetur *inter. O*; reflectatur *R*/BO *inter. E*; EO *R* 284 AD *rep. S* 285 tricesima: trecesima *F*; triscesima *L3* 286 convertitur: reflectitur *R*/ad aliquod punctum: ab aliquo puncto *R* 287 et in tricesima *rep. L3E*/tricesima *corr. ex* tricesesima *F*/*post* tricesima *add.* et tricesima *SOC1*/sexta: quinta *O*/*post* quod *add.* punctus *FP1*/convertitur: convertetur *C1*; reflectitur *R* 288 R: HR *O*; linee GR *R*/*post* nisi *add.* ex *OER* (*inter. O*)/uno *om. ER*/forma[2]. . . puncto (289) *mg. a. m. E* 289 *post* puncti *scr. et del.* R ad Z ex arcu AD *C1*/linee GR *transp. FP1*/GR: HR *FP1O*; GX *L3*/convertitur: reflectitur *R* 291 et *inter. O*/GR: HR *FP1O*; GK *S*/forma . . . AD (292) *om. R* 292 *post* uno *add.* solo *P1*/illud *inter. O*

non erit nisi in arcu FH. Sin autem, convertatur forma C ad Z ex U, quod est in arcu AF, et continuemus lineas ZU, CU, GU, EU.

[7.60] Linea ergo GU erit maior linea GF, et ZU est minor quam ZF; ergo proportio GU ad ZU maior proportione GF ad FZ. Ergo est maior proportione GM ad MZ. Linea ergo que dividit angulum GUZ in duo equalia secat lineam ZM; secat ergo ZE. Angulus ergo GUE est minor angulo EUZ; ergo angulus CUE multo minor est angulo EUZ, et similiter de quolibet puncto arcus AU. Forma ergo C non convertitur ad Z nisi ex arcu HF.

[7.61] Et dico quod non potest converti ex arcu HD. Quod si fuerit possibile, convertatur ex Q, quod est in arcu HD, et continuemus lineas ZQ, CQ, RQ, ZR, EQ, et extrahamus EH ad N. Linea ergo ZQ est maior quam ZH, et linea QR est minor quam HR; ergo proportio ZQ ad QR est maior proportione ZH ad HR, que est sicut proportio ZN ad NR. Linea ergo que dividit angulum ZQR in duo equalia secat lineam NR; secat ergo lineam ER. Angulus ergo RQE est maior angulo EQZ; angulus ergo EQC est multo maior angulo EQZ. Hoc idem sequitur in omni puncto arcus HD; forma ergo C non convertitur ad Z ex arcu HD, neque ex arcu AF.

[7.62] Sed iam patuit quod omnino debet converti ex arcu AD. Forma ergo C non convertitur ad Z nisi ex aliquo puncto arcus FH. Convertatur ergo ex T, et continuemus lineas CT, ET, ZT. Quia ergo T est inter duo puncta F, H, erit linea ZT inter duas lineas ZF, ZH. Linea ergo ZT secat lineam KL. Secet ergo ipsam in I. I ergo est ymago C, et C nullam habet ymaginem nisi I.

[7.63] Et sic declarabitur quod ymago cuiuslibet puncti linee GR erit punctum linee KL. KL ergo est ymago GR, et KL est linea recta,

293 non erit *inter. a. m. E*/FH: IH *FP1*; GH *S*; BH *O*; KG *L3*; KH *E*/sin: sint *S*/autem: aut *FP1*/convertatur: reflectatur *R*/forma *inter. a. m. E* 294 *post* arcu *scr. et del.* ex quod in arcu *E*/lineas: linea *S*/ZU: ZTI *FP1*/CU: OU *S*; EU *R*/GU *om. SL3*/EU: CU *R*; *om. L3* 295 ergo *corr. ex* ego *L3*/linea[2] *om. R*/GF: AGF *FP1* 296 ZU *corr. ex* CZU *F*/*post* ZU *add.* est *SOL3C1ER*/FZ: ZF *L3*/est *om. L3ER* 297 ergo *om. S*/GUZ: GZ ZU *FP1*; GZ UZ *L3E* 298 in duo: per *R*/equalia: equa *C1*/secat[2] *mg. C1*/ergo[2]: GLO *S*; GO *L3* 299 EUZ *corr. ex* EUG *O*/CUE: QUE *FP1*; UE *L3E* 300 *post* puncto *scr. et del.* ? *P1*/arcus: arcu *L3*/convertitur: reflectitur *R* 1 HF: AF *O* 2 et: item *FP1*/non *om. O*/potest: poterit *P1*/converti: reflecti *R*/HD: GD *O*/si *om. S* 3 convertatur: reflectatur *R*/HD *alter. in* GD *O* 4 RQ: IQ *L3*/ZR *om. E*/ZR EQ *transp. R*/EH: ZH *FP1*; EG *O* 5 maior: minor *E* 6 ZQ *inter. C1*/proportione: proportionum *S*/HR: GR *O*/est[2] *inter. a. m. E* 7 ZN: ZM *FP1SL3E* (*alter. in a. m. E*)/ergo . . . lineam[1] (8) *mg. a. m. E* 8 NR . . . lineam[2] *om. L3*/ergo[1] *om. E*/RQE: IQE *FP1*; CQE *L3E* 9 angulus . . . EQZ[2] *om. P1*/ergo . . . idem *mg. a. m. E*/EQC: CQE *R*/est *inter. O*/est multo *transp. O* 10 convertitur: reflectitur *R* 11 ex[1]. . . neque *om. L3E*/neque: nec *C1*/AF *corr. ex* HF *C1* 12 converti: reflecti *R* 13 convertitur: convertur *S*; reflectitur *R*/*post* Z *scr. et del.* ex arcu HD *SC1* 14 convertatur: convertitur *L3E*; reflectatur *R*/CT: ED *FP1*; *alter. in* ET *E*/ET: et *L3ER* 15 inter[2]. . . ZF *om. P1* 16 KL *om. L3*/ergo *om. L3*/*post* ergo *add.* lineam *ER*/I[1]: R *L3*/I[2] *inter. O*/C: T *R* 17 C: T *R*/habet ymaginem *transp. C1* 18 sic *corr. ex* si *O*/GR: GI *S*/GR . . . KL[1] (19) *om. P1* 19 erit: est *ER*

quia est pars dyametri circuli. Et GR est linea recta, quia est etiam pars dyametri circuli. Z ergo comprehendit formam GR recte in speculo AB sperico, et hoc est quod voluimus.

[7.64] **[PROPOSITIO 30]** Et iteremus formam [FIGURE 6.7.30, p. 320], et revolvamus super lineam GR a duobus lateribus duos arcus, quomodocumque sit, scilicet GNR, GQR, et sit arcus GNR non secans lineam GH. Et ponamus in linea GR punctum M, quomodocumque sit. Forma ergo M convertitur ad Z ex puncto arcus FH. Convertatur ergo ex T, et continuemus lineas ZT, MT.

[7.65] Duo ergo anguli ZTE, ETM sunt equales; linea ergo MT secabit arcum GNR. Secet ergo ipsum in N, et extrahamus lineam TM in parte M. Secabit ergo arcum GQR; secet ergo in puncto Q. Et continuemus NE, et extrahatur recte. Secabit ergo ZT sub lineam KL. Secet ergo illam in I. Et continuemus QE et extrahamus ipsam recte. Secabit ergo ZT super KL. Secet ergo ipsam in C.

[7.66] Quia ergo duo anguli T sunt equales, erit I ymago N, et duo puncta K, L sunt ymagines duorum punctorum G, R. Ymago ergo arcus GNR est linea transiens per puncta K, I, L, ut linea KIL. Sed linea KIL est convexa ex parte visus, et arcus GNR est convexus ex parte speculi. Z ergo comprehendet formam linee GNR convexe lineam convexam.

[7.67] Et quia duo anguli T sunt equales, erit C etiam ymago Q, et erit linea LCK ex parte visus concava ymago arcus GQR concavi ex parte superficiei speculi. Z ergo comprehendet formam arcus GQR concavi lineam concavam.

[7.68] In speculis ergo concavis ex quibusdam sitibus comprehenditur linea convexa convexa, et concava concava.

20 quia[1]: que *E*/dyametri: semidyametri *R*/*post* circuli *add.* AE *R*/et . . . circuli (21) *mg. a. m. E*; *rep. P1*; *rep. et del. F*/est etiam *transp. C1*/etiam *om. FP1R* 21 dyametri: semidyametri *R*/*post* circuli *add.* OE *R*/Z *om. R*/ergo: G *FP1*/comprehendit: comprehendet *C1*/formam *om. FP1*/*post* GR *add.* est *L3* 22 AB sperico *transp. R*/est *mg. F*; *om. C1*/voluimus: volumus *E* 23 et: item *FP1*/formam: figuram *R*/*post* formam *scr. et del.* GR *F* 24 revolvamus: constituamus *R* 25 quomodocumque: quocumque *F*/sit[1] *inter. O*; sint *R*/non *inter. E* 26 GH: GPH *FP1*; GDH *O* 27 convertitur: reflectitur *R*/puncto: punctus *FP1*/convertatur: reflectatur *R* 28 T *corr. ex* TE *S*/*post* ZT *add.* et *OR*/MT: ME *FP1* 29 ergo anguli *transp. deinde corr. S*/*post* ZTE *scr. et del.* EC TM *F* 30 GNR: HNR *FP1O*/ipsum *om. R* 31 arcum *om. R*/GQR *corr. ex* GNR *a. m. E*/secet *om. S*/Q: F *O*/*ante* et *add.* de *O* 32 NE: FE *FP1*/sub: super *FP1SOL3E*; supra *C1*/KL *corr. ex* GL *C1*/*post* KL *add.* secabit ergo ZT sub KL secet ergo ipsam in T *FP1* 33 *post* ergo *scr. et del.* in puncto Q *S*/in *mg. F* 34 secabit . . . C *om. FP1*/super: sub *SOL3C1E*; supra *R*/C: P *R* 35 *post* anguli *add.* ad *R*/sunt: inter *L3*/N: H *S*; Z *O* 36 sunt *om. R* 37 ut linea KIL *om. L3*; *mg. a. m. E* 38 *post* visus *add.* Z *R*/GNR *corr. ex* GNT *a. m. E*/est[2] *rep. P1*/convexus: convexa *O*/ex: a *E* 39 Z ergo: ergo visus Z *R*/GNR: GM *L3* 41 *post* anguli *add.* apud *R*/T: R *S*/erit *inter. O*/C: P *R*; *om. S* 42 LCK: LIK *L3E*; LPK *R*/*post* LCK *inter.* concava *O*/concava *om. O*/*post* concava *add.* et est *R*/concavi: concava *O* 43 Z ergo: ergo visus Z *R*/GQR: GR *FP1* 44 concavam: concavi *S* 45 in . . . ergo: ergo in speculis *F*/in . . . concavis *om. P1*/quibusdam: quibus *FP1* 46 convexa[2] *om. FP1SL3*/concava[2] *om. P1*; concavam *F*

[7.69] **[PROPOSITIO 31]** Item sit speculum concavum in quo sit circulus ABD maximus [FIGURE 6.7.31, p. 321], et centrum G, et extrahamus lineam BG, quomodocumque sit, et dividamus ex ipsa lineam GT maiorem medietate. Et extrahamus ex T lineam ETZ perpendiculariter, et sit utraque ET, TZ equalis TG. Et continuemus ET, EG, GZ.

[7.70] Et describamus circa triangulum EGZ circulum. Secabit ergo circulum AB in duo puncta, nam punctus T est centrum huius circuli, et TG est maior TB. Secet ergo iste circulus circulum AB in duobus punctis A, D, et continuemus lineas GA, GD, EA, EB, ED, ZA, ZB, ZD.

[7.71] Quia ergo due linee ET, TZ sunt equales, erunt due linee EB, BZ converse per angulos equales. Et quia duo arcus EG, GZ sunt equales, due linee EA, AZ convertuntur per angulos equales, et due linee ED, DZ convertentur per angulos equales.

[7.72] Et quia GT est maior quam TB, erit GE maior quam EB. Angulus ergo EBG est maior angulo EGB, et angulus EGB est semirectus. Ergo duo anguli EGB, EBG simul sunt maiores recto. Ergo angulus BEG est recto minor, et angulus EGZ est rectus. Ergo due linee EB, GZ concurrent extra circulum in parte BZ. Concurrant ergo in M.

[7.73] Et quia ED est intra angulum MEG, concurret cum linea GM. Concurrant ergo in L. Et quia GB transit per centrum EGZ circuli, erit portio AEG minor semicirculo. Ergo angulus AEG est obtusus, et angulus EGZ est rectus. Ergo ille due linee AE, ZG concurrent in parte EG. Concurrant ergo in F. Si ergo visus fuerit in E, et Z in aliquo visibili, tunc puncta M, L, F erunt ymagines Z. Sic ergo Z comprehenditur in tribus locis.

49 *post* BG *scr. et del.* quocumque sit *S*/quomodocumque: quandocumque *FP1L3E*/ex ipsa lineam (50): lineam ex ipsa *C1* 50 ETZ: ET *C1*/perpendiculariter: perpendicularem *L3E*; perpendicularem super BG *R* 51 utraque ET TZ *mg. O*/*post* utraque *add.* linea *O*/ET² *om. R* 52 EG *corr. ex* TG *a. m. E* 54 AB: ABD *R*/duo puncta: duobus punctis *R*/punctus: punctum *R*/est: etiam *FP1*; et *SOL3E* 55 et: Z *L3E*/TG *om. FP1*; *corr. ex* TH *O*/iste circulus *transp. R*/circulum *inter. E*/AB: ABD *R* 56 duobus *om. R* 58 *post* ergo *add.* iste *L3*/*post* linee² *add.* EG GZ equales et similiter *R* 59 EB BZ: EA AZ *S*/*post* BZ *add.* etiam *E*/converse: convertuntur *S*/converse per angulos *om. R*/angulos *om. S*/et . . . equales² (60) *om. S*/EG GZ: HE HZ *FP1O* (HE: EH *O*) 60 due¹. . . equales² *om. FP1*/EA *corr. ex* A *E*/convertuntur per: reflectentur inter se propter *R*/et . . . equales (61) *om. L3*; *mg. a. m. E* 61 ED DZ: EB BZ *R*/convertuntur per: reflectentur inter se propter *R* 62 maior¹. . . GE *rep. S*/quam¹ *om. C1*/GE: BGE *FP1*; *corr. ex* HE *SO* 63 *ante* angulus¹ *scr. et del.* ergo *E*/EBG: OBH *P1*; EBH *O*; *alter. ex* H *in* OBH *F*/EGB¹: EBH *FP1*; EHB *SO*/EGB et angulus *mg. C1*/EGB²: EHB *FP1SO*; EBG *L3C1E* 64 anguli *om. O*/EGB: EHB *FP1SO*/EBG: EBH *FP1O*/*post* EBG *add.* sumpti *C1* 65 BEG: HEB *FP1S*; BEH *inter. O*/EGZ: EHZ *FP1SO* 67 M: L *R* 68 intra: inter *FP1*/angulum: triangulum *R*/MEG: MEH *FP1SO*; LEG *R* 69 GM: HM *FP1O*/concurrant: concurrat *R*/L: M *R*/GB: GF *O*/EGZ: ZEG *R*; *alter. in* TEG *a. m. E* 70 AEG¹: AG *ER* 72 ergo¹ *om. L3* 73 puncta *corr. ex* punctum *P1*/*post* puncta *scr. et del.* ? *F*/M L *transp. C1*/Z²: et *FP1*/comprehenditur: comprehendetur *R*

[7.74] Item extrahamus ex E lineam ad arcum DZ, quomodocumque sit, et sit EK. Et continuemus GK, et secet arcum DZ in K, et continuemus lineas KZ, GK. Quia ergo arcus EG, GZ sunt equales, erunt duo anguli EKG, GKZ equales. Ergo angulus EKG est maior angulo GKZ. Sit ergo angulus GKN equalis angulo EKG. Due ergo linee EK, KN convertentur per angulos equales. Et extrahamus EK ad Q. Erit ergo Q ymago N respectu E.

[7.75] Et ymaginemur superficiem exeuntem a linea MGF perpendiculariter super circulum ABD, et extrahamus ex Z lineam in hac superficie perpendicularem super GZ, et transeat in utramque partem. Sit ergo CZR. Et ponamus G centrum, et in longitudine GN faciamus arcum circuli CNR. Secabit ergo lineam CR in duobus punctis, et sint C, R. Et continuemus lineas GC, GR. Erunt ergo in superficie perpendiculari super superficiem ABG. Et extrahamus GC, GR recte, et super G, et in longitudine GQ faciamus arcum circuli. Secabit ergo duas lineas GC, GR. Secet in S, O.

[7.76] Quia ergo superficies circuli ABD est perpendicularis super superficiem duarum linearum GC, GR erunt duo anguli EGS, EGO recti. Erit ergo utraque superficies EGS, EGO perpendicularis super superficiem SGO, et utraque istarum superficierum facit in speculo circulum magnum comparem circulo ABD. Punctum ergo compar puncto K circuli quem facit superficies EGC, convertuntur ex ipso secundum angulos equales due linee inter duo puncta E, C.

[7.77] Et linee GC, GR sunt equales, et linee GS, GQ, GO sunt equales, et Q est ymago N, et S est ymago C, et O est ymago R. Yma-

75 ex E *om. L3*/ad *inter. E*/DZ: DB *O*/*post* DZ *scr. et del.* et M B *C1* 76 et[2] *inter. C1*/GK: GB *C1*/et[3] *om. S*/K: H *FP1*; U *O*; B *L3E* 77 lineas: lineam *R*/KZ GK: KZIAEU *O*/GK *om. R* 78 EKG[1]: EUG *O*/GKZ: GZ *FP1*; GUZ *O*; GBZ *L3*; *alter. ex* GZ *in* GBZ *E*/*post* equales *add.* sic ergo duo anguli EUK ZUK erunt equales sed linea EU est maior quam UZ *O*; *add.* et continuemus GK in R et extrahamus ER ZR *R*/ergo *corr. ex* angulo *F*; *om. P1*/EKG[2]: ERG *R*/maior: minor *S* 79 GKZ: EKZ *FP1SOL3C1E*; GRZ *R*/sit ergo *transp. deinde corr. C1*/GKN: GKH *S*; GRN *R*/EKG: ERG *R* 80 *post* linee *scr. et del.* ergo *S*/EK KN: ER RN *R*/convertentur per: reflectentur inter se propter *R*/EK[2]: ER *R* 81 Q ymago *transp. C1* 82 et *om. L3*/ymaginemur: ymaginemus *L3*; ymagine in *S*/superficiem: superficientem *F*/*post* superficiem *scr. et del.* existentem *C1*/perpendiculariter: perpendicularis *L3* 83 *post* super *add.* centrum *L3*/in hac: MH AC *S* 84 utramque *corr. ex* unamque *a. m. E* 85 CZR: CZP *R* 86 CNR: CZR *FP1SOL3E*; *corr. ex* CZR *C1*; CNP *R*/CR *inter. a. m. E*; CZP *R*/sint: situm *S* 87 R: P *R*/GR: GP *R* 88 ABG: ABD *R*/GR: GP *R* 89 G *corr. ex* GI *E*/*post* arcum *add.* cum *C1* 90 GR: GP *R*/secet *om. O* 91 circuli ABD *transp. R*/ABD: ABDG *FSL3C1E*; ABAG *P1*; ABG *O* 92 GR: GP *R* 93 recti *om. P1*/*post* recti *add.* et EG perpendicularis super superficiem GCP *R*/super *om. FP1* 94 superficiem: superficie *FP1*/*post* superficiem *scr. et del.* duarum linearum GT GR erunt duo anguli *S*/SGO *corr. ex* EGO *P1*/*post* istarum *add.* duarum *R*/facit: faciet *C1* 95 ABD: ABO *FP1*; ABGH *O*/compar . . . circuli(96): circuli compar puncto R est *R* 96 circuli: est *E*/quem: quoniam *FP1L3*; quod *R*/*post* superficies *scr. et del.* G *L3*/EGC: EGTO *FP1*; EGO *L3*; EGS *R*/*post* EGC *add.* ergo *R*/convertuntur: concurrunt *FP1SR*; *alter. ex* concurruntur *in* concurrunt *C1* 97 linee *rep. P1*/*post* C *add.* et similiter inter duo puncta D P *R* 98 *post* GC *add.* GN *O*/GR: GP *R*/GS: GC *deinde inter.* S in arabico *O* 99 N *inter. O*/est[2, 3] *om. L3ER*/C . . . ymago[3] *om. S*/R: P *R*/R ymago *mg. C1*

go ergo arcus CNR convexi ex parte speculi est arcus SQO concavus ex parte visus.

[7.78] Et L est ymago Z, et duo puncta S, O sunt ymagines C, R. Ymago ergo linee CZR recte est linea transiens per puncta S, L, O, et talis linea est concava ex parte visus.

[7.79] Et signemus lineam transeuntem per puncta S, L, O, et extrahamus lineam EG ad H. Si ergo speculum non pervenit ad duo puncta B, H, sed alter duorum terminorum suorum fuerit inter duo puncta B, D, et reliquus fuerit infra H, et visus fuerit in E, et due linee RZC, RNC fuerint in aliquo visibili, tunc forma linee RZC recte erit concava, scilicet SLO, et forma arcus RNC convexi erit etiam linea concava, scilicet SQO. Et RZC recta habebit unam ymaginem, et arcus RNC habebit unam ymaginem.

[7.80] Item extrahamus BG ad I, et continuemus lineas EI, IZ. Iste ergo due linee convertentur secundum angulos equales, et EI secabit FG; secet ergo in T. T ergo erit ymago Z. Puncta ergo M, L, T, F sunt ymagines Z. Et si speculum excesserit duo puncta A, I, et visus fuerit in E, et deorsum aspicientis in speculo fuerit ex parte arcus AI, comprehenderit totum arcum IDA.

[7.81] Tunc Z videbitur in quattuor locis, scilicet in L, M, T, F, et videbit duo puncta R, C in duobus punctis S, O, et sic linea RZC habebit quattuor ymagines concavas. Una transit per puncta S, M, O, scilicet linea SMO; secunda pertransibit per puncta S, L, O, scilicet linea SLO; tertia transibit per puncta S, T, O, scilicet linea STO; quarta transibit per puncta S, F, O, scilicet linea SFO.

100 CNR: GT ZR *FP1*; CZR *OL3E*; CNP *R*/SQO: CQO *O*/concavus *corr. ex* concavis *P1* 102 R: P *R* 103 *post* linee *inter.* recte *mg. a. m. E*/CZR: CZP *R*; *corr. ex* EZR *a. m. E*/recte *om. ER*/L: I *FP1*; B *L3* 104 linea *om. L3R* 105 L: I *FP1* 106 H: B *S* 107 B H *transp. O*/duorum *om. R*/terminorum suorum *transp. R*/suorum *inter. a. m. E* 108 D: H *FP1R*/reliquus: reliquis *FP1*/et[2] *inter. O* 109 RZC[1]: IZC *FP1O*; *corr. ex* IZC *E*; PZC *R*/RNC: INC *L3E*; PNC *R*/forma: formam *FP1*/RZC[2]: IZE *L3E*; PZC *R* 110 scilicet *inter. C1*/SLO: SO *FP1*/RNC: ROC *FP1*; *corr. ex* RNO *E*; PNC *R*/erit: erunt *L3*/etiam: et *F*; *om. P1*/linea: linee *L3* 111 RZC: PZC *R*/ et[2]. . . ymaginem (112) *mg. F*; *om. S* 112 RNC: RC *L3E*; PNC *R* 113 I: L *S*/et *om. C1*/IZ: EZ *FP1SOL3E*/iste . . . EI (114) *mg. a. m. E* 114 ergo: autem *O*/convertentur: reflectuntur *R* 115 FG: ZG *SO*/T[1, 2, 3]: U *R*/T[2] *om. O*/ergo[3] *mg. C1* 116 et[1] *om. S*/excesserit: excessit *SE*/A I: AZ AG *FP1SL3C1E*; AL *O*/I: D *R* 117 E *alter. in* I *O*/deorsum: dorsum *L3C1ER*/in speculo *om. R*/ex parte *inter. E*/AI: ABG *FP1SL3C1E*; *corr. ex* AB *O*/*post* AI *add.* et *C1R* 119 in[2] *om. R*/ T: U *R*/et . . . RZC (120) *rep. L3* 120 *post* puncta *add.* scilicet puncta [*inter.* scilicet] *E*/R: I *L3*; P *R*/sic: si *O*/*post* linea *add.* recta *R*/RZC: ZR ZT *FP1SL3C1E*; PZC *R* 121 *post* concavas *add.* et *C1*/transit: transibit *ER*/*post* O *add.* SCL *SO*; *add.* SO *L3E*; *scr. et del.* SCL scilicet per puncta MSO *C1* 122 scilicet . . . SMO *om. C1*/post scilicet[1] *add.* per puncta vel per *FP1*; *add.* per puncta *SL3E*/linea: lineam *FP1*; *om. SL3E*/*post* SMO *add.* secunda pertransibit per puncta SMO *S*/ pertransibit: transibit *OER*/S: F *FE*/S L O *om. P1* 123 SLO: SFO *FP1*/tertia . . . transibit (124) *mg. a. m. E*/tertia . . . SFO (124) *om. FP1*/transibit: pertransibit *L3*/*post* puncta *add.* scilicet *L3E*/ S: F *S*/scilicet . . . O *om. E*/per puncta: linea *O*/T: U *R*/STO: FTO *O*; SUO *R* 124 linea *om. S*

[7.82] Patet ergo ex hac figura quod linea recta in speculis concavis comprehenditur concava, et convexa comprehenditur concava, et quod recta habet plures formas concavas.

[7.83] **[PROPOSITIO 32]** Item sit speculum concavum [FIGURE 6.7.32, p. 322] per cuius centrum transeat superficies, et faciat circulum ABG, et sit centrum D. Et extrahamus ex D lineam, quomodocumque sit, et sit DG, et transeat extra circulum. Et extrahamus ex D in superficie huius circuli lineam perpendicularem super lineam DG, et sit DA. Et abscindamus de angulo ADG recto particulam parvam, quomodocumque sit, et sit angulus GDE, ita quod inter angulum rectum et angulum ADE sit multiplum anguli EDG, et dividamus angulum ADE in duo equalia per lineam DB. Et abscindamus distinctionem equalem angulo EDG, et extrahamus ex D lineam continentem cum DB angulum rectum, et sit DT.

[7.84] Et extrahamus AD in parte D, et sit DK, et extrahamus ex Z lineam continentem cum ZD angulum equalem angulo KDT. Hec ergo linea concurret cum DA, nam duo anguli KDT, ADZ sunt minores duobus rectis. Concurrant ergo in H. Angulus ergo ZHD est equalis angulo ZDT.

[7.85] Et extrahamus ex Z lineam continentem cum ZH angulum equalem angulo BDK obtuso, et sit ZL. Duo ergo anguli LZD, BDZ sunt minores duobus rectis; linea ergo ZL concurret cum DB. Concurrat ergo in L.

[7.86] Et continuemus LH, et circa triangulum HLD faciamus circulum DHL. Transibit ergo per Z, quia duo anguli LZH, LDH sunt equales duobus rectis. Anguli ergo LHZ, LDZ sunt equales, quia basis eorum est idem arcus. Sed angulus ZHD est equalis angulo ZDT; remanet ergo angulus LHD equalis angulo LDT. Et angulus LDT est rectus; ergo angulus LHD est rectus.

126 comprehenditur[1, 2]: comprehendatur *R*/et[1]. . . concava[2] *om. S*; *mg. a. m. E* 127 formas concavas *transp. C1* 128 sit: scit *S* 129 et *inter. a. m. E* 130 lineam . . . D (131) *mg. F*/quomodocumque: quocumque modo *C1R* 131 et[1] *inter. P1* 132 huius *om. S*/super lineam *om. L3* 133 DA: EA *L3* 134 quomodocumque: quocumque *L3*/sit[1]: sint *L3*; *corr. ex* sint *C1*/quod: ut *R*/inter: intra *E*/*post* angulum *scr. et del.* ai *P1* 135 sit: et *S*/multiplum: multi plurii *FP1*; multiplicatio *S*/anguli *om. S* 136 distinctionem: de angulo BDA *R* 137 EDG: EDB *L3E*/continentem *corr. ex* contineam *mg. F*; contingentem *SL3*; *corr. ex* concurrentem *C1* 138 DB: BD *FP1SOL3ER*/DT: DD *FP1*; DX *R* 139 D: TB *E* 140 continentem: contingentem *L3*/KDT: KDX *R*/*post* KDT *add.* et sit ZH *R* 141 concurret: concurrit *FP1L3*/KDT: KDX *R* 142 ZHD: ZDH *S*; DZH *C1*/ZDT: KDT *FP1SL3C1E* 143 ZDT: ZDX *R* 144 continentem: contingentem *SL3*/*post* ZH *inter.* G in arabico *O* 145 ergo *om. FP1L3*/anguli: guli *F*/BDZ: OBZ *FP1*; DBZ *SL3E*; *om. O* 146 duobus rectis *transp. C1*/ZL *corr. ex* LZL *F*/concurret: continet *S*/concurrat: concurrant *OL3C1ER* 148 circa: citra *E*/triangulum: circulum *SOL3E*/HLD: GD *O* 149 DHL: GHL *FP1*; DGL *O*/LZH: LZG *O*/LDH: LDG *O* 150 ergo *om. L3*/LHZ: LGZ *O* 151 ZHD: ZGD *O*/est equalis *transp. FP1*/ZDT: ZDX *R* 152 LHD: LGD *O*/LDT[1, 2]: LDX *R* 153 LHD: LGD *O*; LDH *S*

[7.87] Et abscindamus ex linea DE lineam DM equalem DH, et continuemus LM. Angulus ergo LMD est rectus; circulus ergo LHD transit per M et secat arcum HE in puncto compari Z. Secet ergo in F, et continuemus DF. Angulus ergo LDF erit equalis angulo LDZ, quia arcus LM est equalis arcui LH, et arcus MF est equalis arcui ZH. Ergo arcus FMD est equalis arcui ZHD.

[7.88] Et continuemus lineas HB, HF, FM, FZ, FB. Angulus ergo BHD erit acutus, et angulus GDH erit rectus. Ergo linea HB concurret cum linea DG extra circulum. Concurrant ergo in Q. HF ergo concurret etiam cum DG extra circulum; concurrant ergo in N.

[7.89] Et extrahamus FB quousque secet arcum LZ. Secet ergo in R, et continuemus RM. Angulus ergo FRM, qui est in circumferentia, respicit arcum FM, et angulus FBM est maior angulo FRM, et angulus FBM est in circumferentia ABG. Ergo si BM linea extrahatur in parte M, abscindet de circulo ABG arcum maiorem simili arcus FM.

[7.90] Et arcus FM est duplus similis arcus FE. Et arcus FE est equalis arcui ZA, et arcus ZA est equalis arcui EG, et arcus FE est equalis arcui EG. Ergo arcus GF est duplum arcus GE; ergo arcus GF est similis arcui FM.

[7.91] Si ergo BM extrahatur recte in partem M, abscindet de circulo ABG arcum ultra punctum F maiorem arcu FG. Linea ergo BM secabit lineam DG inter duo puncta G, D. Secet ergo in O. Et extrahamus lineam FM, et secet DO in U; et extrahamus BM in parte B, et secet arcum LR in C. Et continuemus CD.

[7.92] Quia ergo angulus BFZ est in circumferentia ABG, erit angulus BFZ dimidium anguli BDZ. Sed angulus BDZ est multiplus anguli ZDA; ergo angulus RFZ est multiplus anguli ZDH. Ergo arcus

154 abscindamus *corr. ex* abscimdamus *S* / DH: DHG *FP1*; DG *O* 155 LM: LHD *FP1*; LH *SL3E*; LG *O* / circulus ergo *transp. L3* / LHD: LGD *O* 156 per *rep. C1* / et *mg. a. m. E* / secat *corr. ex* sedcat *F* / HE: BE *R* / puncto compari *transp. R* 157 LDF *corr. ex* LDFD *P1* / LDZ: ADZ *FP1* 158 LH: LB *S*; LG *O* / ZH: ZG *O* 159 FMD: LF *R* / ZHD: ZGD *O*; LZ *R* 160 HB: GB *O* / HF: GF *O* / *post* FM *add.* BM *R* 161 BHD: BGD *O*; *corr. ex* HBD *E* / erit[1]: est *R* / GDH: DGH *L3E* / erit[2] *om. R* / HB: GB *O* 162 DG: DH *L3* / concurrant . . . circulum (163) *rep. S* / HF: GF *O* 163 concurrant: concurrunt *FP1*; *corr. ex* concurrent *C1* / N: B *FP1* 164 et *inter. C1* / FB: FH *C1*; *corr. ex* MFB *F* 165 R *corr. ex* K *a. m. E* / RM: NAR *L3* / ergo *mg. C1* 166 FBM: FOM *FP1* 167 FBM: FOM *FP1* / in[1] *inter. a. m. E* / in[2]: ex *R* 168 abscindet: abscondet *FP1* / *post* abscindet *scr. et del.* in L *F* / de: TE *L3* / simili arcus *rep. et del. F* / arcus: arcui *R* / *post* FM *add.* circuli FHD *R* 169 duplus similis: similis duplo *R* / FE[1]: FR *L3* / et arcus FE *om. S* 170 ZA[1]: AZ *L3R*; *alter. ex* AH *in* AZ *E* / ZA[2]: AZ *R* / arcui[2] *om. P1* / EG: ZEG *FP1L3*; *corr. ex* ZEG *E* / et[2]: ergo *R* 171 *post* arcui *add.* FG *FP1C1E* / ergo[1]. . . GE *rep. S* / arcus[1] *inter. a. m. E* / duplum: duplus *R* 173 *post* ergo *add.* arcus *FP1* / de *om. FP1* 174 *post* ABG *scr. et del.* maiorem simili arcus FM et arcus FM est duplus similis arcus FE et arcus FE est equalis arcui EG ergo arcus GF est duplum arcus G ergo arcus GF est similis arcui FM si ergo BM extrahatur recte in partem M [EG *corr. ex* EGD] *S* / F: G *R*; *om. S* 175 secabit *corr. ex* secat *E* / duo *inter. E* / G D *transp. C1* / in O *rep. et del. F* / extrahamus *corr. ex* extramus *F* 176 et[1] *inter. C1* / B *om. P1* 177 LR: LU *L3* 178 est *om. FP1*; *inter. O* 179 BFZ: BFGZ *P1*; *corr. ex* BFGZ *F* / dimidium: dimidius *R*; *corr. ex* dimium *F* 180 ZDA . . . anguli[2] *om. S* / angulus *om. L3* / RFZ: BFZ *R* / ZDH: ZDG *O*

RZ est multiplus arcus ZH, et arcus CZ est maior arcu RZ; ergo arcus CZ est multiplus arcus ZH.

[7.93] Et continuemus CH. Angulus ergo CHD cum angulo CMD sunt equales duobus rectis; ergo angulus CHD est equalis angulo BME. Ergo angulus ZHD addit super angulum CHD angulum CHZ, qui est equalis angulo CDZ, et angulus CDZ est multiplus anguli ZDA. Ergo angulus CHZ est multiplus anguli EDG; ergo angulus ZHD excedit angulum CHD multiplo anguli EDG. Angulus ergo ZHD est equalis angulo FMD, quia arcus FMD est equalis arcui ZHD.

[7.94] Et angulus CHD, ut declaravimus, est equalis angulo BME. Ergo angulus FMD excedit angulum BME multiplo anguli EDG. Ergo angulus FMD excedit angulum OMD multiplo anguli EDG. Et MOG angulus excedit angulum OMD angulo EGD; ergo angulus FMD excedit angulum MOG multiplo anguli EDG.

[7.95] Et angulus FMD excedit angulum MUD angulo EDG solo. Ergo angulus MUD est maior angulo MOG; ergo angulus MOU est maior angulo MUO. Ergo linea MU est maior linea MO. Et quia arcus ZHD est equalis arcui FMD, erunt duo anguli HFD, MFD equales. Due ergo linee HF, FU convertentur equaliter, et similiter HB, BO convertentur equaliter. Q ergo est ymago O, et N est ymago U.

[7.96] Et extrahamus ex M lineam equidistantem linee HQ, et sit MS, et extrahamus ex M etiam lineam equidistantem linee HN, et sit MP. Quia ergo angulus HND est maior angulo HQD, erit angulus MPO maior angulo MSO. P ergo erit inter duo puncta S, U, et quia

181 RZ[1]: RZA *O*/*post* est[1] *scr. et del.* pl *F*/ZH *corr. ex* RZH *F*; ZG *O*/*post* et *scr. et del.* continuemus FH *C1*/RZ[2]: RCZ *L3*; *corr. ex* LZ *E* 182 *post* est *scr. et del.* maior arcu *S*/ZH: ZG *O* 183 CH: CG *O*; LH *L3*; SH *E*/CHD: CDG *O*/angulo: triangulo *L3E* 184 sunt *om. L3E*/sunt equales: est equalis *R*/CHD: CDH *FP1SC1*; CDG *O*/est *om. O*/angulo BME (185) *transp. deinde corr. S* 185 ergo: sed *R*/ZHD: ZDG *O*; *corr. ex* CHD *a. m. E*/super *inter. a. m. E*/CHD: CDG *O* 186 CHZ: CGZ *O*/est[2] *om. FP1* 187 ZDA *corr. ex* CDA *a. m. C1*/ergo[1]. . . EDG *rep. et del. E*/CHZ: CGZ *O*/EDG *corr. ex* ZDG *E* 188 ZHD: ZGD *O*/ZHD . . . ergo (189) *om. L3*/excedit: extendit *FP1O*/CHD: CGD *O*/EDG: EGD *C1* 189 ZHD: ZGD *O*; *corr. ex* ZDH *C1* 190 ZHD: ZGD *O* 191 et . . . CHD *mg. F*/CHD: CGD *O*/ut: aut *L3*; *corr. ex* aut *a. m. E*/declaravimus *corr. ex* declaramuus *P1*; *corr. ex* declarabimus *a. m. E* 192 FMD: FMB *P1*; *alter. ex* FBMB *in* FMB *F*/*post* FMD *scr. et del.* est equalis arcui ZHD *C1*/BME: FME *O*/multiplo: multiplus *L3* 193 OMD: CMD *S*/EDG: ODG *L3*; *corr. ex* edguli *F* 194 MOG: GOM *O*/MOG angulus *transp. OR*/EGD: EDG *L3R*; GDE *C1* 195 MOG: GOM *O*/MOG . . . angulum (196) *mg. a. m. E* 196 angulo: angulus *L3*/EDG: EBG *S*; *corr. ex* AEDG *F*; *corr. ex* ADG *P1*/solo: solum *FP1* 197 MOG . . . angulo (198) *mg. a. m. E* 199 ZHD: GZHD *FP1S*; GZD *O*/est *om. O*/FMD: BMD *O*/HFD: GHFD *FP1*; GLD *O*/MFD: MBD *O* 200 due ergo *transp. L3*/HF FU: GB BO *O*/convertentur[1, 2]: reflectentur *R*/et . . . equaliter (201) *om. S*/HB: GB *O*/BO: BA *FP1*; BQ *O*; BC *L3E*/convertentur[2]: convertuntur *E* 201 O: C *S*/O . . . ymago *mg. S*/N: non *SOL3C1E*/est[2] *om. R* 202 *post* lineam *add.* et similiter HB BO convertentur equaliter *S*/HQ: GQ *O*; HU *L3*; *corr. ex* LQ *P1*/et[2]. . . HN (203) *om. SL3* 203 MS: MF *FP1*/etiam: et *FP1*; *om. C1*/HN: GN *O* 204 MP: MN *O*/HND: GND *O*/HQD: GQD *O* 205 MPO: MDU *O*; CQPO *L3E*/MSO: SMO *E*/S: F *L3*

angulus HDN est rectus, erit angulus HND acutus. Ergo angulus MPD est acutus; ergo angulus MPS est obtusus. Ergo linea MS est maior quam MP.

[7.97] Sed MU est maior quam MO, ut diximus; ergo proportio SM ad MO est maior quam proportio PM ad MU, et proportio SM ad MO est sicut proportio QB ad BO, quia MS est equidistans BQ. Et similiter proportio PM ad MU est sicut proportio NF ad FU; ergo proportio QB ad BO est maior quam proportio NF ad FU. Et proportio QB ad BO est sicut proportio QD ad DO, et proportio NF ad FU est sicut proportio ND ad DU, ut declaravimus in vicesima quinta figura capitulo de ymagine. Ergo proportio QD ad DO est maior quam proportio ND ad DU.

[7.98] Hiis preostensis, iteremus circulum, et perficiamus demonstrationem, ne multiplicentur linee et dubitentur littere. Sit ergo circulus in secunda forma ABG [FIGURE 6.7.32a, p. 323], et centrum D, et extrahamus lineam DQ. Et sit DU equalis DU in prima forma, et DO equalis DO in prima forma. Et DQ sit compar sibi in prima forma, et similiter DN.

[7.99] Et extrahamus super DQ DH perpendicularem super superficiem circuli, et sit DH equalis sibi in prima forma. Angulus ergo HDQ erit rectus, et circulus quem facit HDQ in speculo erit ex circulis ex quibus forma convertitur. Et erit arcus quem mensurant linee HD, DQ equalis arcui AG in primo circulo. Et ex duobus punctis istius comparibus duobus punctis B, F convertentur linee ad duo puncta U, O equaliter. Erit ergo Q ymago O, et N ymago U.

[7.100] Et extrahamus ex U perpendicularem lineam in superficie circuli ABG super lineam DU, et sit ZUE. Et sit D centrum, et in longitudine DO faciamus arcum circuli. Secabit ergo lineam ZUE in

206 HDN: HND *FP1*; GDN *O*/HND: GUD *O*/angulus^{3} *om. L3C1E* 207 MPD: MOP *FP1*; MDU *O* 208 *post* maior *add.* linea *C1*/MP: MD *O* 210 SM^{1}: FM *P1S*/est *rep. et del. F*/est . . . MO (211) *transp. ad* 211 *post* et [SM: FM] *S*/proportio^{1} *om. FP1*/et . . . MU (212) *om. L3* 213 FU: QF *L3E* 214 QB: BQ *C1*/QD: D *FP1*/et . . . DU (215) *mg. a. m. E* 215 ND: QD *O*/*post* in *add.* Z *FP1*/vicesima quinta: secunda *C1*/vicesima . . . figura *om. R* 217 *post* DU *add.* HR *FP1*; *add.* quare patet propositum *R* 218 *ante* hiis *scr. et del.* his *C1*/iteremus *corr. ex* iterremus *S*/perficiamus: proficiamus *FP1*/perficiamus demonstrationem *transp. C1* 219 *post* multiplicentur *add.* et *L3ER*/linee: line *S* 220 forma: figura *L3R* 221 et^{1} *inter. E*/DU^{1}: DB *ER*/DU^{2}: DB *R*/*post* prima *add.* figura *L3*/forma: figura *R* 222 et^{1}. . . forma *mg. F*/DO^{1}: DE *FP1SL3E*/forma: figura *R*/et^{2}. . . forma (223) *om. FP1* 223 forma: figura *R*/et . . . forma (225) *mg. a. m. E*/DN: DU *R* 224 et . . . sit (225) *om. L3*/DQ: D *O*; *om. FP1*/DH *om. R*/perpendicularem: perpendiculariter *C1* 225 forma: figura *R* 226 erit rectus *transp. L3* 227 ex *inter. a. m. E*/*post* forma *add.* punctorum O U *R*/convertitur: reflectitur *R*; *corr. ex* con *O*/*post* convertitur *scr. et del.* et DI *P1*/arcus *om. P1*/quem: que *S*; qui *O*; quam *L3*/mensurant: usurant *S* 228 DQ: QD *C1*/*post* DQ *scr. et del.* et equalis arcui ET *C1*; *scr. et del.* est *E*/AG: HG *L3E*/circulo *corr. ex* ergo *a. m. E*/duobus: duabus *P1* 229 convertentur: reflectentur duo puncta *R*/*post* linee *add.* U P *R*/duo: mo *S*/U: N *R* 230 O^{1}: Q *R*/Q: quasi *F*; *corr. ex* quasi *P1*/O et N ymago *mg. a. m. E* 232 et^{3} *om. S* 233 *post* faciamus *scr. et del.* circulum *E*

duobus punctis. Secet ergo in Z, E, et sit arcus ZOE. Et continuemus DZ, DE et extrahantur extra circulum. Et circa D et in longitudine DQ faciamus arcum TQK. Secabit ergo duas lineas DZ, DE in T, K. Et continuemus TK. Secabit ergo lineam DQ in L.

[7.101] Quia ergo HD est perpendicularis super superficiem circuli, uterque angulus HDT, HDK erit rectus. Et utraque superficies HDT, HDK facit in superficie speculi circulum, et arcus illius qui est inter duas lineas HD, DT erit equalis arcui qui est inter HD, DQ, et similiter arcus qui est inter duas lineas HD, DK. Et utraque linea DZ, DE est equalis linee DO. Ergo hii duo arcus sunt huiusmodi quod ex illis convertentur linee secundum angulos equales ad duo puncta Z, E. Et due linee DT, DK sunt equales linee DQ; ergo punctum T est ymago Z, et K est ymago E.

[7.102] Et quia linee DT, DQ, DK sunt equales, et linee DZ, DO, DE sunt equales, erit proportio DT ad DZ sicut proportio QD ad DO, et sicut proportio KD ad DE. Sed proportio QD ad DO, ut in prima figura preostendimus, est maior proportione ND ad DU. Ergo proportio DT ad DZ est maior proportione ND ad DU, et similiter proportio KD ad DE.

[7.103] Et quia due linee ZD, DE sunt equales, et due linee DT, DK sunt equales, erit linea TK equidistans linee ZE. Ergo utraque proportio DT ad DZ et KD ad DE erit sicut proportio LD ad DU. Ergo proportio LD ad DU est maior proportione ND ad DU; ergo linea LD est maior linea ND. Ergo N est inter L, U. Sed N est ymago U, et duo puncta T, K sunt ymagines Z, E. Ergo ymago linee ZUE recte est linea transiens per puncta T, N, K. Et linea que transit per hec puncta est convexa, ex quibus patet quod linea recta in speculis concavis quandoque videtur convexa in quibusdam sitibus.

[7.104] Item ponamus in linea ZU punctum M, quomodocumque sit, et circa centrum M et in longitudine MU, faciamus arcum RUF.

234 ergo: ? *S* 235 extrahantur: extrahatur *FP1O*; extrahamus *ER*/circa: a *R*; *corr. ex* citra *a. m. E* 236 TQK: THK *FP1*; TQ *R*; *corr. ex* TQH *E* 237 continuemus: continuetur *S*/L: B *L3E* 239 utraque *corr. ex* uterque *F* 240 HDT: HD *S*; HUT *O*; *mg. a. m. C1*/facit: faciet *OL3ER*/illius *om. R*/qui est: quem *FP1*/qui . . . arcui (241) *om. S*/qui . . . arcus (242) *om. L3E* 241 DT: DK *C1*/*post* inter[2] *add.* duas lineas *R* 242 DK: DB *O*; *alter. in* DT *C1* 243 DO: ID *FP1*/*post* quod *add.* est *FP1* 244 convertentur: reflectentur *R*/linee *om. R*/angulos *rep. S*; *corr. ex* angulus *F*/ad: et *S* 245 DT: DE *FP1*/T *om. FP1* 246 *post* E *scr. et del.* ergo *E* 247 DT . . . linee *mg. F*/DK: ZK *O*/linee: linea *L3* 248 erit: ergo *L3*/sicut . . . et (249) *om. S* 249 KD: AD *S*/DO *alter. in* DA *O* 250 ergo . . . DU (251) *om. L3* 251 ND: HD *FP1*; ZD *O*/proportio *om. C1R* 252 DE: D *S* 253 due[1]. . . equales *mg. a. m. E*/equales *corr. ex* est *F*/et[2] *om. L3E* 254 linea *corr. ex* linee *E*/linee *om. R* 255 sicut *om. FP1*/LD: BD *L3* 257 *post* maoir *add.* quam *L3*/ND: HB *FP1*; NED *S*; ED *O*; *corr. ex* NE *C1*/N[1, 2]: non *L3E*/ L U: B O *L3* 258 E: U *O*/linee *corr. ex* linea *C1*/ZUE *corr. ex* ZNE *E* 259 T N K: T C Q K *FP1*; C M K *S*; M K *L3* 260 recta *om. R* 261 in . . . sitibus *mg. a. m. E* 262 quomodocumque: quocumque modo *FP1R* 263 circa: citra *P1*/et[2] *om. S*/MU *corr. ex* LN *a. m. E*/*post* faciamus *add.* ergo *FP1*

Iste ergo arcus secabit arcum UOE in duobus punctis. Secet ergo in R, F, et continuemus lineas DR, DF, et transeant recte usquoque concurrant in arcu TQK in C, I. Superficies ergo duarum linearum HD, DC faciet in speculo circulum a cuius circumferentia convertentur equaliter linee ad R, et similiter superficies duarum linearum HD, DI faciet in speculo circulum a cuius circumferentia convertentur linee ad F. C ergo est ymago R, et I est ymago F, et N est ymago U.

[7.105] Ymago ergo arcus RUF est linea transiens per C, N, I, sed hec linea erit convexa, et arcus RUF est concavus ex parte superficiei speculi. Cum ergo visus fuerit in H et unaqueque linea ZUE, ZOE, RUF fuerit in aliquo visibili, tunc linea ZUE recta comprehendetur convexa, et linea ZOE convexa comprehenditur concava, et concava convexa. Si ergo unaqueque linea ZUE, ZOE, RUF habuerit unam ymaginem, tunc forma illarum linearum erit eodem modo quo declaravimus. Et si habuerit alias ymagines, forte erunt similes aliis ymaginibus, et forte diverse.

[7.106] Patet ergo ex istis figuris quod linee recte in speculis concavis quandoque comprehenduntur recte, quandoque convexe, quandoque concave. Et linee convexe quandoque comprehenduntur convexe, quandoque concave, et concave quandoque comprehenduntur convexe quandoque concave.

[7.107] Forme ergo superficierum visibilium comprehenduntur aliter quam sint in huiusmodi speculis, nam linee recte non sunt nisi in superficiebus rectis, et cum linea recta que existit in superficie plana comprehendatur convexa aut concava, tunc superficies in qua

264 arcum *mg. F*/*post* punctis *scr. et del.* se arcum *S*/ergo^{2} *om. ER* 265 DR: DK *OC1*/et *inter. C1*/usquoque: usquequo *FOE*; usque *S*; quousque *R*/concurrant: currant *FP1L3C1*; *corr. ex* currant *O* 266 arcu *corr. ex* actu *P1*/TQK: DTQK *F*; DTQD *P1*/C: Q *E*; P *R*/I *om. E*/HD: HC *L3*/DC: DP *R* 267 faciet *om. L3E*/circulum: circuli *L3E*/convertentur: reflectentur *R*/equaliter *om. R* 268 R: G *S*/HD: DH *O*/*post* HD *scr. et del.* D *C1*/DI *corr. ex* DY *P1* 269 convertentur: reflectentur *R*/convertentur linee *transp. S*/C: P *R*; *mg. C1*; *corr. ex* E *a. m. E* 270 I: Q *S*; L *L3*/est^{2}. . . N *om. P1*/*post* N *scr. et del.* et *C1* 271 ergo *om. L3*/RUF: MF *L3E*/C: Q *L3E*/N: T *FP1*; Z *O*; M *L3E*/I *om. L3E*/C N I: I P N *R*/sed *inter. O* 272 convexa: concava ex parte visum *R*/*post* est *scr. et del.* linea transiens *S*/concavus: concava *O*; *corr. ex* convus *F*/superficiei: superficies *FP1* 273 unaqueque: unaquaque *SO*/ZOE: ZOZ *FP1*; DOE *O* 274 comprehendetur: comprehenditur *L3E* 275 ZOE: ZOZ *FP1*/comprehenditur: comprehendetur *R*/et concava *om. L3*/*post* et^{2} *add.* RUF *R* 276 linea: linearum *R*/ZOE: ZOZ *FP1*; DOE *O* 277 illarum: istarum *O*; *om. L3E*/linearum: rectarum *L3E*/erit *om. FP1*/modo *rep. et del. C1* 278 *ante* et *scr. et del.* mmmmm *C1*/si *inter. E*/forte *corr. ex* formmnte *C1*/similes aliis *transp. C1* 279 *post* ymaginibus *scr. et del.* mmn *C1* 280 patet *corr. ex* pate *O* 281 comprehenduntur: comprehenditur *FP1* 282 quandoque^{2}. . . convexe (283) *om. S* 283 et concave *mg. C1*/et . . . visibilium (285) *om. L3*/concave quandoque *inter. O*; *transp. E* 284 quandoque concave *inter. a. m. E*/concave *corr. ex* convecave *F* 285 comprehenduntur *inter. O* 286 aliter: alter *P1*/sint *mg. F*; *inter. O*; sunt *ER* 287 nisi *om. FP1*/*post* superficiebus *add.* rebus *FP1* 288 comprehendatur: comprehenditur *R*/aut: vel *FP1*/qua: quibus *FP1SOL3C1E*/*post* qua *add.* ipsa linea *R*

est comprehendetur convexa aut concava. Cum ergo visus comprehendit lineas convexas et concavas et rectas aliter quam sint, comprehendet superficies in quibus sunt aliter quam sint.

[7.108] Patet ergo ex predictis quod in omnibus que in speculis concavis comprehenduntur accidit fallacia, sed in quibusdam accidit semper et in omni positione, in quibusdam vero accidit in aliqua positione. Fallacie autem composite accidunt in hiis speculis eo modo quo in compositis, et hoc voluimus declarare.

CAPITULUM OCTAVUM

De fallaciis que accidunt in speculis columpnaribus concavis

[8.1] In hiis enim accidunt similia eis que accidunt in spericis concavis, accidunt enim fallacie que proveniunt ex conversione, scilicet debilitas lucis et coloris et diversitas situs et remotionis que accidunt omnibus speculis. Accidit autem in eis ex diversitate quantitatis simile illi quod accidit in speculis spericis concavis. Et videtur etiam unum visibile unum, et duo, et tria, et quattuor, et rectum et conversum secundum diversos situs, et planum videtur concavum et convexum. Ostendamus ergo qualiter in hiis speculis diversatur quantitas et numerus rei vise, et qualiter apparet rectum et conversum eo modo quo in speculis spericis concavis declaravimus.

[8.2] **[PROPOSITIO 33]** Iteremus ergo primam figuram ex duabus figuris premissis in fallaciis speculorum columpnalium convexorum, et eisdem litteris. In illa autem figura [FIGURE 6.8.33, p. 324]

289 est: sunt *FP1SOL3C1E* (*inter. E*)/comprehendetur *corr. ex* comprehendet *a. m. E*/convexa aut concava: concava aut convexa *C1*/comprehendit: comprehendat *R*; *corr. ex* comprehendat *E* 290 aliter: alter *P1*/comprehendet . . . sint (291) *mg.* [comprehendet: comprehendit] *C1* 291 aliter: alter *P1*/sint *corr. ex* sunt *E* 293 concavis comprehenduntur *transp. FP1*/*post* concavis *add.* quod *L3C1*/in . . . positione (294) *om. L3* 294 positione: portione *FP1SE*/*post* positione *add.* in quibusdam vero accidit in aliqua positione sed in quibusdam accidit semper et in omni portione *FP1* (sed in quibusdam accidit *rep. et del. F*)/quibusdam[2]: quibus *P1*)/vero *om. ER*/in aliqua positione *mg. F*/positione[2]: portione *L3E* 295 composite: posite *L3* 296 in *om. L3*/compositis: composite *R*/voluimus: volumus *FP1C1* 297 capitulum . . . concavis (298) *om. S*; de erroribus qui accidunt in speculis columpnaribus concavis capitulum octavum *R* 298 in *om. OL3C1*/columpnaribus: columpnalibus *L3C1* 299 enim: autem *R*/*post* accidunt *scr. et del.* in spericis concavis *S*/similia: similes *R*/que: qui *R* 300 proveniunt: perveniunt *S*/conversione: reflexione *R* 1 situs *corr. ex* lucis *a. m. E* 2 *ante* omnibus *add.* in *S*/accidit: accidunt *SOL3*; *corr. ex* accidunt *C1*/*post* autem *add.* in hiis speculis *FP1*/in *om. ER* 3 spericis *om. L3* 4 et[2] *om. E*/et[5] *om. P1*/conversum: convexum *FP1SL3C1ER* 5 videtur: videbitur *O* 6 ostendamus: ostendemus *R*/diversatur: diversificatur *P1* 8 spericis *inter. a. m. E* 9 primam figuram *transp. FP1C1* 10 columpnalium: columpnalis *FP1*; columpnalibus *L3*; columpnarium *C1R*; *corr. ex* columpnarium *E* 11 autem figura *transp. E*

patuit quod linee EG, GT, EB, QB, EA, AH convertuntur secundum angulos equales; et quod linee EO, HA, BQ, TG coniunguntur in O; et quod linea ABG est linea recta extensa in longitudine speculi; et quod linee GZ, BL, AD sunt perpendiculares super superficiem contingentem superficiem que transit per lineam ABG; et quod linea ABG est perpendicularis super superficiem in quo est triangulus EBO; et quod linea TQ est equalis QH, et AB equalis BG; et quod S, C, I sunt ymagines H, Q, T; et quod C est propinquius puncto E quam linea SI; et quod linea SI est in superficie trianguli UHT; et quod due linee UH, UT sunt equales; et quod due linee US, UI sunt equales; et quod due linee ES, EI sunt equales.

[8.3] Et continuemus CU, et secet SI in F. Dividet ergo ipsam in duo equalia, nam HT est divisa in duo equalia in Q, et erit CU in superficie trianguli CUE, qui est superficies circuli B equidistantis basi speculi. Q ergo erit in superficie trianguli CUE, et C est in triangulo CEI. Ergo C est in differentia communi hiis duabus superficiebus. Sed hec differentia est linea EB; ergo C est in rectitudine EB.

[8.4] Et due linee HU, TU sunt sub duobus punctis D, Z, nam due linee HU, TU sunt perpendiculares exeuntes ex H, T super duas lineas contingentes duas portiones in quarum circumferentia sunt puncta A, G. Superficies ergo trianguli UHT est sub axe DLZ.

[8.5] Sed nullum punctum huius axis, quamvis exeat in infinitum, erit in superficie trianguli UHT, nam si esset, tunc, si continuaretur cum aliquo puncto linee HT linea recta, illa superficies in qua esset illa linea recta et linea HT esset tunc superficies trianguli UHT, et illa

12 quod linea *om. S*/linee: linea *FP1OL3C1E*/EG *corr. ex* GEG *F*; EH *O*; AG *L3*/GT: BT *FP1*; HT *O*/QB: RQB *FP1*; KB *O*/convertuntur: reflectuntur *R* 13 *post* linee *scr. et del.* GZ BL EK *C1*/EO: EK *FP1SR*; TK *OL3*; AH *E*; *om. C1*/HA: HK *FP1O*; HL *S*; AB *L3*; TK *E*/BQ: BD *FP1L3E*; LD *S*; BH *O*; QB *R*/TG *om. O* 14 recta *inter. a. m. E* 15 linee: linea *L3*/BL: BK *E* 17 perpendicularis *corr. ex* perpendicalaris *F*/quo: qua *ER*/triangulus: triangulum *R*/EBO: EBD *L3* 18 *post* linea *scr. et del.* ABG est perpendicularis *S*/TQ: TR *FP1S*; TK *O*/QH: RH *FP1S*; KH *O*/*post* BG *inter.* H in arabico *O*/S C I: E S I *FP1*; Q I *S*; C R I *O*; Q L *L3E* 19 Q: R *FP1*; K *SO*/*post* quod *add.* Q *L3C1E*/C: F *O*/SI: CI *O*; SL *L3*/*post* SI *add.* et quod linea S *FP1*/et[2]. . . SI (20) *om. S* 20 SI: CI *O* 21 sunt[1] *corr. ex* sint *FP1*/US: UC *O* 22 ES: EZ *FP1*; EG *S*; ET *O* 23 CU: KU *O*/SI: SIE *FP1L3E*; SEI *S*; CE *O*; *corr. ex* SIE *C1*/F: Æ *R* 24 Q: R *FP1*; K *O*; *scr. et del.* N *S*/CU: KU *O* 25 CUE: KEO *O*; QUE *R*/CUE . . . circuli *om. P1*/qui: que *SOER*/*post* est *scr. et del.* in *C1*/superficies circuli *transp. E*/*post* superficies *scr. et del.* peculi *C1*/circuli *mg. a. m. E*/B: U *L3*; BF *R*/equidistantis *corr. ex* equidistans *a. m. E* 26 Q *om. FP1L3E*; C *SC1R*/Q ergo *transp. R*/erit *om. P1*/trianguli: triangulus *E*/CUE: KEO *O*/et *om. L3E*/C: GT *FP1*; Q *O*; AT *L3*; EIT *E*; *om. R*/est *om. C1*/in[2] *inter. O*/triangulo: superficie trianguli *R* 27 CEI: CEL *FP1*; DEI *L3*/C: G *O*/in *om. L3E*/*post* in *add.* linea que est *R*/communi *alter. in* communis *E* 28 ergo: GQ *S*/C: OC *S*; Q *O*; CQ *L3*; *corr. ex* CQ *C1*; *alter. ex* EQ *in* CQ *a. m. E* 29 et . . . Z *rep.* [et[2]: nam] *L3*/sunt . . . TU (30) *rep. E*; *rep. et del. C1* 30 HU: CN *S* 32 G: H *FP1O*/*post* trianguli *add.* est *E*/UHT est *transp. L3* 33 in *om. L3*; *inter. a. m. E* 34 erit *corr. ex* erat *E*/*post* erit *scr. et del.* circulum *E*/*post* tunc *scr. et del.* si esset *F* 35 linea *om. S*/*post* recta *add.* tunc *R*/illa . . . esset (36) *om. FP1*/*post* superficies *add.* esset *OL3C1E* 36 HT: HZ *L3*/*post* esset *inter.* in *C1*/tunc *om. R*

superficies esset illa in qua sunt due linee equidistantes HT, DZ. Et sic superficies in qua sunt due linee HT, DZ est superficies trianguli HUT, et sic axis erit in superficie trianguli HUT.

[8.6] Sed axis est equidistans linee HT positione, et axis secat duas lineas HU, TU. Et linea TH est in superficie trianguli UEH, que est superficies conversionis, et superficies communis huic superficiei et superficiei columpne est aliquis sector. Superficies ergo EUH secat axem columpne in uno puncto, scilicet in D, ut preostendimus. Et cum axis secet lineam HU, punctus sectionis cum linea HU erit in superficie trianguli UEH. Sed in hac superficie non est punctum per quod axis transeat preter quam D. Ergo linea HU secat axem in D. Et iam ostendimus quod HU secat eum in puncto sub D, quod est impossibile.

[8.7] Ergo axis DZ est extra superficiem UHT et propinquior puncto E quam superficies UHT. Superficies ergo in qua sunt linee HT, DZ est propinquior puncto E quam superficies UHT. Et C est in superficie in qua sunt HT, DZ, quia est in linea QL, et QL est in superficie in qua sunt HT, DZ. Ergo C est propinquior puncto E quam S, I. Sed C est in rectitudine EB. Si ergo EB exiverit in parte B, perveniet ad C; perveniat ergo ad C.

[8.8] Hiis preostensis, dico quod linea SI, que est equidistans axi speculi, cum fuerit in aliquo visibili, et visus fuerit in O ex parte concavitatis columpne, et superficies speculata fuerit superficies concava, tunc SI comprehendetur ex O in speculo ABG concavo, et diversabuntur ymagines eius secundum diversitatem sue distantie ab axe.

[8.9] Cuius demonstratio est quod angulus EBM est acutus; ergo angulus LBC est acutus. Et linea EBC est in superficie circuli B, et LB est dyameter huius circuli. Ergo EB secat circulum; ergo CB est intra concavitatem speculi.

37 et . . . DZ (38) *mg. C1* 38 est: esset *R* 39 erit: esset *R* 40 HT: HDT *S*; HD *O* 41 TH: HT *O*; UH *C1* 42 conversionis: reflexionis *R*/superficies[2]: linea *R* 43 est: et *FP1*/sector: sectio columpnaris *R* 45 cum[1]: si *R*/HU: HTL *FP1*/punctus: punctum *R* 46 est *inter. E* 47 quam *om. ER* 48 eum: EN *P1*; UAD *O* 50 est: esse *S*/UHT *rep. et del. L3*/*post* UHT *scr. et del.* superficies ergo in qua sunt linee HT DZ est extra superficiem NHT et *E* 51 E *om. L3*/superficies[1] *inter. a. m E*/UHT: HUT *FP1L3R* 52 HT *corr. ex* HD *S*/est[2] *rep. P1* 53 in[1] *inter. O*/est: sunt *FP1SOL3C1E*/QL[1]: KL *FP1SOL3C1E*/QL[2]: DL *FP1*; KL *SOL3C1E* 54 C: R *FP1SC1*; K *OL3*; RI *E*/propinquior: propinquius *R*/puncto *om. ER*/S: G *S*; Q *O*; C *C1*/I *om. FP1SOC1* 55 C: Q *O*/exiverit: exierit *P1C1*/perveniet: pervenit *FP1* 56 C[1, 2]: Q *O*/perveniat: perveniet *R*/*post* C[2] *add.* S *FP1* 57 SI: S *FP1*; G *S*; CI *O* 58 fuerit[1] *mg. F*/in aliquo *om. L3*/*post* ex *scr. et del.* tunc *L3* 60 SI: GI *S*; CI *O*; SR *L3*/O: eo *FP1*/in speculo *inter. a. m. E*/ABG concavo *transp. R*/*post* ABG *scr. et del.* in speculo *C1*/*post* concavo *add.* a linea ABG *R* 61 sue *om. R*/*post* sue *scr. et del.* sub *C1*/sue distantie *om. P1* 62 quod: quia *R* 63 angulus *om. R*/LBC: LBQ *O*; LHC *L3*/EBC: ELC *SL3*; EBQ *O*/B: BF *R*/B et: HET *FP1*; BEC *E*/et *inter. O*/LB *corr. ex* LEB *E* 64 huius circuli *transp. FP1*/ergo[1]. . . circulum *rep. et del. F*/EB: EBC *R*/secat *corr. ex* secet *OC1*/CB: QB *O*

[8.10] Et similiter OB est intra concavitatem speculi, quia angulus OBL est acutus, et duo anguli OBL, CBL sunt equales, nam sunt equales duobus angulis EBM, QBM, et LB est perpendicularis super superficiem contingentem columpnam que transit per B. Forma ergo C extenditur per CB et pervenit ad B, et convertitur per BO et comprehenditur a visu ex O.

[8.11] Item in quinto capitulo, cum fuimus locuti de speculis columpnaribus convexis, declaravimus quod superficies contingens columpnam in G erit sub E. Ergo EG secat superficiem contingentem; secat ergo lineam contingentem circumferentiam sectoris in G. Secat ergo sectorem et cadit intra ipsum; cadet ergo intra concavitatem speculi. Ergo due linee OG, GI sunt intra concavitatem speculi, et ZG est perpendicularis super superficiem contingentem columpnam in G, et duo anguli OGZ, IGZ sunt equales. Ergo forma I extenditur per IG et pervenit ad G, et convertitur per GO et comprehenditur ex O per lineam GO. Et similiter S extenditur per SA et convertitur per AO.

[8.12] Et iam declaravimus, cum tractavimus de fallaciis speculorum columpnalium convexorum, quoniam due linee HU, TU sunt perpendiculares super duas superficies contingentes sectores transeuntes per duo puncta A, G. Ymago ergo S est in linea HU. Et OA linea radialis que extenditur ex visu ad punctum conversionis; ergo ymago S est in OA. H ergo est ymago S, et sic patet quod T est ymago I.

[8.13] Et continuemus CL. Quia ergo C convertitur ad O ex circumferentia B, erit ymago Q in linea CL. Et OB est linea radialis que extenditur inter visum et punctum conversionis; ergo ymago C est in linea OB. Ergo ymago C est in puncto sectionis inter QL et OB.

66 et . . . speculi *om. S*/OB: AB *FP1*/est: erit *R*; *om. E*/intra: circa *L3* 67 OBL[1]: ABL *FP1OL3E*/CBL: QBL *O*; *om. P1*/nam sunt equales (68) *om. R* 68 duobus *inter. a. m. E*/QBM: KBM *FP1O*; ABM *S*; TBM *L3*; IBM *E* 69 B: ? *S*; *inter. E* 70 C: Q *O*/extenditur: ostenditur *S*/CB: QB *O*/convertitur: reflectitur *R* 71 ex: in *R*/O: BO *O* 72 item *om. S*/item . . . GO (80) *rep. P1*/item . . . O (80) *rep. F*/speculis *om. P1* 73 columpnaribus: columpnalibus *SE* 74 G: H *FP1SO*/EG: EH *FP1SO*/contingentem . . . lineam (75) *om. C1* 75 secat ergo lineam *om. P1*/*post* contingentem *add.* superficiem *S*/sectoris: sectionis *R*/G: H *FP1SO*; S *E* 76 sectorem: sectionem *R*/ipsum: ipsam *R* 77 OG: OH *FP1SO*/GI: H *FP1*; HI *SO*/et *inter. a. m. E*/ZG: ZH *FP1SO* 78 super superficiem *om. P1*/G: H *FP1SO* 79 OGZ: OBZ *FP1S*; OHZ *O*; *corr. ex* GGZ *E*/IGZ: IHZ *FP1O*; *corr. ex* SIG *E*/I *om. P1*/IG: IH *FSO*; H *P1* 80 G: H *FP1*/convertitur: reflectitur *R*/GO: HO *FP1O*; SG *S*/ex: in *R*/per[2]. . . GO (81) *om. P1* 81 GO: HO *FO*/F: C *O*/SA: EA *S*; CA *O*/*post* et[2] *add.* pervenit ad A et *R*/convertitur: reflectitur *R*/per[2] *mg. E*/AO: CO *O*; SA *L3*; SO *FP1S*; SA *mg. E*/*post* SO *add.* et comprehenditur in O *R* 82 et *inter. O*/tractavimus: detractavimus *S*; *corr. ex* detractavimus *E* 83 *ante* columpnalium *scr. et del.* con *P1*/columpnalium: columpnarium *SC1R* (*alter. in C1*)/quoniam: quando *L3*; quod *R* 84 duas *om. R*/sectores: sectiones *R* 85 G: H *FP1O*/S: C *O*/est *om. L3E*/OA: AO *R*/*post* OA *inter.* est *O* 86 conversionis: reflexionis *R*/ymago *mg. a. m. E*/S: G *S*; C *O* 87 OA: EA *L3*; AO *R*/H: B *SL3*/S: C *O*; *corr. ex* Y *E*/est[2] *inter. O*/I: R *O* 88 CL: QL *O*/C: QC *FP1SL3C1E*; Q *O*/convertitur: reflectitur *R*/circumferentia: circumferentie puncto *R* 89 Q : C *C1R*/Q in: quoniam *FP1L3*/CL: QL *O*; CB *L3*; *inter. E* 90 conversionis: reflexionis *R*/ergo . . . OB[1] (91) *scr. et del. E*/C: CQ *S*; Q *O*/est *om. E*/*post* in *add.* puncto communi CL et OB nempe in puncto Q *R* 91 linea . . . OB[2] *om. R*/C: Q *O*/*post* puncto *scr. et del.* conversionis *C1*/QL: CL *C1*

[8.14] Sed in capitulo de ymagine, cum tractavimus de ymaginibus speculorum spericorum concavorum, patuit quod ymago puncti cuius forma convertitur a concavitate circuli forte concurret cum linea radiali que est inter visum et punctum conversionis ultra circulum, et forte inter visum et circulum, et forte in centro visus, et forte ultra centrum visus, et forte CL equidistans erit OB.

[8.15] Et in illo capitulo patuit quod forte ymago erit unum punctum, aut duo, aut tria, aut quattuor Ymago ergo C forte erit in BQ, forte ultra Q, et forte in BO, et forte in O, et forte ultra O. Et forte ymago C erit unum punctum, aut duo, aut tria, aut quattuor.

[8.16] Si ergo ymago C fuerit Q, tunc HT erit dyameter ymaginis SI. Si ergo omnes ymagines SI fuerint in linea HT, tunc forma eius erit linea recta. Sin autem, erit prope rectam, nam medium eius est in rectitudine duarum extremitatum. Si autem ymago C fuerit ultra Q, tunc ymago SI erit fere concava ex parte visus. Et si ymago visus fuerit in linea BO, tunc ymago SI erit convexa ex parte visus.

[8.17] Et si ymago C fuerit plura puncta, tunc ymago C erit plures linee quarum omnium extremitates coniunguntur in duobus punctis H, T, et media eorum sunt distincta separata. Et HT est dyameter ymaginis SI, quocumque modo fuerit ymago, et dyameter est communis omnibus ymaginibus eius si plures habuerit ymagines, et linea HT est maior quam SI modica quantitate.

[8.18] Patet ergo quod, cum linee recte equidistantes axi columpnali speculi concavi fuerit in aliquo visibili, ymago eius forte erit recta aut concava, et forte una erit aut plures.

[8.19] **[PROPOSITIO 34]** Item iteremus secundam figuram de fallaciis speculorum columpnalium convexorum. In hac autem figura [FIGURE 6.7.34, p. 325] dictum est quod due linee EB, BH conver-

92 *post* sed *scr. et del.* cum *P1* 93 *post* spericorum *inter.* et *C1* 94 convertitur: reflectitur *R*/*post* convertitur *scr. et del.* et *C1* 95 linea radiali *transp. R*/conversionis: reflexionis *R*/circulum: speculum *R* 96 inter *inter. E*/circulum: speculum *R*/et[3]. . . centrum (97) *rep. S* 97 *post* visus *add.* EC *L3*/CL: QL *O* 99 C: CQ *FP1*; Q *O*; *om. ER*/BQ: BK *FP1SO* 100 ultra[1] *om. P1*/Q: R *FP1S*; K *O*; B *C1*; OQ *L3ER*/O[1]: MO *E*/O[2] *om. R* 101 C: CQ *FP1*; TQ *SL3ER*; Q *O* 102 C: Q *O*/fuerit: fuit *FP1S*/Q: R *FP1SO*/*post* tunc *add.* ergo *C1*/HT: HQT *R* 103 SI[1]: S *FP1*; ST *S*; CI *O*/omnes ymagines *transp. deinde corr. S*/SI[2]: S *FP1*; CI *O*/fuerint *corr. ex* fuerit *O*; fiunt *L3*/HT: HQT *R* 104 sin . . . rectam *om. R*/erit[2] *om. FP1*/eius *inter. E* 105 *post* extremitatum *add.* HT *R*/C: CQ *FP1*; Q *O* 106 Q: C *FP1*; R *S*; K *O*/SI: S *FP1*; CI *O*/fere *om. FP1*/et . . . visus (107) *om. R*/ymago[2]. . . visus (107) *om. L3C1E* 107 BO: FO *O*/SI: CI *O* 108 C[1]: CQ *S*; Q *O*; *inter. C1*/fuerit: fuit *L3*; fuerint *R*/C[2]: CI *O*/erit: erunt *R* 109 coniunguntur: coniungentur *R* 110 eorum sunt: earum erunt *R*/*post* sunt *add.* distantia *O*/*post* distincta *add.* et *R* 111 SI: sed *FP1*; CI *O* 112 habuerit: habuerint *S* 113 SI: CI *FP1O*; C *SL3E* 114 cum *om. L3*/linee recte *transp. FP1*; *alter. in* linea recta *C1*/equidistantes *alter. in* equidistans *C1*/columpnali: columpnaris *R*; *corr. ex* columpnalis *F*; *alter. in* columpnalis *O* 115 fuerit: fuit *S*; fuerint *R*/eius: earum *R* 116 aut[1]: et *FP1*/erit *om. R* 118 columpnalium: columpnarium *SR* 119 EB: EG *FP1*/BH: UT *FP1*; HB *ER* (*alter. in E*)/convertuntur: reflectuntur *R*

tuntur secundum angulos equales; et quod due linee EG, GT convertuntur secundum angulos equales; et quod HB, TG perveniunt ad L; et HB continet cum BO angulum acutum. Ergo HB secat superficiem contingentem superficiem columpne in B; BL ergo est sub concavitate columpne. Et similiter GL, et similiter due linee BR, GY.

[8.20] Et duo anguli LBD, DBR sunt equales, et duo anguli LGD, DGY sunt equales. Si ergo RY fuerit in aliquo visibili, et visus fuerit in L, et superficies concava columpne fuerit tersa, tunc forma R extenditur per RB, et pervenit ad B. Et convertetur per BL, et perveniet ad L, et comprehendetur ex L. Et linea HU est perpendicularis super lineam contingentem sectorem ex cuius circumferentia convertentur due linee RB, BL. H ergo est ymago R. Et similiter declarabitur quod forma Y extenditur per YG et convertitur per GL, et ymago eius est T.

[8.21] Et continuemus KU. Secabit ergo RY in M. M ergo est in superficie transeunte per axem et per L, nam L et K sunt in hac superficie; ergo KU est in hac superficie. Et quia duo puncta M, L sunt in superficie transeunte per axem columpne, ideo forma M convertetur ad L in hac superficie. Et linea AZ est differentia communis inter superficiem columpne et superficiem transeuntem per suum axem et per L; forma ergo M convertetur ad L per AZ.

[8.22] Et continuemus EM, que est in hac superficie. Et EL etiam est in hac superficie, et punctum E est elevatum a superficie contingente superficiem columpne in linea AZ. Ergo si AZ extrahatur recte in parte Z, concurret cum duabus lineis EM, EL. Concurrat ergo cum EM in I, et cum EL in N. N ergo est inter duo puncta E, L, quia L est

120 et . . . equales (121) *om. FP1*/EG *corr. ex* EH *O*/GT: UT *L3E*; TG *R*; *corr. ex* HT *O*/convertuntur: convertantur *S*; reflectuntur *R* 121 *post* quod *add.* due linee *O*/TG: TH *FP1O*; G *S*/perveniunt *corr. ex* perperveniunt *P1*/ad *rep. FP1E*/L et (122) *om. S* 122 BO: BD *O* 123 superficiem columpne: columpnam *R*/in *corr. ex* N *E*/in . . . columpne (124) *mg. a. m. E*/B: H *L3*/*post* est *add.* in *FP1C1*/sub *om. FP1* 124 GL: BL *FP1*; HL *O*; GR *L3E*/BR: BS *O*/GY: HY *O* 125 DBR: DBS *O*; BDR *C1*; *alter. ex* GDY *in* DGY *a. m. E*/et[2]. . . equales (126) *scr. et del. E*/duo anguli: similiter *R*/LGD: LD *FP1*; GLD *S* 126 DGY: DG *FP1*; DGB *O*; TGY *E*; GDY *R*/RY: XXY *FP1*; SY *O*/*post* et *scr. et del.* f *P1* 127 R: S *O* 128 RB: RO *FP1SL3*; SB *O*; ID *E*/B: D *P1*/convertetur: reflectitur *R*/perveniet: pervenit *C1* 129 L[1] *om. S*/ex: in *R*/*post* ex *add.* B *O*/*post* HU *scr. et del.* est concava columpne *C1*/*post* perpendicularis *add.* est *L3* 130 sectorem: sectionem *R*/cuius *om. FP1*/convertentur: reflectentur *R* 131 RB: SB *FP1O*; FB *L3E*; BR *R*/BL: DL *L3*/H: HG *FP1SOL3E*/ergo est *transp. FP1L3*/R: S *O*/et *om. ER* 132 extenditur: extendetur *SOC1*; *corr. ex* extendetur *E*/YG: YB *FP1*; YH *O*/convertitur: reflectitur *R*/GL: HL *FP1O*/est *om. C1*/T: CT *FP1*; CS *L3*; OS *E* 133 KU: QU *R*/RY: SY *O*/est *om. O* 134 et[1]. . . axem (136) *om. FP1*/et[2]: Z *SOE*/K: Q *R* 135 KU: QU *R*/M L *inter. E* 136 ideo: vel O *E*/convertetur: reflectetur *R* 137 ad L *om. P1*/*post* et *inter.* quia *a. m. E*/linea: quia *R*/superficiem *inter. a. m. E*/superficiem columpne (138) *transp. ER* 138 superficiem: superficie *L3*/transeuntem: transeunte *L3* 139 convertetur: reflectetur *R*/ad L per: a linea *R* 140 EM: OM *SOL3E*/est *inter. E*/EL: L *L3*/EL . . . superficie[1] (141) *mg. a. m. E*/etiam *om. SER* 141 elevatum: elongatum *R* 143 concurret: concurrat *FP1SL3*; *corr. ex* concurrat *E*/EL: L *L3* 144 EL: L *L3*/N[2] *mg. F*; *om. L3E*/N ergo *transp. R*/L[2]: EL *S*

intra concavitatem columpne, et N est in superficie columpne, et E est elevatum a columpna.

[8.23] Et in demonstratione huius figure patuit quod circulus BZG est medius inter lineam HT et superficiem exeuntem ex E equidistantem basi columpne. Et perpendicularis que exit ex E super AZ est in superficie exeunte ex E equidistante basi columpne. Ergo perpendicularis que exit ex E super lineam AZN cadit extra triangulum EIN et in parte N. Angulus ergo EIN est acutus; ergo angulus MIA est acutus.

[8.24] Extrahamus ergo ex M perpendicularem super AI, et sit MQ. Q ergo erit ultra I respectu N. Et extrahamus MQ ex parte Q, et dividamus QS ad equalitatem QM. S ergo erit extra superficiem speculi et ultra concavitatem eius, et L erit sub concavitate eius.

[8.25] Et continuemus LS. Secabit ergo NQ in F, et ex F extrahamus FX ad equidistantiam QM. Ergo est perpendicularis super AN et in superficie transeunte per axem et per L; ergo est dyameter circuli exeuntis ex F equidistantis basi columpne. Linea ergo XF est perpendicularis super superficiem contingentem columpnam transeuntem per AZ.

[8.26] Et continuemus MF. Erit ergo equalis FS, et duo anguli qui sunt in M, S erunt equales. Et quia XF est equidistans MS, erunt duo anguli F equales duobus angulis qui sunt apud S, M. Due ergo linee MF, FL convertuntur per angulos equales, et XF est perpendicularis super superficiem contingentem superficiem speculi in F. Forma ergo M extenditur per MF, et convertitur per FL, et ymago eius erit S.

[8.27] Et quia due linee RY, HT sunt equidistantes et perpendiculares super superficiem transeuntem per axem et per L (quia HT fuit posita talis), ideo due superficies exeuntes a duabus lineis HT, RY erunt equidistantes. Et quia RY est perpendicularis super superficiem transeuntem per axem et per L, ideo superficies duarum

145 N: non *SL3* 146 elevatum: elongatum *R* 147 BZG: BZH *FP1O*; BZD *S*; BG *R* 149 basi: basibus *R*/et *om. S* 150 basi *om. FP1SL3C1ER*; *inter. O* 151 ex *inter. E*/extra: intra *FP1SL3C1E* (*inter. a. m. E*)/EIN: EIA *S* 152 et *inter. a. m. E* 154 M: EM *S*/super AI *transp. L3* 155 MQ[1, 2]: MK *R*/Q[1, 2]: K *R*/*post* Q[2] *add.* in S *R* 156 QS: QR *O*; KS *R*/QM: KM *R*/S: R *O*/S ergo *transp. R* 158 LS: LR *O*/secabit *corr. ex* secat *E*/NQ: NK *R*/F[2]: S *FP1* 159 FX: BZ *O*/QM: cum *L3E*; MK *R*/ergo est: cum ergo FX sit *R* 160 per[2]: P *FP1* 161 equidistantis basi columpne *om. P1*/XF: FX *R*; ZF *O*; *corr. ex* ZF *C1* 162 super *mg. E*/contingentem: continentem *L3* 164 *post* equalis *add.* quia duo latera SQ QF super equalia duobus lateribus MQ QF et anguli contensi quia uterque rectus *L3*/FS: FR *O* 165 in: apud *R*/S: R *O*/XF: ZF *FP1O*; EXF *S*/MS: MG *FP1L3ER*; MR *O* 166 *post* anguli *add.* apud *R*/equales duobus *transp. deinde corr. O*/qui *corr. ex* quod *O*/S: R *O* 167 convertuntur per: reflectuntur secundum *R*/et *inter. O*/XF: ZF *FP1SOL3E* 168 superficiem speculi: speculum *R*/*post* superficiem[2] *add.* circuli *P1* 169 MF: M *P1*/convertitur: reflectitur *R*/FL: F *FP1*/S: L *FP1SOL3E* 170 RY: XY *FP1S*; SY *O*/HT: HZ *FP1* 171 transeuntem *om. FP1*/*post* axem *add.* C *FP1* 172 *post* HT *scr. et del.* RY erunt *S*/fuit: sunt *L3*/duabus: duobus *S* 173 RY[1]: XY *FP1*; SY *O*/*post* equidistantes *add.* et perpendiculares *R*/RY[2]: SY *O*

linearum RM, MS erit perpendicularis super superficiem transeuntem per axem et per L. Et erit MS differentia communis hiis duabus superficiebus, et quia AQ est in superficie transeunte per axem, et est perpendicularis super MS, que est differentia communis inter superficiem transeuntem per axem et inter superficiem duarum linearum RM, MS, erit AN perpendicularis super superficiem duarum linearum RM, MS.

[8.28] Et linea AN est equidistans axi columpne; ergo axis columpne est perpendicularis super superficiem in qua sunt due linee RM, MS. Superficies ergo ista est perpendicularis super axem columpne. S ergo in superficie exeunte ex linea RY perpendiculariter super axem columpne.

[8.29] Sed linea HT est in superficie perpendiculari super axem columpne equidistanti superficiei exeunti ex linea RY. S ergo est extra HT et propinquior L quam HT. Et duo puncta H, T sunt ymagines R, Y, et punctum S est ymago M; ymago ergo linee RMY est linea transiens per H, S, T.

[8.30] Sed talis linea est arcualis, quia S est extra HT, et transeat per puncta H, S, T linea HST arcualis. Et quia HT, secundum positionem, fuit elevata a convexo columpne, erit HT ultra superficiem speculi respectu L. Et iam declaravimus quod S est ultra concavitatem speculi respectu L; ergo tota linea HST est ultra concavitatem superficiei speculi. Et L est sub concavitate speculi; ergo L est extra superficiem in qua est linea HST. Arcualitas ergo linee HST apparebit visui L manifeste.

[8.31] Et quia F est in superficie columpne, et TH est ultra columpnam, et TH est in superficie trianguli LHT, erit linea LFS altior quam superficies trianguli LHT. Linea ergo LS erit altior duabus lineis LH,

175 RM: EM *FP1SL3E*; CM *O*/MS: MG *FP1SL3E*; MR *O*/super *om. F* 176 per: P *FP1L3*/et² *inter. P1*; *om. S*/MS: MR *FP1SL3E*; ML *O* 177 AQ: AK *R*/superficie: superficiente *S*; *corr. ex* superficiente *F*/*post* transeunte *scr. et del.* in *E*/*post* axem *scr. et del.* est *P1* 178 MS: MR *O* 180 RM: CM *FP1SOL3E*/MS: MG *FP1L3E*; MR *O*/*post* erit *add.* linea *O*/AN *inter. a. m. E*; AKN *R* 181 linearum *om. FP1SC1E*; *inter. O*/RM: CM *FP1SOL3E*/MS: MR *O*; MG *L3E* 183 due linee (184) *om. R* 184 RM: CM *FP1SOL3E*/MS: MG *S*; MR *O*/superficies *corr. ex* superficieses *E* 185 S: R *O*/*post* ergo *add.* est *R*/ex *corr. ex* a *E*/ex linea *rep. et del. F*/RY: SY *O*/perpendiculariter: perpendiculari *ER* 188 columpne *om. R*/equidistanti: equidistante *R*/exeunti *om. R*/RY: SY *O*/S: R *O* 189 propinquior: propinquiori *FP1*; propinquius *R*/L *om. FP1*/*post* quam *add.* sint *R*/HT: H et T *R* 190 R: S *SO*/Y: I *S*/S: R *O*/RMY: RIAY *F*; NAY *P1*; SMY *O*/*post* RMY *scr. et del.* SMY *C1* 191 S: R *O*; A *L3* 192 linea est *transp. R*/quia . . . arcualis (193) *om. S*/S: R *O*/est *om. P1*; *inter. E* 193 S: R *O*; C *L3*/linea HST *rep. et del. F*/HST: HRT *O*; HCT *L3*/*post* HST *add.* est *P1*/et *om. E*/*post* HT *add.* est *P1* 194 elevata: elongata *R*/*post* elevata *add.* et *C1*; *add.* ei *E*/HT: HR *C1* 195 S: R *O* 196 HST: HRD *O*/est: erit *ER* 197 superficiei speculi *transp. S*/L¹: EL *R* 198 HST¹˒²: HRT *O* 199 L: vel *S* 200 columpne . . . superficie (201) *om. L3*/TH: BH *FP1SOL3C1E*/est² *om. R*/ultra: intra *OC1*; inter *E* 201 et TH *om. R*/TH: BH *FP1SOL3C1E*/LHT: BHT *FP1*/LFS: LFR *O* 202 LS: LR *O*/*post* altior *scr. et del.* quod *E*

HT respectu visus L. S ergo est altior quam dua puncta H, T; linea ergo HST apparebit visui L concava.

[8.32] **[PROPOSITIO 35]** Item secemus columpnam per superficiem declinem super axem eius, et non transeat per totum axem. Faciet ergo sectorem. Sit ergo ABG [FIGURE 6.8.35, p. 326]. Sed in prima figurarum de columpnis concavis declaratum est quod in superficie cuiuslibet sectoris columpne erit perpendicularis super superficiem contingentem columpnam ex cuius extremitatibus convertuntur forme. Sit ergo perpendicularis GZ, et sit BE perpendicularis super lineam contingentem circumferentiam sectoris in B, et sit B prope G. BK ergo secabit perpendicularem GZ, et continebit cum ipsa angulum acutum. Secet ergo in E. Angulus ergo BEG erit acutus.

[8.33] Et extrahamus ex G lineam ad equidistantiam linee BK, et sit GD. Angulus ergo DGE erit acutus; ergo GD erit intra concavitatem columpne. Et ponamus angulum EGL equalem angulo EGD. GL ergo concurret cum BE in L. Et signemus M in linea LE. Erit ergo angulus MAG acutus, quia AM est intra sectorem.

[8.34] Et ponamus angulum GAD equalem angulo GAM. Ergo AD concurret cum GD, nam duo anguli qui sunt apud A, G sunt acuti. Concurrant ergo in D. AD ergo secabit BK. Secet ergo in T.

[8.35] Cum ergo BK fuerit in aliquo visibili, et visus fuerit in D, tunc forma L videbitur in G, quia forma L convertetur ad D ex G, et quia DG est equidistans perpendiculari LB. Et forma M videtur in T, quia forma M convertitur ad G ex A, et T est ymago M.

[8.36] Et transeat per D superficies equidistans basi columpne. Secabit ergo superficiem ABG et faciet in superficie columpne circu-

203 HT: T *FP1SOL3E* (*inter. F*); *corr. ex* T *C1*/L *om. R*/S: R *O*/S ergo *transp. R*/altior: altius *R*/dua: duo *OC1ER* 204 ergo HST *transp. deinde corr. C1*/HST: HRT *O*/L: I *S* 206 *ante* declinem *add.* contingentem *E*/et . . . axem[2] *om. R* 207 faciet: faciat *C1*/sectorem: sectionem columpnarem *R* 208 figurarum: figura *R* 209 cuiuslibet: cuiusque *FP1*/sectoris: sectionis *R*; *corr. ex* linee *L3*/*post* columpne *add.* exit a puncto reflexionis *R*/erit: exit *SOC1E*; *om. R*/super *om. L3* 210 columpnam *om. R*/convertuntur: reflectuntur *R* 211 GZ: HZ *FO*; GA *R*/GZ . . . perpendicularis *om. P1*/BE: BEK *R* 212 super: supra *SOL3C1*/sectoris: sectionis *R* 213 secabit: secet *E*/GZ: GA sub axe *R* 214 in . . . ergo[2] *om. S*/angulus ergo *transp. C1*/BEG: BG *E* 216 equidistantiam *corr. ex* equidistantem *C1*/et[2] *om. L3E* 217 ergo[1]: G *S*/DGE: GDE *E*/GD[2]: GDE *R* 218 EGL: EBL *FP1*; EHL *O*/EGD: EG *FP1* 219 GL: DG *FP1*/concurret *corr. ex* concurrat *P1*/*post* signemus *add.* punctum *R*/M: LN *FP1L3E*; FN *S*/ergo *mg. F*/ergo angulus (220) *transp. F* 220 angulus *om. R*/MAG *corr. ex* MAGI *F*/quia: ergo *R*/sectorem: sectionem *R* 221 ergo *rep. et del. F* 222 A: H *O*; *corr. ex* D *a. m. E* 223 AD: LD *O*/*post* AD *add.* D *L3E*/BK: BH *FP1L3E* 224 BK: HK *FP1L3E*; LEK *O*; LK *R*/D: C *FP1* 225 L[1] *inter. E*/videbitur: videbit *FP1*/quia *corr. ex* quod *a. m. E*/convertetur: reflectetur *R*; *corr. ex* contervertetur *F*/D: C *FP1* 226 LB: DB *SO*; BLK *R*/videtur: videbitur *R*/T: CT *FP1* 227 convertitur: reflectitur *R*/G: D *C1R*/est *inter. E*/est ymago *transp. ER* 229 superficiem: sectionem *R*/*post* superficiem *scr. et del.* EG *E*

lum COR. Superficies ergo huius circuli secabit BK, secat enim GD, que est ei equidistans. Secet ergo BK in K, et sit centrum circuli CR punctum H. Et continuemus DH, et transeat ad R. Et continuemus KH, et transeat ad C.

[8.37] Forma ergo K convertitur ad D ex circumferentia ex arcu RC, ut patuit in ymaginibus circulorum. Convertatur ergo ex O, et continuemus KO, DO, HO. Anguli ergo qui sunt apud O sunt equales, et DO secabit HC in N. N ergo est ymago K.

[8.38] Et continuemus KD. KD ergo erit differentia communis inter circulum RC et sectorem ABG, nam duo puncta K, D sunt in utraque superficie, nichil enim de superficie sectoris ABG est in superficie circuli RC nisi linea KD. G ergo est extra circulum, et similiter T, et sunt in superficie sectoris.

[8.39] Et N est in superficie circuli, et forma LMK transit per puncta G, T, N, et linea que transit per hec puncta est arcualis. Sed superficies sectoris est declinis super superficiem columpne; axis ergo sectoris non transit per totam axem columpne, nec est equidistans basi columpne.

[8.40] Patet ergo ex hac figura et duabus premissis quod linee recte equidistantes axi columpne et equidistantes basi eius, et etiam ille que declinantur super superficiem eius, forte videbuntur arcuales, forte recte, forte converse. Item, quia T est ymago M, et N ymago K, erit forma MK conversa.

[8.41] Et si linea etiam fuerit in superficie circuli equidistanti basi columpne, cuius superficies transit per centrum visus, ut dictum est in ymaginibus circulorum in septimo capitulo huius tractatus, forma forte erit equalis recta, forte conversa.

[8.42] Patet ergo quod forma eorum que comprehenduntur in speculis columpnalibus concavis forte erit recta, forte conversa.

230 COR: POR *R*; *corr. ex* COZ *a. m. E* 231 que: qui *C1*/ei *om. S*/ei equidistans *transp. C1*/*post* equidistans *inter.* est *mg. a. m. E*/secet ergo *transp. ER*/ergo *mg. E*/CR: POR *R*; *corr. ex* Q *a. m. E* 233 *ante* KH *add.* ad *S*/C: P *R* 234 ergo *inter. O*/convertitur: reflectitur *R*/ex arcu: arcus *R* 235 RC: RP *R*/in: de *R*/circulorum: speculorum *R*/convertatur: reflectatur *R* 237 HC: HP *R* 239 RC: IR *S*; RP *R*/sectorem: sectionem *R* 240 *post* superficie[1] *add.* et *R*/enim *om. R*/superficie[2]. . . in *rep. P1*/sectoris: sectionis *R*/ABG: AB *S* 241 RC: IT *E*; RP *R*/G *inter. O*; *corr. ex* ergo *P1* 242 T: B *R*/sectoris: sectionis *R* 243 N: enim *P1*/*post* circuli *add.* RP *R*/puncta: punctum *FP1L3E* 244 G T N: G M *FP1SL3E* 245 sectoris[1, 2]: sectionis *R*/declinis: declins *S*/axis . . . columpne (247) *om. S* 246 totam: totum *FP1C1ER* (*alter. ex* totam *a. m. E*)/nec: neque *OC1R*/basi columpne (247) *transp. C1* 248 figura: forma *C1*/duabus: dua *P1* 249 equidistantes[2]: equidistans *L3* 250 *post* ille *add.* linee *R*/declinantur: obliquantur *R*/videbuntur: videntur *C1* 251 converse *alter. in* convexe *O*/item: et *R*/ymago . . . N *om. FP1*/N *inter. O*/est[2] *om. R* 252 erit: est *FP1*/MK: in K *S* 253 si linea: similia *SOC1*/linea: ista *FP1*/fuerit: fiunt *SO*/circuli *inter. a. m. E*/circuli equidistanti *transp. E*/equidistanti basi: equidistante basibus *R* 254 columpne: commune *L3* 255 in[1]: de *R*/circulorum *om. S*; speculorum concavorum *R*/tractatus: contractatus *L3* 256 forte[1] *inter. O*; *om. L3*/forte erit *transp. E*/forte[2] *inter. a. m. E* 257 comprehenduntur: comprehenditur *E* 258 columpnalibus: columpnaribus *P1L3C1ER*

[8.43] **[PROPOSITIO 36]** Item iteremus formam tertie figure de fallaciis speculorum spericorum concavorum, ipsis litteris existentibus. Et sit circulus BZA [FIGURE 6.8.36, p. 327] in superficie speculi columpnalis concavi, et sit visus in D. Erit ergo extra superficiem circuli, et erunt due linee EA, EB perpendiculares super superficiem contingentem superficiem columpne. Et erit superficies trianguli DGE perpendicularis super superficiem circuli, quia DG est perpendicularis super superficiem circuli.

[8.44] Superficies ergo trianguli DGE transit per totum axem et per D, et neutra superficies DBO, DAO, que se secant in linea DO, transit per totum axem. Et in neutra superficie est aliquid de axe columpne nisi E, quod est centrum circuli. Et utraque superficies DBO, DAO facit in superficie columpne sectorem, et forme convertuntur ex hiis sectoribus a duobus punctis A, B.

[8.45] Forma ergo R convertitur ad D ex B, et forma M convertitur ad D ex A, et NU erit dyameter ymaginis MR, et est minor quam MR. Et similiter duo puncta H, L convertuntur ad D ex duobus punctis A, B, et erit TK dyameter ymaginis LH, et est ei equalis. Et erit CI diameter ymaginis FQ, et est maior illa. Et omnes iste ymagines erunt converse.

[8.46] Et si visus fuerit in O, et linee CI, TK, NU fuerint visibiles, erunt econtra, tunc enim dyameter ymaginis CI erit minor ipsa, et dyameter ymaginis NU erit maior ipsa, et erit dyameter TK equalis ei, et omnes ymagines erunt recte. Et omnia ista ostensa sunt in predicto capitulo.

[8.47] Item cum utraque extremitas alicuius harum habuerit unam ymaginem, et aliquod punctum in medio habuerit plures ymagines, tunc illa linea habebit tot ymagines quot punctum medium habet. Et si utraque extremitas vel altera habuerit plures ymagines, et punc-

260 speculorum *mg. a. m. E*/spericorum *om. R*/spericorum concavorum *transp. L3*/ipsis: iisdem *R*/litteris: lineis *FP1*; litteribus *L3*; *inter. O* 261 BZA: BAZ *C1* 262 columpnalis: columpnaris *ER* (*alter. in E*) 263 super *inter. a. m. E*/superficiem: superficiens *S*; superficies *L3ER*/*post* superficiem *scr. et del.* circuli *C1* 264 contingentem: contingentes *SL3ER*/ superficiem *mg. a. m. E*/erit *inter. a. m. E* 265 quia *scr. et del. C1*/DG: GD *OR*; GB *L3C1E*/ *ante* est *inter.* quia *a. m. C1* 267 ergo *inter. a. m. E*/axem . . . axem (269) *scr. et del. E*/et *om. E*/et . . . axem (269) *om. R* 268 se *om. FP1*/secant: secat *FP1* 269 *post* superficie *add.* DBO DAO *R* 271 DBO: ABO *FP1*/facit: faciat *L3*/sectorem: sectionem *R*/et forme *om. L3*/convertuntur: reflectuntur *R* 272 sectoribus: sectionibus *R*/a: ad *C1* 273 R: I *L3E*/convertitur[1, 2]: reflectitur *R*/ex . . . D (274) *om. S* 274 ad D *om. L3ER*/et[2]. . . MR *inter. O*/minor: maior *L3* 275 convertuntur: reflectuntur *R*/D *inter. E* 276 est: erit *R*/CI: PI *R* 278 converse: convexe *O* 279 in O *om. L3*/CI: Q *L3*; PI *R*/TK: TH *FP1*/fuerint *mg. C1* 280 econtra: extra *FP1*; econverso *O*; econtrario *R*/*post* econtra *scr. et del.* extra *C1*/CI: PI *R*; *om. FP1* 281 erit[2]. . . ei: dyameter TK erit ei equalis *R* 286 medium habet *transp. C1* 287 plures *mg. C1*; *inter. a. m. E*

tum medium habuerit unam, tunc linea tot habebit ymagines quot habuerit punctum extremum. Et si utraque extremitas aut altera habuerit multas ymagines, et punctum medium habuerit multas ymagines, tunc linea habebit ymagines secundum maiorem numerum. Et hoc patebit ut de ymaginibus patuit speculorum spericorum concavorum.

[8.48] In speculis ergo columpnalibus concavis accidit fallacia in omnibus que in eis comprehenduntur sicut accidit in speculis spericis concavis, scilicet de formis specierum visibilium, et de quantitatibus et de numero suarum ymaginum, et de rectitudine et de conversione, cum fallaciis que appropriantur conversioni. Et fallacie erunt in hiis ut in speculis predictis, et hec sunt que voluimus declarare in capitulo hoc.

CAPITULUM NONUM

De fallaciis que accidunt in speculis piramidalibus concavis

[9.1] In hiis autem accidunt ille fallacie que accidunt in speculis columpnalibus concavis. Debilitas vero coloris et lucis, et diversitas positionis et remotionis accidunt in hiis sicut in omnibus speculis, nam causa huius est conversio. Accidit etiam in hiis speculis multitudo ymaginum, sicut in speculis columpnalibus et spericis concavis, sicut dictum est in capitulo de ymaginibus. Accidit etiam in eis ut columpnalibus concavis, scilicet quod rectum videtur convexum et videtur concavum.

[9.2] Huius autem demonstratio est quia linee recte que extenduntur in longitudine speculi que transit per caput piramidis, et que sunt prope illas, videntur convexe, et videntur concave, et forte recte.

288 linea: line *S* 289 habuerit: habet *L3ER*/aut: vel *R* 290 habuerit[1]: buerit *S*/multas ymagines *transp. C1* 291 *post* ymagines[1] *add.* et punctum medium medium habuerit multas ymagines *S*/*post* linea *add.* tot *L3ER* 292 hoc: hic *L3*/speculorum spericorum *transp. C1*/spericorum *om. P1* 294 columpnalibus: columpnaribus *L3C1ER* (*alter. in E*) 295 spericis concavis (296) *transp. S* 296 specierum: specie tamen *S* 298 conversioni: reflexioni *R* 299 hec sunt: hoc est *ER*/que: quod *L3ER*/voluimus: volumus *P1E*/capitulo hoc (300) *transp. P1L3ER* 1 capitulum . . . concavis (2) *om. S*; de erroribus qui accidunt in speculis piramidalibus concavis capitulum nonum *R* 2 accidunt in speculis: in speculis accidunt *FP1* 3 *post* in[2] *add.* hiis *P1* 4 columpnalibus: columpnaribus *C1ER*/concavis *inter. E*/coloris et lucis: lucis et coloris *FP1* 5 et *om. L3*/*post* hiis *add.* speculis *P1*/*post* omnibus *add.* predictis *C1* 6 conversio: reflexio *R*/in *om. FP1* 7 columpnalibus: columpnaribus *C1R*/et . . . columpnalibus (9) *om. S* 8 sicut *om. R*/est *om. R*/ut: in *L3*; quod in *R*; ut in *mg. a. m. E* 9 columpnalibus: columpnaribus *R*/quod: ut *R*/videtur: videatur *R* 10 videtur *om. R* 11 autem: aut *FP1*/quia: quod *L3ER*/linee recte *transp. ER*/recte *om. FP1* 12 que: qui *L3*/transit: transeunt *R*/per caput *om. P1*/caput: verticem *R* 13 videntur[1] *corr. ex* videretur *a. m. E*/et videntur concave *mg. a. m. E*; *om. R*

[9.3] **[PROPOSITIO 37]** Et demonstratio super hoc est ut demonstratio in speculis columpnalibus concavis, nam si iteraverimus secundam figuram de fallaciis speculorum piramidalium convexorum, inveniemus dyametrum ymaginis linee recte posite in illo speculo, qui est illic linea AY intra concavitatem speculi piramidalis, et inveniemus punctum quod est sub superficie contingente piramidem transeuntem per lineam ex qua convertitur forma linee recte ad visum, quod illic punctum F.

[9.4] Si fuerit punctum centrum visus, erunt omnia puncta que sunt in dyametro ymaginis conversa ad punctum F, et ymagines duarum extremitatum A, Y erunt extremitates linee recte AN, et loca ymaginis puncti quod est in medio AY diversabuntur. Et hoc declarabitur eadem via qua processimus in demonstratione prime figure speculorum columpnalium concavorum.

[9.5] Patet ergo ex hoc quod si AY fuerit in aliquo visibili, et visus fuerit F, tunc ymago forte videbitur convexa, et forte concava. Et patet etiam in secunda figura de fallaciis speculorum columpnalium concavorum quod linee posite in latitudine speculi apparebunt concave concavitate mirabili, et quod ymagines linearum rectarum que sunt in superficiebus transeuntibus per axem et per centrum visus erunt recte.

[9.6] **[PROPOSITIO 38]** Item iteremus tertiam figuram de fallaciis speculorum spericorum concavorum eisdem litteris. Si ergo aliquod punctum fuerit in axe piramidis, et due linee EA, EB fuerint perpendiculares super superficies contingentes piramidem (et hoc est possibile, quia sunt equales, possunt enim cum axe continere duos angulos acutos equales), cum ergo hee due linee fuerint perpendiculares, et visus fuerit D, tunc superficies in qua sunt linee GE, ED transibit per totum axem et per centrum visus.

15 speculis: spericis *E*/columpnalibus: columpnaribus *R*/iteraverimus: superconverimus *S*; iteravimus *E* 16 figuram: regulam *E* 17 illo *om. S* 18 qui: que *R*; *corr. ex* que *O*/ AY: AI *FP1SOE*; AL *L3*; AN *R*; *corr. ex* AI *a. m. C1*/et *om. L3E* 19 piramidem: piramidalem *S* 20 transeuntem: transeunte *S*/*post* lineam *add.* longitudinis *R*/ex: a *R*/convertitur: reflectitur *R* 21 *post* quod *inter.* est *O*/punctum *scr. et del. E* 22 si *om. FP1*/*post* si *add.* igitur *R*/*post* fuerit *inter.* positum *O*/punctum *om. O*/*post* punctum *add.* illud *R* 23 conversa: reflexa *R* 24 *post* A *add.* P *R*/Y: I *FP1OL3C1E*; L *S*/linee recte *transp. FP1* 25 *post* puncti *add.* P *R*/est *om. FP1O*/*post* medio *add.* puncti *FP1* 26 qua: quia *S* 27 columpnalium: columpnarium *R* 28 ergo *inter. C1*/ex hoc *inter. E*/AY: APY *R* 30 secunda figura *transp. ER*/columpnalium: columpnarium *R* 31 *post* concavorum *add.* et *P1* 32 rectarum *om. L3R* 33 sunt *inter. a. m. E*/transeuntibus: transeuntes *L3* 37 piramidis: piramidalis *S* 38 superficies contingentes: superficiem contingentem *L3*/*post* superficies *add.* concavas *P1* 39 possibile: impossibile *E*/ equales *om. P1* 41 visus fuerit *transp. ER*/GE: HE *FP1SOL3E* 42 transibit: transit *FP1*

[9.7] Et utraque superficies DAM, DBR erit declinis super axem piramidis, et erunt differentie earum due sectores piramidis. Et erit forma punctorum R, H, Q conversa ad D ex B, et forme punctorum L, M, F convertuntur ad D ex A. Cum ergo linee MR, LH, FQ fuerint in aliqua superficie visibili, et visus fuerit in D, tunc NU erit ymago MR, et TK erit ymago LH, et CI erit ymago FQ.

[9.8] Sic ergo ymago MR erit minor se ipsa, et ymago FQ maior se ipsa, et ymago LH equalis sibi ipsi, et omnes ymagines erunt converse.

[9.9] Et si visus fuerit in O et NU, TK, CI fuerint in superficiebus visibilium, tunc ymagines earum erunt MR, LH, FQ. Sic ergo erit ymago CI minor se ipsa, et ymago NU maior, et ymago TK equalis.

[9.10] Et iste ymagines erunt recte, nam iste ymagines erunt ultra centrum visus et comprehendentur ante visum super lineas radiales. Puncta ergo M, L, F comprehenduntur in linea AO, et puncta R, H, Q comprehenduntur in OB, et sic forma revertetur recta.

[9.11] Patet ergo ex hiis que diximus in hoc capitulo quod linee recte quandoque videntur in hiis speculis convexe, quandoque concave, quandoque recte, et quandoque maiores, et minores, et equales, et quandoque recte, et converse.

[9.12] Et in capitulo de ymagine declaravimus quod omne punctum visibile in huiusmodi speculis quandoque habet unam ymaginem, quandoque duas, et tres, et quattuor. Omnia ergo que comprehenduntur in huiusmodi speculis accidit in eis fallacia ut in columpnalibus concavis, et accidunt in eis etiam fallacie composite sicut in ceteris speculis. Et exempla et declaratio eorum sunt sicut in speculis planis. Et hoc intendimus declarare in hoc capitulo. Nunc autem finiamus sextum tractatum.

43 DAM DBR: DAO DBO *R* 44 *post* differentie *scr. et del.* ei *F*/earum *corr. ex* eorum *E*/due: duo *E*/sectores: sectiones *R*/erit forma (45): erunt forme *R* 45 R: B *E*/H: G *FP1OL3E*; E *SO*/R H *transp. R*/conversa: reflexe *R*/B: F *SOL3E*; *om. FP1*/et *inter. P1* 46 convertuntur: reflectentur *R*/D: T *L3*/cum *corr. ex* sum *F*/linee *om. S*/MR: MLR *FP1L3E*; LMR *S*; MLF *R*/LH: LG *O*; *om. FP1SL3ER*/FQ: GFQ *FP1SL3E*; RHQ *R*/fuerint: uerint *L3* 47 *post* D *scr. et del.* ex A *S* 48 et^{1} . . . MR (49) *om. FP1*/TK: CR *L3*/LH . . . ymago^{2} *mg. a. m. E*/CI: PI *R* 49 MR *corr. ex* LH *S* 50 ipsi *om. P1* 52 CI: PI *R* 53 earum *om. P1*/erit *om. L3C1* 54 CI: Q *FP1SL3E*; FQ *R*/*post* CI *add.* erit *C1*/minor: maior *O*; *corr. ex* maior *a. m. E*/minor se ipsa: se ipsa minor *FP1R*/NU: MC *L3E*/et ymago^{2} *inter. O*/TK: THK *S* 55 *post* recte *add.* N *S* 56 comprehendentur: comprehenduntur *R* 57 M L *transp. deinde corr. mg. E*/R: T *L3*; C *E* 58 OB: OBF *FP1SL3E*; OF *deinde inter.* B in arabico *O*/sic: si *FP1L3*/revertetur: reflectetur *R* 59 ex *om. FP1* 60 quandoque^{2} *om. S* 61 et^{1} *om. FP1*/*post* quandoque^{2} *add.* minores *P1*; *scr. et del.* minores *F* 62 et^{1} *om. FP1*/et^{2} *om. L3ER* 63 *ante* et *add.* se *FP1*/declaravimus . . . ymaginem (64) *mg. O* 64 huiusmodi speculis: huius speculum *L3*/*post* habet *scr. et del.* i *P1*/unam ymaginem *transp. C1* 65 *post* quandoque *add.* tres *P1*/et^{1}: quandoque *P1C1*/omnia: in omnibus *R* 66 huiusmodi: huius *OL3ER*/in eis *om. R*/*post* fallacia *inter.* similis *O*/in^{3} *om. FP1*/columpnalibus: columpnibus *L3*; columpnaribus *R* 67 et *om. ER*/accidunt: acciduntque *ER*/in eis etiam: etiam in eis *R* 68 sunt *mg. F*; *om. S* 69 hoc^{2} *om. S*

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A Critical Edition, with English Translation
and Commentary, of Book 6
of Alhacen's *De aspectibus*

VOLUME ONE
Introduction and Latin Text

VOLUME TWO
English Translation

A. Mark Smith

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VOLUME TWO

CONTENTS

VOLUME II

ENGLISH TRANSLATION
Topical Synopsis 155
Chapter 1 161
Chapter 2 161
Chapter 3 162
Analysis of Plane Mirrors:
Proposition 1 163
Chapter 4 164
Analyis of Convex Spherical Mirrors:
Propositions 2-15 165
Chapter 5 188
Analysis of Convex Cylindrical Mirrors:
Propositions 16-19 189
Chapter 6 196
Analysis of Convex Conical Mirrors:
Propositions 20-22 196
Chapter 7 204
Analysis of Concave Spherical Mirrors
Propositions 23-32 205
Chapter 8 221
Analysis of Concave Cylindrical Mirrors:
Propositions 33-36 221
Chapter 9 230
Analysis of Concave Conical Mirrors:
Propositions 37 and 38 230
Notes 233
Figures for Introduction and Latin Text
Introduction 261
Latin Text 304

APPENDIX 330
Latin-English Index 339
English-Latin Glossary 359
Bibliography 371
General Index 387

BOOK SIX OF ALHACEN'S *DE ASPECTIBUS*

Topical Synopsis

CHAPTER 1: ***Prologue*** 161

[1.1] The agenda for book 6: explaining image-distortion in the seven baisc types of mirrors.

CHAPTER 2: ***General Review of Misperception*** 161

[2.1-2] Just as in direct vision, so in reflected vision, misperception can arise from an excess or defect in the threshold conditions governing veridical vision. Misperceptions in reflected vision are magnified by the weakening of light caused by reflection itself. **[2.3]** In reflected vision misperception is exacerbated by three factors: first, the object appears to lie where it actually does not; second, the object's color mingles with that of mirror; third, the object's light and color appear more dimly because of the weakening caused by reflection. **[2.4]** Because of this weakening, the limits for the threshold of veridical vision are truncated in reflected vision.

CHAPTER 3: ***Plane Mirrors*** 162

[3.1] Introductory paragraph. **[3.2-4]** PROPOSITION 1: The image in a plane mirror lies the same distance below the reflecting surface as its object, and it is the same size and shape as its object. **[3.5-6]** Because of the weakening of light and color due to reflection, a viewer can misperceive light, color, number, and distance in ways that would not occur were the object viewed directly. **[3.7]** Image-reversal, which involves spatial disposition, is a misperception unique to reflection. **[3.8]** Reflection invariably causes misperception of light, color, and spatial disposition; misperceptions stemming from these misperceptions are like those in direct vision, only they occur more easily. **[3.9]** Reflected vision is subject to diplopia in the same way as direct vision. **[3.10]** Images seen in plane mirrors appear to lie even farther away than they would if they were objects lying at the same distance but seen directly. **[3.11]** Misperception of shape can occur in reflected vision for the same reason as in direct vision, but that misperception is more frequent and more exaggerated in reflected vision. **[3.12]** Misperception of disjunction or separation can occur in reflected vision as it can in direct vision.

CHAPTER 4: ***Convex Spherical Mirrors*** 164

[4.1] Introductory statement: Aside from the misperceptions these mirrors have in common with plane mirrors, they also diminish the size of objects appearing in them; the only thing about the object that appears unmodified in these mirrors is the arrangement of its parts. **[4.2-10]** PROPOSITION 2: An object should always appear smaller than it is in a convex spherical mirror. **[4.11-71]** PROPOSITION 3: It is possible for the image to appear the same size as, or larger than, its object in such a mirror. **[4.72]** Transitional paragraph. **[4.73-79]** PROPOSITION 4, LEMMA 1: If two points that are equidistant from the mirror's center lie different distances from the center of sight, then the image of the point lying farther from the center of sight will lie farther from the mirror's center than the image of the point lying nearer the center of sight, and the endpoint of tangency for the farther point will lie farther from the mirror's center than the endpoint of tangency for the nearer point, no matter whether these points lie in the same plane as the center of sight or not. **[4.80-85]** PROPOSITION 5, LEMMA 2: Given a line AB cut at points G and D so that AB:BD = AG:GD, if three lines are erected to points B, G, and D so as to intersect above line AB, and if a line is drawn through them from point A, that line will be cut in the same ratio as line AB. **[4.86-88]** PROPOSITION 6, LEMMA 3: Given line AB cut at points G and D so that AB:BD = AG:GD, if a line is drawn from A and cut according to the same ratio, and if lines are drawn through the respective points of division on both lines, those lines will intersect at a point. **[4.89-90]** PROPOSITION 7, LEMMA 4: Given line AB cut at points G and D so that AB:BD = AG:GD, if parallel lines are erected to that line from points B, G, and D, and if a line is passed through them from A, that line will be cut in the same ratio as AB. **[4.91-101]** PROPOSITION 8: In convex spherical mirrors the curvature of the image of an arc accords with the mirror's center, not with its surface. **[4.101-104]** PROPOSITION 9: The more sharply curved a line is, the less sharply curved it may appear in such a mirror. **[4.105-107]** PROPOSITION 10: The images of straight lines whose endpoints are equidistant from the center of curvature appear curved in such a mirror. **[4.108-110]** PROPOSITION 11: The images of straight lines whose endpoints are not equidistant from the center of curvature are curved when those lines or their extensions do not touch the mirror's surface. **[4.111-120]** PROPOSITION 12: If a straight line or its extension is tangent to the mirror, its image will be curved. **[4.121-125]** PROPOSITION 13: The image of a straight line that intersects the mirror's surface will be curved. **[4.126-140]** PROPOSITION 14: If a straight line lies in the same plane as the center of sight, then, depending on how it is poised with respect to the mirror's surface and the center of sight, all, some, or none of that line may be seen in the mirror. **[4.141-152]** PROPOSITION 15: When the visible line, the center of sight, and the center of curvature all lie in the same plane, the image of that line will appear curved. **[4.153-154]** Summary of conclusions for chapter 4.

CHAPTER 5: ***Convex Cylindrical Mirrors*** 188

[5.1] Introductory statement: in convex cylindrical mirrors the same misperceptions occur as occur in convex spherical mirrors, except they are more pronounced in convex cylindrical mirrors. **[5.2]** The agenda for this chapter according to whether the plane of reflection forms a line of longitude, a circle, or an ellipse on the cylinder's surface. **[5.3-11]** PROPOSITION 16, LEMMA 5: In an elliptical section formed on a convex cylindrical mirror, if a point is chosen on it that is not a point of reflection and if a line is drawn from that point to intersect the normal dropped from the point of reflection at the cylinder's axis, then when a another line is dropped through that point perpendicular to the tangent at that point, this perpendicular will intersect the normal dropped from the point of reflection beyond the cylinder's axis and beyond the intersection of the first line with the normal dropped from the point of reflection. **[5.12-20]** PROPOSITION 17: If a straight line lies outside a convex cylindrical mirror within a plane passing through a line of longitude on the cylinder and its axis, its form will be reflected from that line of longitude, and if the center of sight lies within that same plane, the image will be a straight line. **[5.21-27]** PROPOSITION 18: Given the previous situation, if the center of sight does not lie in the same

plane as the visible line, the resulting image will be curved. **[5.28-5.41]** PROPOSITION 19: If a straight line lies outside the cylinder, and if it is perpendicular to the plane containing the center of sight and the cylinder's axis, the image of that line will appear curved. **[5.42-43]** Summary of conclusions for chapter 5.

CHAPTER 6: ***Convex Conical Mirrors*** 196

[6.1] Introductory statement: misperceptions in these mirrors are like those in convex cylindrical mirrors. **[6.2-9]** PROPOSITION 20, LEMMA 6: If a point of reflection is found on a conic section produced on the mirror's surface, and if a point is chosen on that section farther from the cone's vertex than the point of reflection, then when a normal is dropped from this latter point to the tangent passing through it, that normal will intersect the normal dropped from the point of reflection at a point outside the axis. **[6.10-18]** PROPOSITION 21: If a straight line is located outside the cone's surface in such a way that it or its extension intersects the cone's vertex, its form will be reflected from a line of longitude on the cone. **[6.19-40]** PROPOSITION 22: Under the foregoing circumstances, the image of that line will be curved. **[6.41-42]** The images of lines that face convex conical mirrors widthwise are noticeably curved, and the more those lines approach an upright position with regard to the cone, the less curved their images appear. **[6.43-46]** Overall, the images of objects seen in such mirrors take the form of the mirror, shrinking toward the vertex of the cone and expanding toward the base; also, the closer the object is brought toward the reflecting surface, the larger it appears in the mirror. **[6.47]** The images of objects seen in such mirrors take the form of the mirror, shrinking toward the vertex of the cone and expanding toward the base, but the compound misperceptions arising in these mirrors are common to all the others.

CHAPTER 7: ***Concave Sperical Mirrors*** 204

[7.1-7] Introductory statement: While some of the misperceptions in these sorts of mirrors are common to the rest, concave mirrors cause a host of misperceptions that are unique to them; these involve the misperception of size, shape, number, and the arrangment of parts, which includes image-reversal and inversion. **[7.8-13]** PROPOSITION 23: If the center of sight and a visible line both lie in the plane of a great circle on a concave spherical mirror, and if

they both lie closer than half the radius to the reflecting surface, then the image of that line will be larger than the line itself. **[7.14-21]** PROPOSITION 24: If the center of sight and the visible line do not lie on the same great circle but do lie closer than half the radius to the reflecting surface, then the image of that line will be larger than the line itself. **[7.22-30]** PROPOSITION 25: Under certain circumstances, the image of a visible line will be the same size as the line itself when seen in a concave spherical mirror; the image may be inverted, however. **[7.31-35]** PROPOSITION 26: Under certain circumstances, the image of a visible line can be smaller or larger than the line itself when seen in a concave spherical mirror; when the image is smaller, moreover, it will be inverted. **[7.36-39]** PROPOSITION 27: Under certain other circumstances, the image of a visible line can be smaller or larger than the line itself when seen in a concave spherical mirror; but it is the larger rather than the smaller image that is inverted. **[7.40-49]** PROPOSITION 28: If an eye faces the mirror such that the mirror's center of curvature lies between the eye's surface and the surface of the mirror, its image will lie in front of the mirror, it will be smaller than the eye itself, and it will be inverted. **[7.50-52]** Summary of conclusions for propositions 25-28. **[7.53-63]** PROPOSITION 29: In concave spherical mirrors, the images of straight lines may be straight, and they may be oriented the same way. **[7.64-68]** PROPOSITION 30: Under certain circumstances, the image of a convex line appears convex, and the image of a concave line appears concave in concave spherical mirrors. **[7.69-82]** PROPOSITION 31: Under certain circumstances, the image of a straight or convex line will appear concave in a concave spherical mirror, and a straight line may have as many as four images. **[7.83-105]** PROPOSITION 32: In concave spherical mirrors the images of straight lines may appear convex, the images of convex lines may appear concave, and the images of concave lines may appear convex. **[7.106-108]** Summary of conclusions.

CHAPTER 8: ***Concave Cylindrical Mirrors*** 221

[8.1] Introductory statement: The misperceptions arising in these mirrors are essentially the same as those arising in concave spherical mirrors. **[8.2-18]** PROPOSITION 33: When a straight line parallel to the axis is viewed in a concave cylindrical mirror, it may yield one or more images, and those images may be straight, convex, or concave. **[8.19-31]** PROPOSITION 34: When a straight

line facing the axis breadthwise is seen in a concave cylindrical mirror, its image may appear concave. **[8.32-42]** PROPOSITION 35: When straight lines are seen in concave cylindrical mirrors, their images may be properly oriented or reversed. **[8.43-47]** PROPOSITION 36: Depending on where the center of sight lies with respect to a given visible line, the images that line yields in a concave cylindrical mirror may be properly oriented or reversed; that line will also yield a plurality of images depending on how many images its endpoints and midpoints yield. **[8.48]** Summary of conclusions for chapter 8.

CHAPTER 9: ***Concave Conical Mirrors*** 230

[9.1-2] Introductory statement: The misperceptions arising in these mirrors are essentially the same as those arising in concave cylindrical mirrors. **[9.3-5]** PROPOSITION 37: The image of a straight line seen in a concave conical mirror may be straight, convex, or concave. **[9.6-10]** PROPOSITION 38: The image of a straight line seen in a concave conical mirror may be properly oriented, or it may be reversed, depending on the position of the line and the center of sight. **[9.11-12]** Summary of conclusions for chapter 9.

BOOK SIX

This book is divided into nine chapters. The first chapter [describes] the basic purport of the book; the second [explains] that error occurs in sight because of reflection; the third [focuses] on error that arises in plane mirrors; the fourth [focuses] on error that originates in convex spherical mirrors; the fifth [focuses] on convex cylindrical mirrors; the sixth [focuses] on convex conical [mirrors]; the seventh [focuses] on concave spherical [mirrors]; the eighth [focuses] on concave cylindrical [mirrors]; the ninth [focuses] on concave conical [mirrors].

CHAPTER 1

[1.1] It was shown in the preceding books how forms are apprehended in mirrors by the visual faculty, how the lines of reflection or incidence are disposed, [and] how images are disposed and where they are located. However, the form is not always perceived as it actually exists by means of reflection. For in concave mirrors the image of [one's] face appears distorted, and its proper disposition is obscured from sight, so it is obvious that error occurs in the perception of forms through reflection. In the present book it is [our] purpose to explain how this error occurs and the reason for it, as well as to discuss the different types of errors due to the different types of mirrors.

CHAPTER 2

[2.1] The second book showed how forms are perceived in direct vision, and the third book carefully analyzed the particular factors that lead to error in that [kind of] vision when the [conditions for proper vision] exceed or fall short of the [appropriate] threshold. The perception of forms by means of a reflection [of rays] occurs in the same way [as it does] in direct vision, and [so] the things that are apprehended in direct vision are also apprehended in reflected vision—such things as light, color, shape, size, distance, and the like [i.e., the full range of visible intentions].

[2.2] Moreover, just as happens in the direct visual apprehension of things [whose forms] are already ensconced [in the soul] and known, so in reflected vision there is a correlation [of the form] to something else [like it] so that a conclusion is drawn and a judgment is made in the soul. Hence, any excesses or defects in the threshold conditions [for proper sight that] cause an error in direct vision likewise cause [an error] in reflected vision. And according to each case [of excess or defect in the threshold condition], the error is magnified in reflected vision because of the diminished light that results from the weakening caused by the actual reflection.

[2.3] Furthermore, to generalize, we should say that the proper disposition of the form cannot be perceived in reflected vision as it can be in direct vision because of a threefold constraint specific to reflection. The first is that in reflection the form of the object appears to the viewer to lie directly in front of the eyes when this is not actually the case. The second [is] that the light and color in the visible object are mingled with the color of the mirror, and the visual faculty perceives that mingled [color] rather than the actual color or light belonging to the visible object. The third is that, as has been pointed out earlier [in book 4], reflection itself weakens light and color, so the actual light and color will be less clearly seen in reflected vision than in direct vision.

[2.4] In addition, earlier discussions showed that the range of the [limits of the] threshold conditions [whose excess or defect] leads to error depends on the intensity of the light and color, for that range will be greater in stronger light or color [and] less in weaker [light or color]. And, since light and color will be weakened by reflection, the range of the [limits of the] threshold conditions [whose excess or defect] leads to particular kinds of error will be less in reflected vision than in direct vision, and the shortening of that range leads to an increase in the number of errors. Besides, certain tiny features of objects can be perceived through direct vision that are in no way perceptible through reflected vision. It is therefore evident that reflected vision exceeds direct vision in the degree and number of errors.

CHAPTER 3
[On Plane Mirrors]

[3.1] In each kind of mirror a misperception of forms occurs, but the variety of errors [that occur] depends on the variety of mirrors [in which the forms are perceived]. In plane mirrors less error occurs than in the others. For in these [kinds of mirrors] the proper shape, spatial disposition and size [of the object] are perceived, just as [they are] in direct vision, which will be shown by [the following] demonstration.

[3.2] **[PROPOSITION 1]** Imagine a plane mirror, and let line AB [in figure 6.3.1, p. 97] on that mirror's surface be the common section of the mirror's surface and a plane perpendicular to the mirror's surface. Let H and Z be two points in that perpendicular plane, [let] E [be] the center of sight, and draw perpendicular HL from point H to the mirror's surface. Extend it so that LG = LH. Likewise, extend perpendicular ZF so that DF = FZ.

[3.3] It is clear from earlier discussions [i.e., book 5, proposition 1, in Smith, *Alhacen on the Principles*, 399] that [the form of point] H is reflected to [point] E from a point on the mirror, and its image-location G lies as far from the mirror's surface [below it] as H [lies above it]. By the same token, [the form of point] Z is reflected to [point] E, and its image-location is D.

[3.4] Now when line ZH is drawn, and likewise line GD, [the form of] any point on line ZH is reflected to [point] E. Its image-location lies the same distance from the mirror's surface as the point itself, and so any point on line ZH appears to lie the same distance [from the mirror's surface] as it will [actually] lie [from that surface]. Hence, if line ZH is straight, line DG will be straight. If it is curved, DG will be an arc of the same curvature, so line ZH will appear the same size and shape as it is, which is what was set out [to be proven].

[3.5] However, if there are various colors that are only slightly different from one another at points along line ZH, the variation [among them] may not be perceived; instead, a single blend of color will be presented to sight. Hence, because of reflection there will be an error involving light and color, and in addition [an error] concerning number. For that difference among the colors and lights might be perceptible in direct vision, but the [perceptibility of the] color has exceeded the threshold with respect to reflected vision, although not with respect to direct vision. Likewise, tiny features that could be discerned in direct vision are either hidden or confused in reflected vision.

[3.6] Moreover, because of the weakening of light or color by reflection, an error arises in [perception] of distance that would not arise in direct vision.[1]

[3.7] In the case of spatial disposition an error clearly arises from reflection alone, for in the image we perceive things on the left-hand side of the visible object that we would see on the right-hand side if the object were [actually placed in front of us] at the image-location. For, when something faces something else, its corresponding spatial disposition is opposite because what is the right-hand side of the one will be the left-hand side of the other. Accordingly, the right-hand side of the visible object is the left-hand side of the image, whereas the left-hand side of the image will be its right-hand side to the viewer, but it is perceived on the left-hand side of the image.[2]

[3.8] Overall, in the case of light, color, or spatial disposition, error invariably arises from the very reflection itself. In these cases, as well as in others, the things that lead to error in direct vision likewise lead to error in reflected vision, and more easily because the [range of] threshold conditions for each is smaller in reflected vision than in direct vision. One example for all of these [cases] may be applied, and the same should be understood [to apply] to the rest.

[3.9] In direct vision, when the visible object lies far outside the visual axes, it may appear double; the same thing happens in mirrors when the visible object lies far outside the visual axes.

[3.10] In mirrors, the object will appear smaller than it should at a given distance, whereas at such a distance it may look smaller than it should in direct vision, but not to such a great extent.[3] And this increased diminution [which happens] in mirrors is due to the decrease in the [range of] threshold conditions [for the perception] of distance.

[3.11] In [the perception of] shape error sometimes arises in mirrors for the same reasons it does in direct vision, but [it does so] more significantly and more frequently according to spatial disposition.

[3.12] If a rope or something like it faces a mirror at a given distance, and if its ends cannot be perceived by the visual faculty, it may appear to lie on the very surface of the mirror. The same thing happens in direct vision. If some rope is placed facing a window and the ends of the rope cannot be seen, the separation between rope and window will not be apparent, even if it is significant, and [this] is due to spatial disposition.[4] Moreover, if one of the ends is visible but the other is not, that end may appear to lie in the plane [of the window]. In each case, where [error] occurs in direct vision, it occurs likewise in reflected vision.

CHAPTER 4
On [Convex] Spherical Mirrors

[4.1] The entire range of errors that occur in plane mirrors also occurs in convex spherical [mirrors], and besides this, a visible object looks smaller than it should in [convex] spherical mirrors. Overall, in these [kinds of] mirrors nothing about the visible object is perceived as it actually is except the arrangement of its parts, which appears in the mirror as it actually exists in the visible object.

[4.2] **[PROPOSITION 2]** That an object should always appear smaller than it is in this [sort of] mirror is demonstrated [as follows].

[4.3] Let AB [in figure 6.4.2, p. 97] represent a visible line [on some object, let] ZP be the mirror, D the center of the [great] circle [produced by the plane of reflection on the mirror], and E the center of sight. Let [the form of point] A be reflected to [point] E from point H, and [let the form of point] B [be reflected to E] from point N. When it is extended, line AB will pass through the center of the mirror, or [it will] not.

[CASE 1]

[4.4] Let it pass through. From point N draw line NL tangent to the circle, and from point H [draw] tangent HM. Draw the line[-couple]s of reflection BN and EN, and AH and EH, extend lines EH and EN until they fall on normal AD, and let T and Q be the points where they fall. It is evident that T is the image-location for A, and Q is the image-location for B. I say that AB > QT.

[4.5] It is evident from previous discussions [in book 5, prop. 7] that AD:DT = AM:MT. Likewise, BD:DQ = BL:LQ.[5] But AD > BD, and DT < DQ [so AD:DT > BD:DQ and thus > BL:LQ]. Hence, AM:MT [which = AD:DT] > BL:LQ.

[4.6] Cut AM at point F so that FM:MT = BL:LQ. Therefore, BM:MT < BL:LQ. Cut MT at point K so that BM:MK = BL:LQ. K will necessarily fall between M and Q because LQ < MQ, and BL > BM. Accordingly, since FM:MT = BL:LQ, as well as BM:MK, FB:KT = BL:LQ.[6] But BL > LQ [because we know by previous conclusions that BD:DQ = BL:LQ, and BD > DQ, so BL > LQ]. Hence, FB > KT, so [visible line] AB > [its image] QT [since AB > FB, and QT < KT], which is what was set out [to be proven].

[CASE 2]

[4.7] But if line AB, when it is extended, does not reach the center [of the circle], then from point A [in figures 6.4.2a and 6.4.2b, p. 98] draw line AG to the center, let G be the center, and from point B draw line BG. Let point D be the image-location for [point] A, let [point] E be the image-location for [point] B, and draw line ED, which is the image of line AB. I say that [object] AB > [image] ED because ED is either parallel to AB or not.

[4.8] If it is parallel [as in figure 6.4.2a], it is clear that it is smaller [i.e., ED < AB because triangles EDG and BAG are similar, so BA:ED = BG:EG, and BG > EG]. If it is not parallel [as in figure 6.4.2b], extend [ED] until it meets AB. Let Z be the [point of] intersection, and from point E draw EH parallel to AB. Angle EDH is acute, right, or greater [than a right angle].

[4.9] If it is right or greater [than a right angle], side EH > [side] ED. But [by previous conclusions] EH < AB, and so [we have demonstrated] what was set out [to be proven].[7]

[4.10] If it is acute, it could happen that the form [i.e., ED] is larger than the object [AB] whose form it is, which, although it may be larger, will happen rarely. And when it does happen, the form may be perceived from such a distance that it will appear smaller than it should because the object itself may appear smaller [than it should] at that distance [in direct vision].[8]

[4.11] **[PROPOSITION 3]** It will now be demonstrated that in these [kinds of] mirrors a form may sometimes appear larger than the visible object, i.e., when it [actually] is larger, and that it may be perceived [as larger] from such a distance that its size can be discerned with proper certitude.

[4.12] Let A [in figure 6.4.3, p. 99] be the center of the mirror, and take a plane of reflection that will cut the mirror along a [great] circle. Let that circle be EDB, let ED be the diameter of that circle, and extend diameter ED to Z so that rectangle EZ,ZD is not greater than AD^2, which is clear[ly possible], since it is possible for a line to be added to diameter ED such that the rectangle formed by the whole and the added part equals AD^2 [by Euclid, III.36].[9] Bisect line ZD at point H. Hence, AH will be half of EZ. Accordingly, [since AD < AH, which = half EZ, while DH = half DZ] rectangle AD,HD will not be greater than one-fourth AD^2 [because it is no greater than one-fourth rectangle EZ,ZD, which is no greater than AD^2], and since AH,HD > HD^2, let AH,HT = HD^2.[10]

[4.13] Produce a circle according to length AH [as radius], and from point H draw chord HQ equal to one-half line HD. Draw lines QA and QT, and at point Q form angle HQN equal to angle QAH. Accordingly, since these two angles in these two triangles [HQN and QAH] are equal, and since one [angle], i.e., QHA, is common, the third [angle] = the third [angle], i.e., [angle] AQH = angle HNQ. And [so] the triangles will be similar [by Euclid, VI.4], and [according to proportional sides] AH:HQ = HQ:HN. Therefore, AH,HN = HQ^2 [by Euclid, VI.17].

[4.14] But HQ^2 is one-fourth HD^2, since HQ is one-half HD [by construction]. Therefore, AH,HN = one-fourth AH,HT [which = HD^2, by construction], so HN is one-fourth HT. Accordingly, N lies between H and T. It follows that HT,TN is three-fourths HT^2.

[4.15] Angle QHD is acute, however, and [it is] equal to angle HQA, since they are subtended by equal sides in the larger triangle [i.e., QA and HA, which are radii of the larger circle]. Therefore, angle QHN [in triangle AQH] = angle HNQ [in triangle HQN, which is similar to triangle AQH, by previous conclusions], and so HQ = QN.

[4.16] Also, angle HNQ is acute, so [adjacent] angle QNT is obtuse. Hence, TQ^2 exceeds $QN^2 + TN^2$ by TN,NH because, as Euclid claims [in

II.12], the square on the opposite side of an obtuse [angle] exceeds the squares on the [other] two sides by twice the rectangle formed by one of the sides and the adjoining segment that extends to where the perpendicular is dropped [to it] from the endpoint of the other side. And if a perpendicular is dropped from point Q to line HT, it will fall at the midpoint of line HN [because triangle QNT is isosceles], and twice the rectangle formed by TN and one-half HN equals TN,HN.

[4.17] Therefore, TQ^2 exceeds $QN^2 + TN^2$ by TN,NH [i.e., $TQ^2 - QN^2 - TN^2$ = NH,NT, so NH,NT + $TN^2 = TQ^2 - QN^2$]. But HN,NT + NT^2 = HT,TN [by Euclid, II.3]. Therefore HT,TN = $TQ^2 - HQ^2$ [because $HQ^2 = QN^2$, since HQ = QN in isosceles triangle QHN].

[4.18] Now let AI:AH = QT:QH [by Euclid VI.12]. The square [of AI] to the square [of AH] will be as the square [of QT] to the square [of QH—i.e., $AI^2:AH^2 = QT^2:QH^2$], and $(AI^2 - AH^2):AH^2$ = HT,TN [which = $QT^2 - QH^2$]:QH^2 [by Euclid, V.17]. And since $4QH^2 = HD^2$ [by previous conclusions], while 4HT,TN = $3HT^2$ [by previous conclusions], HT,TN:QH^2 = $3HT^2$ [which = 4HT,TN]:HD^2 [which = $4QH^2$].

[4.19] Moreover, let HC = 3HT. [Therefore,] CH,HA = $3HD^2$ [since HT,HA = HD^2, by construction], but since AH:HD = HD:HT [because AH,HT = HD^2, by construction, so HD is the mean proportional between AH and HT], HT:HA = $HT^2:HD^2$.[11] But CH:HA = CH,HT:HA,HT, and so CH:HA = $3HT^2:HD^2$ [because CH:HA = 3HT:HA]. But this [i.e., $3HT^2:HD^2$] was as $(AI^2 - AH^2):AH^2$.[12] Therefore, CH:HA = $(AI^2 - AH^2):AH^2$. Therefore, CA [which = CH + HA]:AH = $AI^2:HA^2$, for $(AI^2 - HA^2) + HA^2 = AI^2$.

[4.20] Hence, IA will be the mean proportional between CA and HA, whose converse we touched upon a bit earlier.[13] Accordingly, CA:IA = IA:HA, and the remainder will be in the same proportion to the remainder, i.e., CI:IH [= CA:IA = IA:HA], and since IA > HA, CI > IH.

[4.21] Furthermore, AH,HD < one-fourth AD^2 [by previous conclusions]. Therefore, [line] HD < one-fourth line AD [because AH > AD]. So it is less than one-fifth AH [because AH = AD + HD]. Therefore, since AH > 5HD, and since AH,HT = HD^2 [by construction], HT < one-fifth HD, and so HT < one twenty-fifth HA. But, as was [just] claimed, CI:IH = IA:HA. Thus, CH [which = CI + IH]:IH = (IA + AH):AH. Hence, one-third the first [term is] to the second as one-third the third [term is] to the fourth [i.e. one-third CH:IH = one-third (IA + AH):AH].

[4.22] But [line] HT is one-third line CH [by construction]. Therefore, TH:IH = one-third (line IA + AH):line AH. Accordingly, TH:IH = (two-thirds line AH + one-third line IH):line AH.[14] However, since CI > IH [from previous conclusions], IH < one-half CH, and one-third IH < one-sixth CH, and so one-third IH < one-half TH [which = one-third CH]. Therefore, (two-thirds AH + less than one-half HT):AH = TH:IH. So, conversely, IH:HT = AH:(two-thirds AH + less than half HT).

[4.23] But HT < one twenty-fifth AH [by previous conclusions], and its half < half of one twenty-fifth. But line AH is divided into twenty-five parts, [so] two-thirds [of those 25 parts, i.e., < 17] + half of one twenty-fifth [part] does not add up to 18 parts. Therefore, IH:HT > 25:18.

[4.24] Furthermore, since HT < one twenty-fifth AH, AT > twenty-four twenty-fifths AH. But line IH < one-half CH, so [IH] < (HT + one-half HT), so [IH] < one and one-half of [one of] the twenty-five parts comprising AH, and so IA < twenty-six-and-one-half of the given 25 parts into which HA is divided. Therefore, IA:AT = (less than twenty-six-and-a-half):more than 24. Thus, IA:AT < twenty-six-and-a-half:24. But IH:HT > 25:18. Therefore, IH:HT > IA:AT.

[4.25] [By Euclid VI.12] let IM:MT = IA:AT [in figure 6.4.3a, p. 100]. M will therefore fall between I and H. Moreover, IM:MH > IA:AT, and so it is greater than IA:AH. So let IL:LH = IA:AH [by Euclid VI.12]. L will of course fall between M and I.

[4.26] Now from points L and M draw tangents LB and MG, and draw lines IB, HB, IG, TG, AB, and AG, and extend the last [two] to the outer circle [to intersect it at points Z_2 and Z_1, respectively].[15]

[4.27] From the fifth [proposition] of the fifth book you will [then] have that angle IBZ_2 = angle HBA, for, since IL:HL = IA:AH [by construction], H will be the image-location [of object-point I] in the case of reflection from point B [when HB, extended beyond B to H', forms the line of reflection].[16] And if the contrary is claimed, so that some other image-location is chosen, you will disprove it by a *reductio ad absurdum*, given that it is impossible for the ratio of IA to the line from point A to the image-location not to be as [the ratio of] IL to the line from point L to the image-location.

[4.28] Therefore, since H is the image-location, and since LB is tangent [to the mirror] on AB, then when it is extended [to H'], HB will form an angle of reflection [$H'BZ_2$] equal to its vertical [angle HBA], and because LB is perpendicular to ABZ_2, it will follow that angle IBL = angle LBH. By the same token, angle IGZ_1 = angle TGA, and since MG is perpendicular [to AG], angle IGM = angle MGT.

[4.29] Now draw line HP from point H to line AB parallel to [line] IB, and from point T [draw] TR parallel to IG. Angle IBZ_2 = angle HPB. But, as was claimed [earlier], angle IBZ_2 = angle HBA, and so the two angles HBA and HPB are equal, so the two sides HB and HP [of triangle HBP] are equal. Likewise [in triangle TGR, side] TR = [side] TG. But angle HPB is acute, since it is equal to the angle of reflection [IBZ_2, so adjacent] angle HPA will be obtuse, and HA > HP [by Euclid, I.19], so it will be greater than HB [since HB = HP]. So too, TA > TG.

[4.30] Moreover, since HP is parallel to IB, then IA:AH = AB:AP [by Euclid, VI.2]; likewise, IA:AT = GA:AR, and [conversely] AH:AI = AP:AB. But IA:AT = AB:AR, since AB = AG. Therefore, from the first, AH:AT = AP: AR [by Euclid, V.22].[17]

[4.31] But since angle HPA [in triangle HPA] is obtuse, HA^2 will exceed (HP^2 + AP^2) by twice the rectangle formed by AP and the line [segment] extended from point P to the perpendicular dropped [to AP] from point H [by Euclid, II.12]. But the perpendicular dropped from point H [to AP] will fall to the midpoint of line PB, since HB and HP are equal [as established earlier], and so HA^2 will exceed (HP^2 + AP^2) by AP,PB [i.e., HA^2 – HP^2 – AP^2 = AP,PB, so HA^2 – HP^2 = AP^2 + AP,PB]. Accordingly, AH^2 exceeds HP^2 by AB,AP [i.e., AH^2 – HP^2 = AB,AP] because (AP,PB + AP^2) = AB,AP. Likewise, AT^2 exceeds TR^2 by AG,AR, or AB,AR, which is identical [since AG = AB, both being radii of the circle].

[4.32] Accordingly, combine line AB with the two lines AP and AR [to form rectangles AB,AP and AB,AR], and the two remainders will be formed. Hence, remainder [AB,AP] is to remainder [AB,AR] as AP:AR, so (AH^2 – HP^2 [which = AB,AP]):(AT^2 – TR^2 [which = AB,AR]) = AH:AT [since AB,AP:AB,AR = AP:AR = AH:AT, by previous conclusions]. And since HP = HB, and TR = TG, (AH^2 – HB^2):(AT^2 – TG^2) = AH,AT.

[4.33] But [let us cut line AH at point U such that] AH,HU = HB^2. [However, AH^2 = AH,AU + AH,HU (by Euclid, II.2), so AH^2 – AH,HU (which = HB^2) = AH,AU]. Therefore, AH,AU = AH^2 – HB^2. So AH:AT = AH,AU:(AT^2 – TG^2) [since AH:AT = (AH^2 – HB^2):(AT^2 – TG^2), by previous conclusions, and (AH^2 – HB^2) = AH,AU]. And if the two lines AH and AT are combined with AU [to form rectangles, then] AH:AT = AH,AU: AT,AU. Hence, AT,AU = AT^2 – TG^2. So AH,HU = HB^2 [by construction], and AT,TU = TG^2 [since AT^2 = AT,AU + AT,TU, and, consequently, AT,AU = AT^2 – AT,TU. But because AT,AU = AT^2 – TG^2, then AT,TU = TG^2].

[4.34] Now bisect arc BG at point O [see inset to figure 6.4.3a, p. 100], drop the three perpendiculars BF, OY, and GK to line HA, draw line GS from point G parallel to HA, and drop perpendicular BX from point B to AG. If BX were extended to the circle [DGB], line AG would bisect it, as well as the arc whose chord it would form [when extended]. Accordingly, it[s other half after extension] would cut off another arc equal to arc BG, since its other arc would subtend angle GBX, and so angle GBX is half the angle [GAB] at the center [of the mirror] subtended by that same arc, according to Euclid [III.20]. Hence, angle GBX is one-half angle BAG, which line OA bisects. Therefore, angle GBX = angle OAG. Moreover, the two angles BSG and BXG are right [by construction].

[4.35] If a circle is imagined on BG [as diameter, and if it] passes through S, it will pass through X [by Euclid, III.31], and arc SX will be formed such that the two angles XBS and XGS [i.e., AGS, since X lies on AG] will fall upon it. These two angles will therefore be equal [by Euclid, III.27]. But angle GAY = [alternate] angle XGS because of the parallelism of lines [AY and GS], so angle GAY = angle XBS. And, as has [already] been claimed, angle GBX = angle OAG. Angle OAY = angle GBS, and [so] triangle OAY will be similar to triangle GBS.[18] Hence, GB:BS = OA:AY.

[4.36] Now since angle AHB [in triangle AHB] is acute, AB^2 is less than $AH^2 + HB^2$ by 2AH,HF [i.e., $AH^2 + HB^2 - AB^2$ = 2AH,HF], according to what Euclid claims [in II.13]. Therefore, [since] DA = AB [because they are radii of circle DGB], $AH^2 + HB^2$ is greater than DA^2 by 2AH,HF [i.e., $AH^2 + HB^2 - DA^2$ (i.e., AB^2) = 2AH,HF], and thus by 2AH,HD + 2 AH,DF [i.e., $AH^2 + HB^2 - DA^2$ = 2AH,HD + 2 AH,DF, or $AH^2 + HB^2 = DA^2$ + 2AH,HD + 2AH,DF]. But 2AH,HD + $AD^2 = AH^2 + HD^2$ [by Euclid, II.7]. So, if the common [term] (AB^2 [+ 2AH,HD]) is subtracted, it will follow that HD^2 + 2AH,FD = HB^2.[19]

[4.37] But AH,HT = HD^2 [by construction], and AH,HU = HB^2 [by construction]. Hence, AH,HU = AH,HT + 2AH,DF. Having subtracted AH,HT, which we designate as common to both rectangles, it will follow that AH,TU = 2AH,DF. So TU = 2DF.

[4.38] Moreover, since angle ATG is acute, then according to previous reasoning [based on Euclid, II.13], $AT^2 + TG^2 = AD^2$ [which = AG^2] + 2AT,TK, and so [$AT^2 + TG^2 = AD^2$] + 2AT,TD + 2AT,DK. And it is proven by previous reasoning that $TG^2 = TD^2$ + 2AT,DK.[20] But AT,TU = TG^2 [by previous conclusions], and so AT,TU = TD^2 + 2AT,DK.

[4.39] Let [point E′ be chosen on line AT such that] AT,TE′ = TD^2. It follows, then, that AT,E′U [which = AT,TU − AT,TE′] = 2AT,DK [which = TD^2 + 2AT,DK − AT,TE′ (which = TD^2)], if the common term AT,TE′ is subtracted. Thus, E′U = 2DK. But it has already been claimed that TU = 2DF. It follows, then, that TE′ [which = TU − E′U] = 2FK [which = 2DF − 2DK].

[4.40] Furthermore [by Euclid, V, definition 9], AH:HT = AH:HD is a duplicate ratio [i.e., AH:HD = HD:HT], since HD is the mean proportional between them [i.e., AH and HT] because HD^2 = AH,HT [by construction]. Likewise, AT:TE′ = AT:TD is a duplicate ratio [i.e., AT:TD = TD:TE′ because AT,TE′ = TD^2 by construction]. But AT:TE′ > AH:HD.[21] Therefore, AT: TE′ > AH:HT, and since AH > AT, HT > TE′. But TE′ = 2FK [by previous conclusions].

[4.41] Moreover, as has been claimed [earlier], BG:GS = OA:OY. [So] BG:OA = GS:OY. But OA = BA [since they are radii of circle DGB], and GS = FK, according to the parallelism [of lines GS and FK and lines GK and FS]. Hence, BG:BA = FK:OY.

[4.42] In addition, IH < one-half CH [by previous conclusions], and CH = 3HT [by construction]. [So] IH < one-and-one-half HT. But HT < one-fifth HD [by previous conclusions]. Accordingly, IH is smaller yet than TD, so IH is even smaller than ND, and so MI < ND [since MI < IH]. From this it is evident that I will lie between H and Z.

[4.43] Furthermore, EZ,ZD is not greater than AD^2 [by construction, with EH as the diameter of circle QHZ], so EM,MD < AD^2. But since MG is tangent [to circle DGB], EM,MD = MG^2, according to what Euclid claims [in III.36]. Thus, MG < AD, so MG < AG.

[4.44] Also, the two triangles AGM and MGK have a common angle [i.e., AMG], and both of them have a right angle [i.e., MGA and MKG]. Hence, they are similar [by Euclid, VI.4], so MK:KG = MG:GA, and so MK < KG [since MG < GA, by previous conclusions]. And since OY > GK, HD < OY [since HD < GK < OY].

[4.45] Moreover, AH:HD = HD:HT [by construction], so AH:HD = one-half HD:one-half HT. Hence, AH:HD = QH:one-half HT, since QH = one-half HD [by construction], so AH:QH = HD:one-half HT, and so QH:AH = one-half HT:HD. But one-half HT > FK [since HT > 2FK by previous conclusions], and HD < OY. Accordingly, one-half HT:HD > FK:OY, so QH:AH > FK:OY.

[4.46] Furthermore, line AQ cuts circle EBD [represented in figure 6.4.3, p. 99]. Let Q′ [in figure 6.4.3a, p. 100] be the point of intersection, and draw line DQ′, which will be parallel to [line] QH. So QH:HA = Q′D:DA [in similar triangles AQH and AQ′D], and so Q′D:DA > FK:OY. But FK:OY = GB:BA [by previous conclusions]. Therefore, Q′D:DA > BG:BA, so Q′D > BG [since DA = BA], and arc Q′D > arc GB.[22]

[4.47] Extend AQ to point S so that AS = AI, and draw line SI, which will be parallel to [line] QH, and [so] SI:QH = IA:AH. But it was posited earlier that IA:AH = TQ:QH, so SI = TQ.

[4.48] Now because of the lack of letters for designating the key points, let us revise the diagram to avoid the excessive tangle of lines. Accordingly, since IA = the line we have designated as AS, construct circle [NRZ in figure 6.4.3b, p.101] according to their length [as radius]. Let us replace S with the letter N, let AG and AB be extended to [points R and C on the circumference of] this circle, let [the resulting lines] be ABC and AGR, and let us replace the letter Q′ with F. It has been claimed [earlier] that [arc] DF [formerly DQ′] > arc BG. Let arc BM = arc DF, and draw line AMU, as well as lines IM, NM, and [draw] line QM, and extend it to the outer circle. Let it fall to point Z, and draw lines ZA and ZG.

[4.49] Since arc BM = arc DF, then if common arc [DM] is added [to both], arc MF = arc DB. [Accordingly] angle NAM = angle IAB, the [corresponding] sides [NA and IA, and MA and BA, of triangles NAM and IAB] will be equal [and will contain equal angles, so the triangles will be equal, by Euclid, I.4], and [so] MN = IB. And since AQ was assumed earlier to be equal to AH [because they are radii of the same circle passing through Q and H, sides] AQ and AM [of triangle AQM] = [corresponding sides] HA and AB [of triangle HAB], and angle [QAM contained by sides AQ and AM in triangle AQM is equal] to angle [HAB contained by equal corresponding sides HA and AB in triangle HAB, so the two triangles are equal, by Euclid, I.4]. [Hence] QM = HB, and angle QMN = angle HBI, since both the sides containing them [i.e., QM and NM, and HB and IB, respectively] are equal [as concluded earlier]. [Accordingly] base IH = base NQ, and angle NMU = angle IBC.

[4.50] But angle IBC [which is IBZ_2 in figure 6.4.3a] = angle HBA [by previous conclusions], and angle HBA = angle QMA, [so] angle NMU = angle QMA. And since QMZ is a straight line, as we stipulated, angle QMA = [vertical] angle UMZ, so [angle of incidence NMU = angle of reflection UMZ, and so the form of] point N is reflected to [point] Z from point M, and its image-location is [point] Q. This still falls short of a proof to show that all of [line of reflection] MZ lies outside the circle, which will be demonstrated as follows.

[4.51] It is clear that the tangent drawn from point B will fall between I and H [since HBA and IBC are acute, by previous conclusions], and [it is clear] that point B lies as far from point H as point M lies from point Q, and IH = NQ [from previous conclusions]. Therefore, the tangent drawn from point M will fall between N and Q. QM therefore intersects the circle, so all of MZ [lies] outside the circle, and so what we set out [to be proven is demonstrated].[23]

[4.52] Furthermore, because angle NMU = angle UMZ, arc NU = arc UZ. [So] angle NAU = angle UAZ. But it has already been shown that angle NAU [which is the same as angle NAM] = angle IAC [which is the same as angle IAB]. [Hence] angle IAC = angle ZAU.

[4.53] Angle BAG will be equal to, less than, or greater than angle GAM.[24] Let it be equal [as represented in figure 6.4.3c, p. 102]. Accordingly, if angle [C]BAG[R] is subtracted from angle IAB[C], and angle [U]MAG[R] from angle ZAU, angle IAG[R] will be left equal to angle ZAG[R]. [Thus], IG = ZG, triangle [IAG equals] triangle [ZAG], and angle IGA = angle ZGA. It will follow that angle IGR = angle ZGR. But angle IGR = angle TGA [from previous conclusions]. [So] angle TGA = angle ZGR. Hence, if TG is extended, it will reach [point] Z [on the circle passing through points I and N in figure 6.4.3c], so TGZ is a straight line. [The form of point] I is therefore reflected to [point] Z from point G, and [point] T is its image-location.

[4.54] So let Z be the center of sight. [The forms of] the two points N and I will be reflected to it from the two points M and G, and the [respective] image-locations will be points T and Q. Thus, TQ will be the image of line IN, and it was proven earlier that TQ = IN [i.e., IS in figure 6.4.3a], and so it can happen in these kinds of mirrors that the image is the same size as the visible object.[25]

[4.55] If, however, angle BAG > angle GAM [in figure 6.4.3d, p. 103], then angle ZAG > angle IAG. Let angle KAG = angle IAG. Since point K is lower than [i.e., to the left of] point Z, and point M is lower than [i.e., to the left of point] G, line KG will intersect line ZM. Let it intersect at point L. Accordingly, if the center of sight lies at point L, [the form of point] N is reflected to it from point M, and [point] Q is its image location; [the form of point] I is reflected to it from point G, and [point] T is its image-location, according to the preceding proof. And so TQ is the image of IN, which is what was set out [to be proven].[26]

[4.56] On the other hand, if angle BAG < angle GAM [as in figure 6.4.3e, p. 104], angle ZAG < angle IAG. Let angle OAG = angle IAG, and extend line OG. It is clear that [the form of point] I is reflected to [point] O from point G. Line OG will either intersect line ZMQ outside the [great] circle [FDGB] of the mirror, or it will not.[27]

[4.57] If it intersects outside, and if the center of sight lies at the point of intersection, [the forms of] the two points I and N are reflected to it, and T and Q are the image-locations [for those points], and so what was proposed is arrived at.[28]

[4.58] If line OG should happen to intersect line ZMQ inside the circle [as represented in figure 6.4.3e, p. 104, where X is the intersection-point], the foregoing proof cannot be applied. Instead, I say that a point can be found outside that entire surface to which [the forms of] the two points I and N are reflected from two points [on the mirror], and [that] TQ [forms] the image.

[4.59] For instance, from the foregoing it is evident that angle NAZ is twice angle IAB [since angle IAB = angle NAM = angle MAZ, by construction], and angle IAO is twice angle IAG [since angle IAG = angle OAG, by construction]. Furthermore, angle NAZ exceeds angle IAO by an amount no greater than angle NAI.[29] In addition, the two angles OAI and ZAN [together] are greater than the third [angle], which is IAN, the two [angles] OAI and IAN are greater than the third [angle] NAZ, and the two [angles] ZAN and NAI are greater than the third [angle] IAO. We therefore have three angles [IAO, IAN, and NAZ], any two of which [together] are greater than the third.

[4.60] From these [angles], therefore, a solid angle can be formed [by Euclid, XI.23].[30] Form that angle at A [in figure 6.4.3f, p. 105], let line SA be erected at A, let angle IAS = angle IAO, [and let] angle NAS = angle NAZ. Angle NAI will remain where it is, and line AS will be formed equal to lines AN and AI, which are both equal.[31]

[4.61] Then draw lines TS and QS. It is evident that angle TAS = angle TAO [by construction], and the two [corresponding] sides [TA and OA are equal] to the two [corresponding] sides [TA and AS]. [Hence] base TS = base TO, and triangle [TAS] = triangle [TAO], and so angle [O]GTA = angle STA [since G lies on TO]. Likewise, angle QAS = angle QAZ, and the [corresponding] sides [AS and AQ are equal] to the [corresponding] sides [AZ and AQ]. [So] triangle [QAS] = triangle [QAZ], and angle [Z]MQA = angle SQA [since M lies on line QZ].[32]

[4.62] Bisect angle TAS with line AY, and let Y be the point at which that line will intersect line TS. Since angle IAG is one-half angle IAO, it is evident that angle TAG = angle TAY, whereas angle GTA = angle YTA, and one side, i.e., TA, is common [to both triangles TAG and TAY. Accordingly] TG = TY, triangle [TAG] = triangle [TAY], AY = AG, and so Y will lie on the surface of the sphere [from which the mirror is formed]. Thus, angle IAG = angle IAY, and the [corresponding] sides [IA and AG] = the [corresponding] sides [IA and AY. So] triangle [IAG] = triangle [IAY], [angle] AGI = angle AYI [and] line IY in its full extent = [line] IG.[33]

[4.63] Extend AY beyond the sphere to point P [in figures 6.4.3f and 6.4.3h, pp. 105 and 107]. Angle IGR will be left equal to angle IYP. But, since TS = TO, and TY = TG, it follows that GO = YS. Therefore, AY and YS are equal to AG and GO [respectively], and base AS = base AO. [Hence] triangle [AYS] = triangle [AGO], and [so] angle AYS = angle AGO. It follows that angle SYP = angle OGR. Thus, the two angles IGR and OGR are equal to the two angles IYP and SYP [respectively].

[4.64] But line AS intersects the sphere [of the mirror]. Let O' [in figure 6.4.3h] be the point of intersection. Accordingly, the three points O', Y, and D lie on the surface of the sphere, so line O'YD is a segment of a [great] circle of the sphere, and it is the common section of the sphere's surface and plane ITASP, so [the form of] point I is reflected to point S from point Y [within plane of reflection ITASP], and T is the image-location.

[4.65] Likewise, if angle NAS [in figure 6.4.3f. p. 105] is bisected by [line] AZ'Z", it will be proven in the preceding way that QZ' = QM, AZ' = AM, Z'S = MZ, and the two angles NZ'Z" and SZ'Z" are equal to the two angles NMU and ZMU. And so [the form of point] N is reflected to [point] S from point Z', and Q is the image-location, so TQ is the image of IN, which is what was set out [to be proven].

[4.66] Now if a perpendicular is dropped from point I to NA, it should fall between N and Q, not beyond N because angle INA is acute, since it is equal to angle NIA, and if that perpendicular were to fall beyond N, an acute [angle] would be greater than a right angle.[34] Therefore, that perpendicular will form a right angle on NQ, and that angle will be subtended by line IN, so line IN > that perpendicular, and so that perpendicular < TQ [since TQ is equal to IN by the initial construction].

[4.67] The [form of the] point on line NQ where the perpendicular falls is reflected to point S, and its image will lie on line NA above point Q because the farther away [from the mirror's surface] the points that are reflected lie, the more their image-locations approach the center of the circle, according to the tenth [proposition] of the fifth book.[35]

[4.68] Moreover, any line drawn from point T to any point on NQ above Q will be longer than TQ. Therefore, the image of the perpendicular will be longer than the perpendicular itself [which is shorter than IN]. By the same token, no matter what line is drawn to NQ from point I between this perpendicular and IN, its image will be longer than it.

[4.69] But these claims may be determined more definitively [as follows. The form of] point N [in figure 6.4.3m, p. 108] is reflected to [point] Z from point M, and Q is the image-location. Line QM cuts the circle at point E. Therefore, the tangent drawn from point Z to the circle will fall at some point on arc ME, and that tangent will fall above Q, since the point where it will fall [on NQ] will form the endpoint of tangency [X on cathetus NA] and [thus] the limit of images, and points below that endpoint of tangency cannot be reflected, [whereas points] above it can.[36]

[4.70] Therefore, if the perpendicular dropped from point I falls above the endpoint [X] of tangency, the point where it falls is reflected, and the image of the perpendicular will be longer than the perpendicular [itself]. But if the perpendicular should fall at or below the endpoint of tangency, the point where it falls is not reflected, so there will be no image of the perpendicular. However, since the endpoint of tangency lies below N, there will be an infinitude of points between the endpoint of tangency and N, and any of them will be reflected, and the image of any of them [will lie] on NQ. And the image of any line drawn from point I to any of those points will be longer than the line of which it is the image.

[4.71] In these [sorts of] mirrors, then, the image may sometimes be the same size as the visible object and sometimes larger, which is what was set out to be explained. Moreover, we have not read an explanation of this matter in any text, nor have we heard anyone who has discussed it or thought about it.

[4.72] Moreover, in these [sorts of] mirrors straight lines appear curved, such that, in many cases, the curvature [of their images] does not correspond to that of the mirror but is opposite. Likewise, curved [things] will appear curved in these [sorts of] mirrors, and if the curvature corresponds to that of the mirror, it will appear in an opposite orientation, but this must be understood not [to hold] in all cases but in several, [but] for the sake of explaining this, certain preliminary points must be set out, one of them being as follows.

[4.73] **[PROPOSITION 4, LEMMA 1]** If two points lie the same distance from the center of the mirror and different distances from the center of sight, the image of the point lying farther from the center of sight will lie farther from the center of the sphere [forming the mirror] than the [image of the point] lying nearer [the center of sight], and the endpoint of tangency for the farther [point will lie] farther from the center [of the circle] than the endpoint of tangency for the nearer [point, and this will be the case] whether those points lie in the same plane as the center of sight or in different planes.

[4.74] The proof [is as follows]. Let T and D [in figure 6.4.4, p. 109] be two points equidistant from G, the center of the mirror, [and let] E be the center of sight. Plane DGT will cut the mirror along [great] circle AB. Let angle EGD = angle TGZ, [let] angle EGT = angle TGH, and find point Q on the circle from which [the form of point] T is reflected to [point] Z [by book 5, proposition 25, in Smith, *Alhacen on the Principles*, 427-432].[37] I say that [the form of point] T is not reflected to [point] H from any point on BQ.

[4.75] It is obvious that [it does] not [do so] from point Q [itself]. Moreover, if some point is taken on BQ, the line [of reflection] drawn to that point from point H will intersect line QZ. [The form of point] T is therefore reflected to that point of intersection from the point selected on BQ, and it is [also] reflected to that same point of intersection from point Q. So [the form of] point T is reflected to the same point from two points on that circle, which is impossible in these [sorts of] mirrors, as was shown in [proposition 16 of] the fifth book [in Smith, *Alhacen on the Principles*, 412-414].

[4.76] It follows that [the form of point] T may be reflected to H from some point on QA. Let that [point] be M [as found by book 5, prop. 25], and from point M draw MN to line GT tangent to that circle. N will be the endpoint of tangency for T with respect to H [as center of sight].[38]

[4.77] Then from point Q draw tangent QO, which will necessarily lie below MN. Extend ZQ until it falls on GT at point C. C will be the image-location [of T] for Z [as center of sight]. Thus, GT:TO = GC:CO [by book 5, proposition 7, in Smith, *Alhacen on the Principles*, 404]. So GT:TN > GT:TO. *A fortiori*, then, GT:TN > GC:CN. Accordingly, let GT:TN = GL:LN.

GL > GC, and L will be the image-location [of T] for [center of sight] H [according to book 5, prop. 7].

[4.78] So let lines HG, EG, and ZG be equal, [let] GF = GC, [and let] GS = GO. Therefore, since angle EGD = angle TGZ [by construction], and since D lies as far from point E as Z does from point T [given that DG = TG, and EG = ZG, by construction], the image of D with respect to G will lie as high on line GD as the image of T on line GT [with respect to G, by book 5, proposition 17, in Smith, *Alhacen on the Principles*, 414-415]. Thus, the image of [point] D [with respect to G lies] at point F [since GF = GC, by construction]. Likewise, the endpoint of tangency for D with respect to E will lie at the same height as the endpoint of tangency [at point O] for Z, so the endpoint of tangency for D [lies] at point S [since GS = GO, by construction].

[4.79] But since angle EGT = angle TGH [by construction], and since HG = EG [also by construction], L will be the image of T with respect to E, just as it is with respect to H. And N is the endpoint of tangency with respect to E, so the image [L] of the point farther from E [i.e., T] lies farther from the center than the image [F] of the nearer [point D], and the endpoint of tangency [N] for the farther [point T lies] farther from the center than the endpoint of tangency [S] for the nearer [point D], which was what was set out [to be proven].

[4.80] **[PROPOSITION 5, LEMMA 2]** Furthermore, given line AB [in figure 6.4.5, p. 109] divided at points G and D such that AB:BD = AG:GD, I say that, if three lines, i.e., GE, DE, and BE, are drawn from the points of division to intersect at one point, and if a line is drawn from point A to intersect those three lines, that line will be cut according to the aforesaid proportion.

[4.81] The proof [is as follows]. Draw line AT to cut the three sides GE, DE, and BE at the three points Z, H, and T. I say that AT:TH = AZ:ZH.

[4.82] From point H draw HQ parallel to AB. It is clear that AB:BD is compounded from AB:HQ and HQ:BD [i.e., AB:BD = (AB:HQ)(HQ:BD)]. But since QH is parallel to AB, triangle TQH will be similar to triangle BTA, and [so] AB:QH = AT:TH. Likewise, triangle QEH is similar to triangle BED. Therefore, QH:BD = HE:ED. Hence, AB:BD is compounded of AT:TH [which = AB:QH] and HE:ED [which = QH:BD].

[4.83] Extend QH until it falls on EG at point M. AG:GD is compounded from AG:HM and HM:GD. But since angle EMH = [alternate] angle ZGD, then angle HMZ [adjacent to EMH] = angle ZGA [adjacent to ZGD], and [so] triangle AZG will be similar to triangle HZM, and AZ:ZH = AG:HM.

[4.84] But triangle HEM is similar to triangle GED. [Hence] HM:DG = HE:ED. Accordingly, AG:GD is compounded from AZ:ZH and HE:ED, and

AG:GD = AB:BD [by construction].[39] Thus, that same [proportion AG:GD] is compounded from AT:TH and HE:ED, and it is likewise compounded from AZ:ZH and HE:ED. Hence [if we drop the common term HE:ED], AT:TH = AZ:ZH, and so what was set out [has been proven].

[4.85] The same proof will hold no matter what line is drawn from point A to intersect those three intersecting lines. And if three other lines are drawn from the three point G, D, and B to intersect at some point other than E, and if any line is drawn from A to intersect those [three lines], it will be cut according to the aforesaid ratio. And so, however the three lines may intersect, if the [resulting] three lines [represented by] EG, ED, and EB are extended beyond the three points B, D, and G, on the other side [away from the point of intersection], and if lines are drawn from point A to intersect them on that other side, those lines may never be cut according to the aforesaid ratio.[40]

[4.86] **[PROPOSITION 6, LEMMA 3]** Moreover, given line AB [in figure 6.4.6, p. 110] cut in the preceding way, if another line, such as AT, is drawn from point A so as to be cut according to the same ratio, and if lines are drawn from the points of division on AB to the points of division on AT, those lines not being parallel, I say that those three [lines] will intersect at the same point.

[4.87] The proof [is as follows]. Let AT:TH = AZ:ZH. BT and DH are not parallel, so they will intersect at point E. Line GZ will either intersect [them] at the same point, or it will not. If [it does intersect] at that point, we have what was set out [to be proven]. If not, then draw line EG. It will intersect line AT in some point other than Z. Let that point be L. Thus, AT:TH = AL:LH, according to the previous proof. But it has been supposed that AT:TH = AZ:ZH, and so it is impossible [for EG to intersect AT at some point other than Z].

[4.88] Likewise, if it is supposed that line GZ intersects DH at point E, it will be proven in this way that line BT will intersect [it] at the same [point]. So too, if it is supposed that GZ and BT intersect at point E, it will be indisputable that DH will intersect at the same [point].

[4.89] **[PROPOSITION 7, LEMMA 4]** Furthermore, given that AB [in figure 6.4.7,p. 110] is divided according to this ratio [AB:BD = AG:GD], if lines GZ, DH, and BT are parallel, and if AT is drawn to cut them, AT will be divided according to this ratio.

[4.90] The proof [is as follows]. Since DH is parallel to GZ, AZ:ZH = AG:GD, and since BT is parallel to DH, AB:BD = AT:TH. But AB:BD = AG:GD, [so] AT:TH = AZ:ZH, and so [we have demonstrated] what was set out [to be proven]. With these points established, let us proceed to what was proposed [in paragraph 4.72, p. 175-176 above].

[4.91] **[PROPOSITION 8]** First of all, it must be shown how in these [sorts of] mirrors the image of an arc is curved with a curvature that accords not with the [surface of the] mirror but with its center.

[4.92] For instance, let AB [in figure 6.4.8, p. 110] be an arc facing the mirror [composed from sphere YZ on whose surface Y′Z′ is a great circle within the plane of arc AB], let G be the center of that arc as well as of the mirror, [and let] D [be] the center of sight. Draw lines DG, AG, and BG. Take E at random on arc AB, and draw line EG. Let line DG not lie in plane ABG. Line DG will either be perpendicular to plane ABG, or [it will be] inclined [to it].

[4.93] Let it be perpendicular. Angles DGA, DGE, and DGB will be equal, and the [corresponding] sides [of triangles DGA, DGE, and DGB will be equal] to the [corresponding] sides, so the bases [DA, DE, and DB will be] equal. Hence, all the points on arc AB will lie the same distance from the center of sight, so the images of all of them [will lie] the same distance from the center.

[4.94] Let Q, M, and L be the images of A, E, and B. Accordingly, GQ will be equal to GM and GL, so QML will be an arc, and its convex curvature accords with the center [of curvature], not with the [surface of the] mirror, or with the points of reflection, which is what was set out [to be proven].[41]

[4.95] If, however, line DG is not perpendicular to plane AGB [as in figure 6.4.8a, p. 111], and if a perpendicular [DX] is dropped from point D to that plane, then, since that perpendicular is the shortest of all [possible] lines extending from point D to this plane, the angle this perpendicular forms with respect to G will be smaller than any angle imagined at point G formed by any other line drawn from point D to the plane, and the farther the line drawn from point D to the plane will lie from the perpendicular, the longer it will be and the greater the angle it will form. Accordingly, if this perpendicular does not fall on arc AEB but on one side of it, all the lines drawn from point D to this arc will be slanted to one side, and the ones that lie farther away will be longer and will form a larger angle.[42]

[4.96] Let [this] be [the case], then, and take three points, i.e., E, C, and B, on the arc [in figure 6.4.8b, p. 111].[43] Let L be the endpoint of tangency for point B, and let M be the endpoint of tangency for point C, for, since C is nearer than B to D, M will be nearer than L to G, and so CM > BL.[44]

[4.97] Let Q be C′s image, let T be B′s image, and draw TQ. Then draw lines CB and ML, which will intersect when extended, for if a line were drawn from M parallel to CB, it would cut a line from GB equal to CM [and thus be shorter than GL, which means that angle MLG < angle CBG]. Let them intersect at point O.

[4.98] Since GC:CM = GQ:QM [by book 5, prop. 7], and likewise, since BG:BL = GT:TL [by the same proposition], line QT will intersect lines CB and ML [all at the same point]. Let the intersection be at point O.[45]

[4.99] Let N [in figure 6.4.8b, p. 111] be the endpoint of tangency for point E. Since point N is lower than point M [by proposition 4, lemma 1], EN > CM. Hence, if they are extended, lines EC and NM will intersect. Let the intersection be at point P, draw line QP, and extend it until it falls upon EG at point F. Then extend line [O]TQ to EG, and let it fall at point K.

[4.100] It is evident that K will lie above F. However, since GC:CM = GQ:QM, and since three lines [EP, NP, and FP] are drawn through the points of division to meet [at point P] when extended to the other side, they will cut line EG according to the previous ratio [by proposition 6, lemma 3], so GE:EN = GF:FN. But N is the endpoint of tangency [for E, by construction], so F is the image-location [for E]. Hence, line FQT will be the image of arc ECB, and it will be a curved line rather than a straight one, for TQK is straight [by construction], and the curvature of the [resulting] line [TQF] is not oriented with [that of] the mirror.

[4.101] Likewise, if the perpendicular dropped from point D falls on the other side of the arc [i.e., to the left of perpendicular D'G], the proof will be identical. But if the perpendicular falls on the midpoint of arc AB, the lines drawn to the arc from point D to opposite sides and equidistant from the perpendicular will be equal and will form equal angles at G. Moreover, their images will be equidistant from G, and so will the endpoint[s] of tangency,[46] and it can be proven in the foregoing way for either side of the arc by itself, according to how it is cut by the perpendicular, that its image is a curved line in the way prescribed, which is what was set out [to be proven].

[4.102] **[PROPOSITION 9]** Now take a circle [containing arc ADB in figure 6.4.9, p. 112] whose center [F] is not the center [G] of the mirror [containing arc HK]; nonetheless, let it lie in the same plane as the mirror's center. I say that, if an arc [ADB] is taken on the [larger] outer circle on the side of the mirror's center, i.e., nearer that center [G and thus directly opposite it], its image will be curved.

[4.103] For, given this arc, draw a line [FG] from the center of the mirror to the center of the outer circle, and extend this line to the given arc [ADB]. The line drawn from the center of the mirror to this arc [i.e., GD], which is a segment of the diameter [FD] of the larger circle, will be shorter than all the [other] lines drawn from the same centerpoint of the mirror to that arc.

Moreover, two equal lines [GA and GB] can be drawn to the given arc from the mirror's centerpoint on opposite sides of this shortest line, and they will of course be longer [than it]. And if a circle is drawn according [to the length of] either of them from the center of the mirror, arc [AEB on it] will pass through the endpoints of these two lines, and it will be longer [and of a sharper curvature] than the given arc [ADB].

[4.104] It is clear that the image of this longer arc will be a curved line, according to previous conclusions [in proposition 8]. [It is] also [clear that] the images of the points [A and B] common to this arc [AEB] and the given arc [ADB will be] the same and [that] the midpoint [E] of the longer arc lies farther from centerpoint [G] than the [mid]point [D] on the given arc that corresponds to it, so its image [will lie] nearer to the centerpoint [G] than the image of point [D] on the given arc that corresponds to it [by book 5, prop. 17, in Smith, *Alhacen on the Principles*, 414-415]. And so, the image of any point on the outer arc [will lie] nearer the centerpoint than the image of the point on the given arc that corresponds to it. Accordingly, the image of the given [less sharply curved] arc [ADB] is more sharply curved than the image of the [more sharply curved] outer arc [AEB], so the image of the given arc is curved, which is what was set out [to be proven].

[4.105] **[PROPOSITION 10]** In addition, it is proven as follows that the image of a straight line is curved in these [sorts of] mirrors.

[4.106] Let A[C]B [in figure 6.4.10, p. 112] be the [straight] line that is seen, [and let] G [be] the center of the mirror. Draw lines AG and BG. They are either equal or not. If [they are] equal, then construct [the] circle [containing arc] AEB on centerpoint G according to their length. Line A[C]B will obviously fall inside [this] circle. From the previous [proposition] it is clear that the image of arc AEB will be curved. Accordingly, let its image be ZTH. Let Z be the image of A, let H be the image of B, and let T be the image of E.

[4.107] Extend line GE to cut AB at point C. It is clear that E lies on the same line with C [and] farther from the centerpoint [G] than C. Its image will [therefore] lie nearer to centerpoint [G] than the image of C [by book 5, prop. 17]. So let it be M. It is clear that line ZMH is the image of line AB, and it is a curved line, which is what was set out [to be proven].

[4.108] **[PROPOSITION 11]** On the other hand, if lines AG and BG are not equal, then, when it is extended, line AB will either intersect the mirror or not. Let it not intersect [as in figure 6.4.11, p. 113], let AG > BG, construct circle AEQ at [centerpoint] G according to the length of AG [as radius], and extend AB until it touches the circle on the side of B. Let it fall at point Q.

[4.109] It is clear from the foregoing [analyses in propositions 8 and 9] that the image of arc AE is curved. Let Z be the image-point for A, and let M be the image-point for E. ZM will [therefore] be the image of arc AE, and since the image of point B lies farther from centerpoint [G] than the image of point E [by book 5, prop. 17], the image [TNZ] of line AB will be curved, [and] this can be demonstrated according to the midpoints of arc AE and line AB, which is what was set out [to be proven].

[4.110] Note that in the preceding figure, if a segment is cut from line AB on the side of A, and a segment is cut on the side of B equal to it, the remaining portion of the line will have a curved image, and the proof [of this] will be the same as it is for [the whole of] line AB. Also, if in this figure another segment of line AB is cut on the side of B, the same proof will hold for the remainder [of the line] as holds for [the whole of] line AB.

[4.111] **[PROPOSITION 12]** But if line AB touches the mirror, it will either intersect it or be tangent to it. Let it be tangent [as in figure 6.4.12, p. 114]. Let G be the center of the mirror [containing arc PE in gray], and draw lines AG and BG. Plane ABG cuts the mirror along the circle SEZ [that forms their] common [section]. It is clear that line AB will be tangent to the mirror on this circle. Let it be tangent at point E. Accordingly, extend AB to E. Let D be the center of sight. The plane containing lines DG and AG cuts the mirror along a [great] circle [that forms the] common [section] of the plane and the mirror. Let ZP be an arc on that circle. Likewise, let HP be an arc on the [great] circle [that forms] a common section of the plane containing DG and BG [and the mirror].[47]

[4.112] It is clear [from proposition 4, lemma 1, paragraphs 4.74-76] that [the form of point] B is reflected to [point] D from some point [F′] on arc HP. If a tangent is drawn from that point, it will intersect line BG, and the point of intersection will be the endpoint of tangency [for point B on cathetus BG]. Let M be that point [on the resulting tangent F′M].

[4.113] It is also clear that, if a tangent is drawn from point M to circle SEH, that tangent will fall in front of E because AB is tangent at point E, and point B lies above point M. It will therefore fall at point F, and, when it is extended, the tangent [MF] will intersect line AE. Let it intersect at point T. It will intersect line AG on the other side. Let it intersect at point C.

[4.114] Form angle BGS = angle BGD, and extend GS to point L so that it is equal to line DG. Accordingly, arc HS = arc HP, and just as [the form of point] B is reflected to [point] D from a point on arc HP, it[s form] is reflected to L from some point on arc HS. Moreover, the reflection will occur from point F just as the reflection on arc HP occurs from the point [F′] from which the tangent [F′M′] is drawn to point M, and those two points lie the same distance from point M [which means that the reflections from B to D and from B to L are perfectly equivalent]. So draw lines BF and LF.

[4.115] [The form of point] A is reflected to D from some point [R] on arc ZP. However, in triangle HZP the two arcs HZ and HP are longer than the third, i.e., ZP.[48] But HP = HS. Therefore, ZP < ZS. Cut an equal segment from ZS at point Y [so that ZY = ZP], and draw line GY, which, when it is extended to the same length as GD, will necessarily intersect line FL. Let it intersect at point X, and let GXK = GD.

[4.116] It is clear that, just as [the form of point] A is reflected to D from some point [R] on arc ZP, it is likewise reflected to K from some point [R'] on arc ZY.[49] I say that it is reflected to it [i.e., K] only from a point that is below F on the side of Z.

[4.117] For if it is claimed that it can [be reflected] from point F or from some point on arc FY, the line drawn from point A to the point of reflection will intersect line BF. [The form of] point K is reflected to that point of intersection, and [the form of] point L is reflected to the same point, and so two points are reflected in these [sorts of] mirrors to the same point on the same side, which is impossible [by book 5, prop. 16]. It follows that [the form of] point A is reflected to [point] K from some point [R'] on arc ZF.

[4.118] If a tangent is drawn from that point, it will intersect line AZ, and it will fall between C and Z because point F is lower than any [other] point on arc ZF, so the tangent from point F is higher than the rest that are drawn from points on arc ZF. So let that tangent [R'N] fall at point N, and draw line NM, and since it passes through the vertex of triangle BMT and cuts the angle when extended, that line will necessarily intersect BT. Let it intersect at point Q, and draw line GQ.

[4.119] Now let I be the image of point A; let O be the image of point B; and let U be the image of point Q. Since B lies nearer than A to point G, O will lie farther than I from point G [by book 5, prop. 17]. So draw line IO. It is also clear [from book 5, prop. 7] that AG:AN = GI:IN, while BG:BM = GO:OM. Thus, since lines AG and BG are each cut in three points according to this ratio, and since two of the lines, i.e., AB and MN, extended from the points of division [A and N on base AG] intersect at the same point, i.e., at the same point Q, the third [line extending from point I on base AG] will necessarily intersect [these two] at that same point [by proposition 6, lemma 3].

[4.120] Therefore, when it is extended, IO will fall upon Q, so IOQ [forms] a straight line. Thus, IOU will not be a straight [line]. But IOU is the image of line AQ, so the image of line AQ will be curved. Furthermore, if point B replaces point Q, and if some point on line AB replaces point B, it will be demonstrable in exactly the same way that the image of line AB is curved, and this is what was set out [to be proven].[50]

[4.121] **[PROPOSITION 13]** If, however, AB intersects the circle [in figure 6.4.13, p. 116], let it intersect at point E, [and let] M [be] the endpoint of tangency on line BG. [The form of point] B is reflected to [point] D from some point [F'] on arc HP. The arc [extending] from that point of reflection to H [i.e., HF'] is either equal to, longer than, or shorter than arc HE.

[4.122] If it is equal (but it is evident that that arc is equal to arc HQ), let Q [in figure 6.4.13] be the point on the circle where the tangent drawn from point M falls on the side of E. Thus, AE passes through point Q, so MQ intersects AE through point E.[51]

[4.123] But if that arc [HF'] < arc HE [as in figure 6.4.13b, p. 118], MQ will intersect line AE beyond point Q [at point T] so as to form triangle EQT.[52]

[4.124] On the other hand, if that arc [HF'] > arc HE, line MQ will intersect line AE below point Q.[53]

[4.125] Whether the latter or the former is the case, repeat the earlier proof, and it should be demonstrated in precisely the same way that the image of line AB is curved, which is what was set out [to be proven].

[4.126] **[PROPOSITION 14]** Furthermore, if the center of sight lies in the plane containing the visible line and the center of the sphere—in the previous cases it was stipulated that the center of sight does not lie in that plane[54]—then the straight visible line will either intersect the [great] circle [forming] the common section of that plane and the mirror, or it will not intersect [it].

[4.127] If it will intersect, it will be either perpendicular to the mirror['s surface] or inclined to it. If [it is] perpendicular, the angle [formed by] those lines will fall on the center of the mirror, and [the image of] that line will appear straight, for the image of any point on that line will appear on that line, and so the image of that line [will be] straight.

[4.128] But if the given line [AB] is slanted, its slant will either be toward the center of sight or away from it. If it slants away from it [as in figure 6.4.14, p. 120],[55] find the point [R] on the circle from which [the form of] some [point on it, such as B'] is reflected to the center of sight [D], and find the line of reflection [RD]. Any of the slanted lines may fall on this line of reflection, and if it does, then [an image of] this slanted line will not be seen.

[4.129] Having extended a line [DG] from the center of sight to the center of the mirror, take a point [R'] on the arc of the circle in front of this line, such that [the form of] some point [B'] on the slanted line is reflected from it to the center of sight. But [the form of] that point is reflected from the previously designated point [R], which is the endpoint of the line of reflection, since the slanted line lies upon the line of reflection, and so [the form of] that point on the slanted line is reflected to the center of sight from two points on the arc, which is impossible [by book 5, prop. 16].

[4.130] Moreover, even though [the form of] that point may be reflected from the point that is initially selected, it[s image] is still not seen, since it lies on the line of reflection, so it[s image] is occluded by points in front of it [on the object-line], and so [the image of] a line lying upon the line of reflection is not seen.[56]

[4.131] But if a slanted line [AB in figure 6.4.14a, p. 120] is taken with its slant not toward the center of sight, and if it lies below the line of reflection [AB] and cuts it at a point [B] on the circle, I say that no [image of any] point on that line will be seen.

[4.132] For, given [such a] point [e.g., A], if it is claimed that [the form of] that point can be reflected from some point [R′] on the arc lying between the line of reflection [DB] and line [DG] extended from the center of sight to the center of the mirror, and if a line [of incidence AR′] is extended from that point to the point chosen on the arc, this line will intersect the line of reflection [BD], and the point of intersection [X] is reflected to the center of sight from two points on the arc, which is impossible.[57]

[4.133] On the other hand, if it is claimed that the [form of the] point [A] taken on the [slanted] line is reflected from a point on the arc of the circle below that line [i.e., to the right of B], it will be impossible [for it to be seen], since that whole arc is occluded by the line.

[4.134] If, however, the chosen line does not reach the circle, it can indeed be seen, but it is quite small. But if a line with the previous slant [i.e., away from the eye] is selected between the line of reflection and the line initially assumed to pass through the point of reflection to the center [of the circle], this line can in fact be seen, and the curvature of the image of this line will decrease as it approaches the line passing through the point of reflection to the center [of the circle].[58]

[4.135] But if lines are chosen between the line passing through the point of reflection to the center [of the circle and the mirror], they will appear, whether their slant lies toward the center of sight or not. And the way they [i.e., lines slanting toward the eye] are seen is like the way lines [slanting away from the eye] between the line of reflection and the line passing to the center are seen. But these things must be understood for lines that meet the arc of the visible part of the circle, i.e., on the arc lying between the two [lines] drawn tangent to the circle from the center of sight.

[4.136] On the other hand, among lines that meet the circle on the side of the circle that is invisible, one of them will be parallel to the line of reflection. That one will not be seen. Likewise, any one that borders on the parallel and lies below it will be invisible, whereas one that borders on the parallel [and lies] above it can be seen.[59]

[4.137] If, however, a line is selected between the parallels but not bordering on any of them, and if it is slanted toward the center of sight, it will be seen. If it slants in the other direction, it will sometimes be seen, and sometimes not because, if a parallel to the line of reflection is extended from its endpoint, and if that line lies below the parallel, then it will not be seen, whereas [if it lies] above it can be seen.[60]

[4.138] But if the lines do not meet the circle, they will either intersect the line drawn from the center of sight to the center of the mirror, or they will be parallel to it. If any of them intersects it, that line will either intersect it on the side of the center of sight, i.e., between the center of sight and the mirror, or [it will intersect it] beyond the mirror [and the center of sight]. If [it intersects] beyond [i.e., above the head], that line will not be visible, but its ends may appear. If it cuts the visual axis on the side of the center of sight, it will in fact appear the same [i.e., not visible in the mirror]. If it is parallel to the visual axis, it can be seen.[61] Moreover, the images of all these lines are curved.

[4.139] And if the center of sight lies in the same plane as the center of the mirror and the visible lines, they appear diminished, and the one that appears most clear in this case is the one that is most slanted and that corresponds to the center of sight. By the same token, the images of arcs that appear in these [sorts of] mirrors and that lie in the same plane as the center of the mirror and the center of sight appear curved according to the curvature of the mirror.[62]

[4.140] And these things must be understood [to apply] when both eyes lie in the same plane as the center of the mirror and the object that is seen. For if either eye lies slightly outside [that plane], the object will be perceived in another way. And if the eye lies outside the plane containing the visible object and the center of the mirror, it will be perceived more clearly than if the eye lies in that plane.[63]

[4.141] **[PROPOSITION 15]** That the image of a visible object is curved when the center of sight lies in the plane containing the mirror's center and the visible object will be proven [as follows].

[4.142] Let D be the center of sight and G the center of the mirror. Let HE be the visible line. Let HE not intersect the circle but be parallel to line DG [as in figure 6.4.15, p. 122], or let it intersect it on the side of D [as in figure 6.4.15a, p. 122]. Take the plane containing line DG and line HE, and let circle AB be the common section of this plane and the mirror.

[4.143] Draw line HG. Let Z be the image of H, let B be the point on the circle from which [the form of point] H is reflected to [point] D, let the tangent be drawn from point B, and let it intersect line HG at point T. T will be the endpoint of tangency [on cathetus HG].

[4.144] Draw line GB, which will necessarily intersect HE when it is extended, for, if HE is parallel to DG, it will necessarily intersect. If, however, DG intersects HE, then *a fortiori* GB will intersect it. That intersection will lie either on line HE, or beyond that line.

[4.145] Let it lie beyond. Let it intersect at point M, let Q be the image of point M, and let S be the endpoint of tangency [on cathetus MG]. Draw line ZQ, as well as line TS, and draw tangent AU from point A. It is clear that [arc] AB is less than one-fourth [the circumference of the circle, since GD lies on a diameter of the circle and DB intersects the circle], so [the eye at point] D should see less than half the circle [when the corresponding arc below A is included], [and] so angle AGB is acute, while angle UAG is right. Hence, AU will intersect GB. Let it intersect at point U. I say that point U should fall above point S.

[4.146] For, since [the form of] point M is reflected from some point [X] on arc AB, and since A lies below that point, the endpoint of tangency for A [as a point of reflection for the form of any point on cathetus GM] will lie higher than the endpoint of tangency for that point [X as a point of reflection for any point on cathetus GM]. And so S [lies] below point U. Accordingly, extend TS until it intersects line AU, and let the intersection be at point K.

[4.147] Draw line GK, and let it intersect HM at point C when it is extended. [The form of] point C is reflected to [point] D from some point on arc AB. Let F be that point, and from it draw a tangent to GC, that tangent being lower than line AK, and [any] point [on it, such as] O will be lower than point K.

[4.148] Let O be the endpoint of tangency. Extend line DF until it falls on GC. Let it fall at point R. Extend ZQ to line GC, and let it fall at point L. I say that L lies above R.

[4.149] For either lines HC, TK, and ZL are parallel, or they will intersect. Let them be parallel [as in figure 6.4.15a]. Accordingly, since they are parallel, let them intersect line CG at the three points C, K, and L, and let them intersect both lines MG and HG. HG:HT = GZ:ZT [by book 5, prop. 7], and likewise MG:MS = GQ:QS [because HG and GM are cut equiproportionally by parallels HMC, TSK, and ZQL, and for that same reason] GC:CK = LG:LK [all according to proposition 7, lemma 4].

[4.150] But it is clear that R is the image of C because line of reflection DF intersects CG at point R, and O is the endpoint of tangency, so GC:CO = GR:RO [by book 5, prop. 7]. However, GC:CK [which = GL:LK] > GC:CO [which = GR:RO], and so GL:LK > GR:RO. Accordingly, OR:RG > KL:LG, and so OG:RG > KG:LG [by Euclid, V.18]. But KG > OG, so LG > RG. Hence, R lies lower than point L. But ZQL is a straight line. Therefore, ZQR is a curved line, and so the image of line HC is curved. So if some point on line HC replaces point M, and if point E replaces point C, it will be demonstrable that the image of HE is curved.

[4.151] But if lines HC, TS, and ZQ intersect, the intersection will either be on the side of D, or [it will be] on the side of HG. Let it be on the side of D [as represented in figure 6.4.15b, p. 123], and let the intersection be at point C. ZQC will be a straight line, so ZQR will be curved, and so the image of line HE [will be] curved, which is what was set out [to be proven].[64]

[4.152] If an arc is posed outside the mirror, however, it will be possible from this to prove that its image is curved just as it was proven [in proposition 11] when the center of sight did not lie in the same plane as the arc and the center of the mirror, and this is what was set out [to be proven].

[4.153] Therefore in these [sorts of] mirrors straight lines appear curved, and likewise curved lines appear curved. Moreover, if a curved object is placed before the eye in these mirrors, and if it is long and has some slight breadth, the curvature of that object will appear clearly, since it can be detected by those features lying on or within the body. In fact, unless it is considerable, the curvature is not clearly detected when the boundaries of the length or breadth are hidden [so that the image extends beyond the visible face of the mirror], so when an object of slight curvature and considerable size is placed before the eye, its curvature is not clearly detected, even though its image is curved, since the boundaries along the length and breadth of the object do not appear [in the mirror].

[4.154] Moreover, all of the errors that occur in plane mirrors occur in these mirrors as well,[65] and besides those [errors] it happens that the images of straight lines are curved, which is far from the case in plane mirrors.

[CHAPTER 5
On Convex Cylindrical Mirrors]

[5.1] Now the same errors occur in convex cylindrical mirrors as occur in convex spherical mirrors, for [in the former] as in the latter, straight lines appear curved and the size of the visible object appears diminished, but far more pronouncedly because in [convex] spherical mirrors a large object will appear smaller, to be sure, but not very small, whereas in convex cylindrical ones even a very large object will appear greatly diminished [in size]. Likewise, a straight line will appear curved in [convex] spherical mirrors, but if it is [even] slightly curved [it will appear] extremely so in [convex] cylindrical [mirrors], so the errors [that occur] in a [convex] spherical mirror are compounded in a [convex] cylindrical mirror.

[5.2] However, in [convex] cylindrical [mirrors] reflection sometimes occurs from a straight line, i.e., [when it occurs] from [a line of] longitude on the mirror, sometimes from a circle [on the mirror's surface], and sometimes from a [cylindric] section [i.e., an ellipse, on that surface]. When the visible line is parallel to a [line of] longitude on the mirror, the reflection will occur from [that] line of longitude, and a straight visible line will appear [only] slightly curved. These things, moreover, will be demonstrated, but for that demonstration a preliminary point must be made, as follows.

[5.3] **[PROPOSITION 16, LEMMA 5]** If a cylindric section [e.g., elliptical section APEBR in figure 6.5.16, p. 124] is assumed and some point [E] is taken on it that is not a point of reflection, then, when a line [ED] is extended from that point to the normal [BD dropped] from the point of reflection [B] to the axis [of the cylinder on which the ellipse is chosen]—and that line should form an acute angle [EDB] with the normal—if a line [EU] is drawn perpendicular to the tangent [QEL] at that point [E], this line will intersect the normal [BD] outside the axis and outside the intersection of the previous line [ED] with the normal [BD].[66]

[5.4] For example, let AEB be the [assumed cylindric] section, E the given point [that is not the point of reflection], N the visible [object-]point, B the point of reflection, BD the [given] normal [dropped from the point of reflection to the axis], EDB an acute angle, and QEL the tangent [to the cylinder as well as to the elliptical section at the chosen point E].

[5.5] At point B form a circle, i.e., BTO, parallel to the cylinder's base, and draw a line of longitude through point E on the cylinder, i.e., ET. Draw axis DH [of the cylinder], and draw line DC perpendicular to BD.

[5.6] It is obvious that plane HDC is orthogonal to the plane of the circle [BTO]. But the plane tangent to the cylinder at point B will be parallel to this plane [HDC] because the line of longitude extended from point B will be parallel to the axis, and the tangent at [point] B [along the line of longitude dropped through it] will be parallel to CD. Therefore, the plane containing lines LE and ET is not parallel to plane HDC. It will therefore intersect that [plane HDC]. Let it intersect along line LC, and draw line TC, which will of course be tangent [to the cylinder], since plane LET is tangent [to it, by construction]. Moreover, when line TD is drawn, angle CTD will be right because TD is a [a radial segment of the] diameter [of circle BTO, and CT is tangent to the circle at its endpoint].

[5.7] Now at point E form a circle, i.e., ESP, on the cylinder parallel to the base. Let K be the point on the [cylinder's] axis [where it intersects] this circle, and draw line KE. Draw line DL, as well, and it will certainly intersect the plane of circle ESP. Let it intersect at point F, wherever that point may lie either outside or inside the circle, and draw lines KF and EF.[67] Then from point F draw FM perpendicular to the plane of circle BTO, and draw line TM.

[5.8] It is evident that KD is parallel and equal to FM [since they are perpendicular to parallel planes], and so KF is parallel and equal to DM. Likewise, KD is parallel and equal to ET, and KE is parallel and equal to DT. Hence, TE will be parallel and equal to FM, and so EF [will be] parallel and equal to TM.

[5.9] But plane KDL is perpendicular to plane BEO of the [cylindric] section, and it is perpendicular to the plane of circle ESP. Therefore, it is perpendicular to common section EF of the [cylindric] section and the circle. So angle EFK is right. Likewise, angle TMD is right [since DM and KF are parallel, as are TM and EF].

[5.10] Therefore, since angle DTC is right [by construction], rectangle DM,MC = rectangle TM,FE,[68] but since FM is parallel to CL [because CL is necessarily parallel to TE, given that it is the common section of planes TCLE and HDT, which are both perpendicular to the plane of circle ESP], then [triangles DFM and DLC will be similar and will have corresponding sides proportional (by Euclid, VI.4), from which it follows that] DF:FL = DM:MC. But DF > DM, so FL > MC. Consequently, rectangle DF,FL > rectangle DM,MC, so, since TM = EF, rectangle DF,FL > rectangle EF,FE [which = rectangle TM,FE, which = rectangle DM,MC], so angle LED > a right angle, for if it were a right angle, then because line EF is perpendicular to LD, rectangle DF,FL would be equal to EF^2.[69] It therefore follows that angle DEQ [adjacent to angle LED] is acute. Hence, the perpendicular [EU] dropped from point E, that perpendicular being perpendicular to tangent QL, will fall outside line ED and will intersect normal BD outside point D, which is what was set out [to be proven].[70]

[5.11] Now that these things have been set out, it is time to get to the proposition.

[5.12] **[PROPOSITION 17]** Assume a cylinder [in figure 6.5.17, p. 126], and let TH be a [visible] line parallel to the [cylinder's] axis [ZK]. TH will of course be parallel to the line of longitude [AG in the same plane with TH and the axis] of the cylinder.

[CASE 1]

[5.13] Therefore, if the center of sight [E] lies in the same plane as the axis and line TH, the [form of the] line can be reflected, and the reflection will occur from the line of longitude on the cylinder, which is the common section of the plane containing the center of sight and the axis and the surface of the cylinder, as was shown in [proposition 28 of] book 5. Line TH will thus appear as a straight line [T'H'] because any normal dropped from a point on line TH [such as TT' or HH'] will lie in the same plane as the center of sight and the axis, and it will be proven that the image of line TH is straight, just as it is proven for [straight] lines seen in plane mirrors.

[CASE 2]

[5.14] Let the center of sight [E in figure 6.5.17a, p, 126] lie outside the plane containing line TH and the axis, and [let] TH be parallel to the axis, which is ZK. Project a plane that passes through the center of sight and cuts the cylinder's surface parallel to the base. It will of course cut a circle [on that surface]. Let that circle be BF. [The form of] some point on line HT is reflected to the center of sight from some point on this circle. Let it be [reflected] from point B, and let E be the center of sight.

[5.15] Let Q be the point on line TH [whose form is reflected to E from B], draw lines EB and QB, draw line of longitude ABG from point B, and draw the normal ML through point B that intersects the axis at point L. Then from point E extend line EO parallel to ML, and extend QB until it intersects [EO]. Let the intersection be at point O.

[5.16] It is obvious that angle QBM = angle EBM [by construction], but angle QBM = [alternate] angle BOE because LM is parallel to OE [by construction]. Likewise, angle MBE = angle BEO, since [it is] alternate [given the parallelism of ML and EO]. Therefore, angle BOE = angle BEO, so BO and BE are equal [in isosceles triangle BEO].

[5.17] Now choose another point on line TH, let it be point T, and draw line TO. It is clear that line TH is parallel to line of longitude AG [by construction]. Therefore, they lie in the same plane, and line QBO lies in that plane, so line TO will lie in [that] same [plane]. Hence it will intersect line AG. Let it intersect at point G, and draw line EG.

[5.18] It is also clear that line AG is perpendicular to the plane of circle BF, as is the axis [ZK] to which it is parallel, and its plane [is] EOBF [which] cuts the cylinder parallel to its base. Thus, angle GBO [is] right, and [so] angle GBE is right. Consequently $GO^2 = GB^2 + BO^2$ [by Euclid, I.47]. Likewise, $GE^2 = GB^2 + BE^2$, and since BE and BO are equal [by previous conclusions], while GB is common, GO = GE. Hence, angle GOE = angle GEO [in isosceles triangle GEO].

[5.19] Moreover, if normal ZGN is drawn, it will be parallel to EO, since it is parallel to MBL [to which EO was made parallel by construction]. Therefore, angle TGN = [alternate] angle GOE, and angle NGE = [alternate] angle GEO, so angle TGN = angle NGE. Furthermore, since E, O, N, G, and Z lie in the same plane, and since G lies in that plane, E, G, and T will lie in the same plane, and so lines EG, NG, and TG lie in the same plane [which is therefore the plane of reflection]. Thus, [the form of] T is reflected to E from point G.

[5.20] Furthermore, if point H is taken on line TH at the same distance from point Q as point T, and if line HO is drawn, it will pass through [some] point on line AG. Let it pass through point A. When normal DA[Z'] and lines EA and HAO are drawn, it will be a matter of proving as before that the two angles ABO and ABE are right, that the two sides AO and AE are equal, and that the two angles HAZ' and EAZ' are equal. And so [the form of point] H is reflected to [point] E from point A. Likewise, if any [other] point on line TH is chosen, it will be a matter of proving that [its form] is reflected to E from another point on line AG, so [the whole form of] line TH is reflected from line of longitude AG.

[5.21] **[PROPOSITION 18]** It remains to demonstrate that the image of line TH is curved. It is clear from the preceding [theorem] that [the form of] Q [in figure 6.5.18, p. 127] is reflected to E from point B, which is a point on circle [FB]. But since it is reflected in this way from the circle, if a line is drawn from point Q to the center of that circle, it will meet the normal [MBL] dropped from point B, and the intersection [of these two lines] will lie at a point on the axis. So draw QL intersecting ML at point L on the axis, and [this] is the center of circle FB. Then extend EB until it meets QL. Let the intersection be at point C. C will be the image of Q, and C lies in the same plane with lines QH and the axis [ZK], and [with] line of longitude AG.[71]

[5.22] It is also evident that [the form of] T is reflected to E from a point on a [cylindric] section of the cylinder, namely, point G. Moreover, [as established in proposition 16, lemma 5] a line can be drawn from point T perpendicular to a line tangent to another point on the [cylindric] section, and it will intersect normal NGZ dropped from point G outside the axis, that is, outside point Z, which is the intersection of normal NZ and the axis, for if line TZ is drawn, angle TZN will be acute [as stipulated in proposition 16]. Accordingly, draw TX [normal to the cylindric section, as prescribed, and] intersecting NZ at point X, and extend EG until it intersects TX at point I. I will be the image of point T.

[5.23] Likewise, when the line [HP] orthogonal at a point on the [cylindric] section from which reflection [of the form of point H] occurs is drawn from H, it will intersect normal DAZ' outside of point D, which is a point on the axis. Let it intersect at point P, and extend EA until it intersects HP at point S. Point S will be the image of point H. Now draw line SI.

[5.24] It is clear that, since line TI intersects normal NZ, which is parallel to line EO, it will intersect line EO. The same holds for line HS; because it intersects normal DAZ, which is parallel to EO, it will intersect EO. But since T's location with respect to point E is equivalent to and the same distance [from E] as H's location [by construction], the location of point T and of point H [will] likewise [be] equivalent with respect to point O, and [that of] points I and S is also equivalent with respect to O. The location of lines TI and HS will also be equivalent with respect to line EO.[72]

[5.25] Lines TI and HS will therefore intersect at the same point on line EO [since each lies in a plane with it, and both are inclined toward one another]. Let them intersect at point U. TUH will [therefore] be a triangle, and [straight] line IS will lie in the plane of this triangle. The axis, however, does not lie in this plane [since normals TIU and HSU bypass it].

[5.26] But TH does lie in the same plane as the axis so that plane [TZKH] intersects the plane of the triangle [TUH] along common section TH, not along some other [line of section]. Therefore, since point C lies in the plane of line TH and the axis, but not on line TH [itself], it does not lie in the plane of triangle TUH, whereas the two points I and S do lie in the plane of that triangle, so line ICS is curved, and the image of line TH will [therefore] be curved, which is what was set out [to be proven].

[5.27] But its curvature is slight because the perpendicular dropped from point C to the plane of the circle is extremely small,[73] and the closer the visible line is to being parallel to the line of longitude on the mirror, the less sharply curved it[s image] is, [whereas] the farther [it is from such parallelism] the more [sharply curved its image is].[74]

[5.28] **[PROPOSITION 19]** Furthermore, if line TH intersects the plane containing the center of sight and the axis, and if it is orthogonal to it, the center of sight will either lie in the plane of line TH intersecting the plane of the axis and the center of sight orthogonally, or [it will lie] outside [that plane].

[CASE 1]

[5.29] If [the center of sight] lies in that plane, it will lie beyond or in front of line TH. If [it lies] beyond, then, since that line has bodily dimensions, it will block the mirror from the center of sight, and so it[s form] will not be reflected [to the eye], although perhaps its terminal segments will appear and be reflected from the circle on the cylinder that forms the common section of the plane of line TH that cuts the cylinder and the cylinder [itself]. And the image of these terminal segments will [appear] just as [they do] in convex spherical [mirrors, as described in proposition 14, paragraph 4.138].[75]

[5.30] Likewise, if the center of sight lies in front of TH, part of that line will be hidden by the head containing the center of sight. Nonetheless, the visible part of the line is reflected [to the center of sight] from the circle [formed by the plane of reflection] in exactly the same way as in convex spherical [mirrors, according to proposition 14, paragraph 4.138].

[CASE 2]

[5.31] But if the center of sight lies outside the plane of TH that cuts the plane of the center of sight and the axis orthogonally, then let E [in figure 6.5.19, p. 128] be the center of sight [above line TH] and XZG the cylinder.[76] [The form of point] H is reflected to E from some point on the cylinder. Let [it be reflected] from B. Let T lie the same distance [as H] from point E. I say that [the form of point] T is reflected to E from another point [i.e., other than B] on the cylinder, and that, since points H and T are equivalently situated and the same distance from point E, their points of reflection, i.e., B and G, will be equivalently situated and the same distance from point E. Therefore, the two points B and G will lie on a circle.

[5.32] Let BZG be the circle, with D its centerpoint. Draw lines HB, BE, TG, and GE, and from the centerpoint [D] draw normals to the tangents at B and G, i.e., [normals] DBO and DGS. Then draw line ED, and extend HB and TG until they intersect line ED.

[5.33] Since points H and T are equivalently situated and the same distance with respect to E and with respect to D, and since, by the same token, points B and G are equivalently situated with respect to D and with respect to E, lines HB and TG will be equivalently situated with respect to line ED, and so they will intersect at the same point on that line. Let [that intersection] be at point L.

[5.34] Produce the line of longitude on the cylinder that contains point Z, let this line lie in the plane containing the center of sight and the axis, let it be AZ, and draw [lines] LZN and DZC. Let Q be a point on line TH, that is, the point [on it] that lies in the plane of the center of sight and the axis, and from point Q draw a line parallel to line DZC. This line will fall on the axis, and LZN will fall on this line beyond point Q. Let it fall at point N.

[5.35] It is clear from the foregoing that angle [of incidence] HBO = angle [of reflection] OBE [by construction]. But angle HBO = angle LBD because they are vertical [angles], and angle OBE = the two angles BED + BDE because it is external [to triangle BDE and therefore equal to the two opposite interior angles, by Euclid, I.32]. Therefore, angle LBD = the two angles BED + BDE. So form angle MBD = angle BDE. It follows that angle MBL = angle BEL [i.e., BED], so the rectangle EM,ML = BM^2.[77]

[5.36] Draw line MZ. Since angle BDM > angle ZDM, and since the two sides ZD and DM [of triangle ZDM] are equal, respectively, to the two sides BD and DM [of triangle BDM], MB > MZ, so the rectangle EM,ML > MZ^2.[78] Let the rectangle EM,MI = MZ^2 [which means that EM:MZ = MZ:MI] and draw lines IB and IZ. Hence, angle MZI = angle ZEI [because triangles

MZE and MZI are similar according to the proportionality of sides EM and MZ in triangle MZE and sides MZ and MI in triangle MZI] , so [angle] MZL > angle ZED.

[5.37] But since angle MBD has been posited equal to angle BDM [by construction], line MD = line MB [in isosceles triangle DMB]. But MB > MZ [by previous conclusions], so MD > MZ. Therefore, angle MZD > angle MDZ [since it is subtended by a longer line], so angle DZL > the two angles ZDE + ZED.[79] But angle DZL = [vertical] angle NZC, and [exterior] angle CZE = the two [interior and opposite] angles ZDE + ZED [in triangle ZED], so angle NZC > angle CZE.

[5.38] Let [angle FCZ] equal [to angle CZE] be cut [from NCZ] by line FZ, which will intersect line NQ [at point F] beyond point N. Therefore, since angle FZC = angle CZE, [the form of] F is reflected to E from point Z. [The form of point] Q is reflected to E from a point on the line of longitude passing through Z, that is, from a point on AZ beyond [i.e., below] Z. For if [it occurs] from a point this side of Z, i.e., nearer E, the line extended from point Q to that point of reflection will cut line FZ, and so the point of intersection is reflected to E from two points, which is impossible.[80]

[5.39] Take point K below Z from which [the form of] Q is reflected to E, then, and extend line EK until it intersects line NQ [i.e., the cathetus dropped from object-point Q] at point P. P will be the image of Q. But [the form of] H is reflected to E from a point on the cylindric section [formed on the cylinder by plane of reflection HBE]. Therefore, if a normal is dropped from point H to the line tangent to the [cylindric] section at some point [on it], that normal [i.e., the cathetus] will intersect normal CZD outside the axis [by proposition 16, lemma 5]. Let it intersect at point U.

[5.40] Likewise, from point T a normal can be drawn to the [cylindric] section from a point on which [its form] is reflected to E. And since points H and T are equivalently situated with respect to line CZD, the same also holds for the points on the [cylindric] section through which the normals [i.e., the catheti] pass, so those two normals will intersect at the same point on line CZD. Accordingly, let them intersect at point U.

[5.41] [Therefore, the extension of] line EB will intersect line HU. Let R be the point of intersection. By the same token, let EG intersect TU at point Y, and draw line RY.[81] It is obvious that R is the image of H, [and] Y is the image of T, and we have triangle ERY. Point Z lies outside the plane of this triangle, so the plane of this triangle is higher than line EP, and so P lies outside [that plane]. Hence, line RPY will be curved, and it is the image of line TH, and the curvature of this image is certainly not inconsiderable, which is what was set out [to be proven].

[5.42] It is therefore clear that in these [sorts of] mirrors, if a straight visible line is parallel to a line of longitude on the cylinder, its image will be either straight or verging toward straightness. But if a straight visible line is parallel to the width of the mirror [i.e., the plane through it is perpendicular to the cylinder's axis], its image will be curved, and its curvature will not be inconsiderable.

[5.43] Furthermore, among [visible] lines oriented between these two [extremes], the images of those that verge more closely toward an orientation parallel to the longitude of the cylinder will be closer to straight, whereas the images of those that are nearer to an orientation parallel to the [cylinder's] width will be more curved. And the curvature of the images will diminish or augment depending on how close or far the lines are from either of these orientations, and this is what was set out [to be demonstrated].

[CHAPTER 6
On Convex Conical Mirrors]

[6.1] Moreover, in convex conical mirrors the same errors occur as happen in convex cylindrical [mirrors],[82] for [straight] visible lines that are parallel to the longitude of the cone appear straight, whereas those parallel to the width [of the cone appear] curved, and for those at intermediate positions, their curvature augments or diminishes according to how near or how far [they are from those extreme positions], and this will of course be demonstrated. However, we must set forth something beforehand, and it is [as follows].

[6.2] **[PROPOSITION 20, LEMMA 6]** If a point of reflection is taken on the surface of a cone and a [conic] section is produced [on that surface] to pass through that point, and if a point is taken on that [conic] section farther from the vertex of the cone than the point of reflection and a normal is dropped from the selected point to a line tangent to the [conic] section, this normal will intersect the normal dropped from the point of reflection [at a point] outside the axis.

[6.3] For instance, let ABGZ [in figure 6.6.20, p. 129] be a cone standing upright on its bases [i.e., a right cone], A the cone's vertex, BFZ the [conic] section [produced on its surface], E the point of reflection, and Z the point on the [conic] section farther from [vertex-]point A than E. At point Z let there be a plane cutting the cone parallel to its base. It will of course cut it along a circle [forming the] common [section of the cutting plane and the cone's surface]. Let that circle be GBRZ, draw lines AZ and AE, and extend

AE until it is equal to AZ. It will reach the circle. So let it fall at point O on it.

[6.4] Let C be the center of the circle, draw axis AC, and from point E draw the normal [ED] to the plane tangent to the cone [at that point]. It will of course intersect the axis in the vicinity of the circle's centerpoint C. Let [that intersection] be at point D, and draw line DZ.[83]

[6.5] Then from point O draw a normal [OK] intersecting the axis at point K, and draw lines DZ and KZ. At point Z produce [line] TQ tangent to the [conic] section, and [at the same point produce] another [line] ZY tangent to circle BGZ.

[6.6] Next draw line BCZ, and from point C draw CR perpendicular to line BCZ. It will of course be perpendicular to the axis, since the axis is perpendicular to the circle [in whose] plane [CR lies], so CR is perpendicular to plane ACZ. It will also be parallel to tangent ZY, so ZY is perpendicular to plane ACZ, [and] so TQ is not perpendicular to that same plane.

[6.7] However, because K is the [the endpoint of] pole [KC] in circle BRZ, then, since lines KO and KZ are equal [because they are lines of longitude on a right cone with its vertex at K and its base circle passing through Z and O], and since axis AK is common [to triangles AOK and AZK], angle AOK = angle AZK, and so angle AZK is right [since angle AOK was constructed as a right angle]. Therefore, since line KZ is perpendicular to [line] AZ, which is a line of longitude [on the mirror], it will be perpendicular to the plane tangent to the cone['s surface] along this line of longitude. But TQ lies in the [same] tangent plane because it is the common [section] of the tangent plane and the [conic] section. Accordingly, KZ is perpendicular to TQ.

[6.8] Furthermore, draw HZ in the plane of the [conic] section perpendicular to line TQ [and therefore normal to the section itself]. Since line KZ lies outside the plane of the [conic] section, it will intersect line HZ and will [therefore] not form a single line with it. Hence, plane KZH intersects the plane of the [conic] section along common section HZ, and it intersects line TQ at point Z. In addition, plane AZK intersects plane AZH along common section KZ.

[6.9] But DZ lies in the plane of the [conic] section, and it is intersected by line KZ at point Z, and point T [lies] above plane KZH, point Q below [it, i.e., on the other side of it from T]. And so plane KZH cuts plane DZQ along a common section, and that common section is perpendicular to line TQ because that line lies in plane HZK to which TQ is perpendicular. And since plane HZK intersects plane DZQ, and since plane HZK slants in the direction of [segment] ZE [of the conic section], the common section [HZX] of those planes will lie between lines QZ and DZ, and so it will intersect normal ED [at point X] outside the axis. That it [i.e., the normal to the section at Z] must necessarily intersect it [i.e., normal ED dropped from center of sight E] has been demonstrated in book 5, proposition 26,[84] and so what was set out [to be demonstrated has been shown].[85]

[6.10] **[PROPOSITION 21]** So let there be a [right] cone [in figure 6.6.21, p. 129] with its vertex at A, AH being its axis and AZ a line of longitude, and from point Z to the plane tangent to the cone along line AZ drop a perpendicular, which will perforce intersect the axis [at point H]. Let it be line TZH.

[6.11] From point A extend line AN outside the cone [and] above the plane tangent to the cone along line AZ so that it forms an acute angle with the axis, as well as with line AZ of longitude. From point H within plane AHN, draw line HO forming an angle [AHO] equal to angle AHZ, that line necessarily intersecting line AN [at point O. Consequently] when a circle is produced through point Z parallel to the [cone's] base, HO will pass through [that] circle just as HZ passes through it.

[6.12] Now draw line OZ, and extend it to point F. Since line OZ lies above the plane tangent to the cone along line AZ, then because HZ is perpendicular to that plane [by construction], angle OZH will be greater than a right angle [because it intersects the plane tangent to AZ from above]. Consequently, [adjacent] angle FZH is acute [so that ZF lies inside the cone].

[6.13] From point Z draw ZM tangent to the circle, and from point F draw a line perpendicular to AZ to fall at point E on it, and when it is extended, it will intersect AO because angle OAZ is acute [by construction, and because AO, AZ, and OZF lie in the same plane]. Accordingly, let it fall at point N, and from point E draw line QE parallel to line TH.

[6.14] Then from point E draw LE parallel to line MZ. It is evident that MZ is perpendicular to AE because it is perpendicular to TH, as well as to the diameter of the circle, to which it is tangent. Therefore, LE is perpendicular to AE [since it is parallel to MZ, by construction].

[6.15] Now produce plane LQD cutting the cone. It will of course form a conic section [because the cutting plane intersects the axis below the circle passing through E]. Hence, since AE is perpendicular to FN [by construction], as well as to QD and LE, FN will lie in that plane [QEDL], which cuts the cone [along the aforementioned conic section]. Accordingly, produce CF parallel to QE. It will of course be parallel to TZ.

[6.16] But since angle OZT is acute [by previous conclusions, adjacent] angle TZF is obtuse. From point Z draw a line [ZC] that forms with TZ an angle [TZC] equal to angle OZT, and that line will necessarily intersect FC. Let it intersect at point C, and draw line EC. Therefore, since CZ and OZ lie in the same plane, and since angle OZT = angle TZC [by construction, the form of] point O is reflected to C from point Z.

[6.17] Since, however, angle OZT = [alternate] angle ZFC, and since angle OZT = [alternate] angle ZCF, sides ZC and ZF [of triangle ZCF] will be equal, and given that angle FEZ is right [by construction], $FZ^2 = EZ^2 + EF^2$ [by Euclid, I.47], while $CZ^2 = EZ^2 + EC^2$ [by the same theorem]. Therefore, CE and FE are equal, and so angles ECF and EFC are equal, whereby angles NEQ [which is alternate to EFC] and QEC [which is alternate to ECF] are equal. And since C, E, and N lie in the same plane [i.e., the plane producing the conic section through point E, the form of] point N is reflected to C from point E.

[6.18] Likewise, draw some line from point F to some point on line ZE, and extend it to ON. As to the point on line ON where it falls, it will be proven that [its form] is reflected to C from the [corresponding] point on ZE because it cuts that line. In the same way, as well, for all such lines, the proof will take its start from perpendicular FE with respect to line EZ, which will be the common terminal [base-line], and so [the form of] any given point on line ON is reflected to C from some point on line EZ.

[6.19] **[PROPOSITION 22]** Having demonstrated this point, we should state [that], when the eye perceives straight lines that pass through the vertex of a right convex conical mirror at a slant to the mirror's axis, the forms of those [lines] will be somewhat convex in that mirror.

[6.20] Accordingly, let the right conical mirror be ABG [in figure 6.6.22, p. 130], with A as its vertex and AD as its axis, let us produce line AZ at random on its surface, and let point Z be marked on it at random. Let a plane pass through Z parallel to the base of the cone, and let it form circle ZU. Then from Z let us drop ZH perpendicular to AZ. Hence, this line will intersect the cone's axis, so let it intersect [that axis] at H.

[6.21] From Z let us extend line ZM tangent to the circle [ZU], and let us extend a line from A that forms an acute angle with both lines AZ and AH, and let it lie outside the plane tangent to the cone that passes along line AZ, which is possible. Let it be AO, then, and let us produce a line from point H within the plane containing AO and AH that forms an angle [AHO] with AH equal to angle ZHA. This line [HO] will therefore intersect AO because the two angles at A and H [i.e., OAH and AHO] are acute. So let them [i.e., AO and HO] intersect at O.

[6.22] Accordingly, line HO will intersect the circumference of circle ZU because angle AHO = angle AHZ [by construction]. So let it intersect at U, and let us extend AU in a straight line. Let us also extend perpendicular HZ to T, let us produce OZ and extend it directly to F, and extend AZ to E. Therefore, angle FZH will be acute because line OZ cuts the plane tangent to the cone and passing along AZ. Hence, line FZ lies below the common section of plane OZH and the [aforementioned] tangent plane [passing along AZ], and this common section forms a right angle with line HZ. Thus, angle OZH is obtuse, [and] so [adjacent] angle FZH is acute.

[6.23] So take point F on ZF, from it extend FE perpendicular to AE, and continue it in a straight line. It will therefore intersect line AO because angle OAE is acute. Let it intersect at N, then, and extend line ED from E parallel to line ZH. ED will thus be perpendicular to the plane tangent to the cone passing along AE.

[6.24] Then from E draw line EL parallel to line ZM, and produce the plane containing LE and ED. It will therefore intersect the surface of the cone and will form a [conic] section, for this plane is oblique to axis AD.

[6.25] Let the [conic] section be BEG'. MZ is perpendicular to plane AZH [since it was constructed tangent to the circle at point Z and is therefore perpendicular to the diameter passing from Z through axis AH of the cone], and this was established earlier [in proposition 21]. Therefore, line LE is perpendicular to plane AED, so angle AEL is right, angle AEN is right, and likewise angle AED is right [all three by construction]. Consequently, lines LE, NE, and DE lie in the same plane [to which AE is perpendicular]. Hence, line FEN lies in the plane of the [conic] section [BEG'].

[6.26] From point F extend line FR parallel to line DE. Accordingly, this line will be parallel to line HZ [to which DE was constructed parallel]. In plane OZH draw a line [ZR] from Z that forms with ZT an angle equal to OZT. This line will therefore intersect FR because it will intersect ZH, which is parallel to FR [by construction] and lies in the same plane with it, since ZF lies in that plane. So let it intersect at R.

[6.27] Accordingly, the two angles at R and F [i.e., ZRF and ZFR] are equal, for they are equal to the two angles [OZT and TZR] at Z.[86] So the two lines RZ and FZ [within isosceles triangle ZRF] are equal. But it has been shown that line FEN lies in the plane of the [conic] section [BEG'], and line FR is parallel to line ED [by construction]. It [i.e., FR] therefore lies in the plane of the [conic] section.

[6.28] Let us then draw RE. It will thus lie in the plane of the [conic] section. Extend DE to K, and it has been shown that EA is perpendicular to the plane of [that] section. Hence, each of the angles AER and AEF is right, and the two lines FZ and RZ are equal. Consequently, the two lines RE and FE are equal, so the two angles ERF and EFR are equal.

[6.29] Accordingly, the form of N will be reflected to R from E, and the form of O will be reflected to R from Z. Moreover, every line extended from F to some point on line AN will intersect AE. But it is clear that that line will be equal to the line extended from R [to the same point on AE where the line from F intersects it] because AE is perpendicular to the plane in which lines RE and FE lie, since this plane is the plane of the [conic] section, and the two lines RE and FE are equal. Hence, both lines extended from R and F to a given point on line AE are equal.

[6.30] It is therefore evident that the form of [any] point on AN will be reflected to R from a point [such as] that on line AE. And the same holds for any point lying on AN beyond N; if it is connected with F by a straight line, that line will intersect AE beyond E. It is also evident that the form of a point on AN will be reflected to R from a point on AE. From this, therefore, it is evident that the form of line AN, as well as any [line] continuous with it, will be reflected to R from a straight line on the surface of cone ABG, and the same holds for every line extended from A at a slant to the cone's axis.[87]

[6.31] Let us draw ND [in figure 6.6.22a, p. 131, abstracted from figure 6.6.22].[88] It will therefore intersect the periphery of the [conic] section because the two points N and D lie in the plane of [that] section, and N lies outside the section['s periphery], whereas D lies inside the section['s periphery]. So let it intersect the periphery of the [conic] section at C, and since triangle AOH lies in the same plane, ND will lie in the same plane as triangle AOH.

[6.32] [Point] C is therefore in the plane of triangle AOH, and the two points A and N lie in the plane of this triangle. Hence, points A, N, and C lie in the plane of triangle AOH. But points A, U, and C lie on the surface of the cone. Accordingly, points A, U, and C lie on the common section of the surface of the cone and plane AND. But this common section is a straight line. So points A, U, and C lie in a straight line.

[6.33] Extend AU directly to C, then, and extend RZ in a straight line. Accordingly, it will intersect OH [because O, Z, R and H all lie in the same plane of reflection]. Let it intersect at P. P therefore lies in the plane of triangle AOH. So extend AP, and let it continue in a straight line. It will therefore intersect ND at G, and since F lies below the plane tangent to the cone that passes along line [of longitude] AZE, angle FED will be acute, whereas [adjacent] angle DEN is obtuse. Hence, [interior] angle ENC [of triangle NED] is acute [because it is smaller than opposite exterior angle FED, which is acute].

[6.34] Furthermore, let line CZ' be tangent to the [conic] section. It is clear, then, as [shown] in an earlier proposition, that angle DCZ' is obtuse[89] and that the perpendicular erected on CZ' at C cuts angle DCZ' and will intersect ED beyond D. So this perpendicular will intersect ED at S.

[6.35] Hence, the perpendicular extended from N to the line tangent to the [conic] section [at the point where that perpendicular intersects the conic section] will intersect the [conic] section beyond C, that is, farther from E than C [lies from it], for these perpendiculars [i.e., NQ and ED] will intersect outside the periphery of the [conic] section [i.e., on the other side of that periphery from N]. Hence, the perpendicular extended from N to the line tangent to the [conic] section will not cut angle DCZ'. It will therefore lie farther from NE than CD [does], and this perpendicular cuts ED beyond D.

[6.36] So let the perpendicular dropped from N to the line tangent to the [conic] section be NQ. Also, RE intersects EN, and it intersects the periphery of the [conic] section and lies in its plane, while NQ [also] lies in the plane of the [conic] section. Hence, if RE is extended in a straight line, it will intersect NQ. Let it intersect at Y, then.

[6.37] Plane AND intersects the plane of the [conic] section. Since point E lies outside plane AND because plane AND is not [in] the plane of the [conic] section [whereas E is], and because A lies outside the plane of the [conic] section, since AE is perpendicular to the plane of [that] section, whereas E lies on its periphery, then ND is the common section of plane AND and the plane of the [conic] section, and NQ will intersect [that] section beyond C [i.e., on the opposite side from C and E]. Hence, NQ lies outside plane AND. Y therefore lies outside line APG.

[6.38] Accordingly, if the center of sight lies at R, and if line AON lies on some visible object, then P will be the image of O, Y will be the image of N, and A will appear at its [actual] location, since it lies at the vertex of the cone.[90] And the image of line AON will be the line passing through points A, P, and Y, but this line is convex because it lies outside [straight line] APG.

[6.39] So let that [image-]line be APY, and it has already been shown that the forms of all the points on AN are reflected to R from AE. Therefore, the radial lines according to which those forms are reflected lie in the plane of triangle RZE, so all the images of [the points on] line AN lie in that plane.

[6.40] Hence, convex line APY lies within that plane, and P lies closer to R than Y does, and the convexity of this image will be toward the center of sight, and it will [therefore] be of slight [apparent] convexity.[91] Moreover, the [length along the] cross-section [AY] of this image will be slightly smaller than the line itself [i.e., AN of which it is the image]. Consequently, the images of straight lines that are extended from the vertex of the cone at a slant to the axis are perceived by sight as convex in such a mirror, and the forms of these lines are reflected from straight lines among the lines extended along the cone's longitude, and this is what we wanted to prove.

[6.41] On the other hand, the forms of lines that are parallel to the width of a conical convex mirror are reflected from convex lines on the mirror's surface, and the convexity of these lines is obvious, as [it is] in a convex cylindrical mirror, and for the same reason, and it will likewise be evident that the images of these lines will be quite convex and manifestly [so] to the [visual] sense. Also, the center of sight will lie outside the plane that contains the convexity of the forms of these lines, and the cross-sections of the images of these lines will be considerably shorter than the lines themselves.

[6.42] As to lines that are slanted between these two extremes, however, those whose orientation approaches that of lines extended along the length of the cone have slightly convex forms, whereas those that approach lines parallel to the width of the cone have forms that are clearly convex.

[6.43] But also, curved lines that approach the vertex of the cone have smaller, narrower, and more convex forms, whereas those that approach the base of the cone have larger forms, according to what was demonstrated for convex spherical mirrors—i.e., that the smaller the mirror, the smaller the circles that fall on its surface—and so the images [falling on those smaller circles] will lie closer to the center [of curvature], from which it follows that they will be smaller.

[6.44] By the same token, sections that lie on a conical mirror toward the cone's vertex are narrower and shorter [than those that lie farther from it], and so the image [within such a section] will be nearer the point where the normals dropped from the visible line to the lines tangent to the sections, which form the common section [of the plane of reflection and the plane tangent to the mirror at that point], intersect, and so those images will be smaller.

[6.45] On the other hand, the opposite holds for sections that lie toward the base of the mirror, so it happens that a form perceived in a convex conical mirror will take on a conical form, i.e., what lies toward the vertex of the mirror will be narrower, whereas what lies toward the base will be broader, and the convexity of a form along the width [of the mirror] will be evident.

[6.46] It also happens in these [sorts of] mirrors that the closer the visible object approaches the mirror, the larger it will appear, whereas the farther away it will be, the smaller it will appear.

[6.47] Therefore, the misperceptions that occur in these sorts of mirrors are in every way like those that occur in convex cylindrical mirrors except for the conical shape of the form. And without exception the form of a visible object that is perceived by reflection will always take the shape of the surface of the mirror from which the form is reflected, and the reason for this is that the image-location is invariably determined by the shape of the mirror's surface and by the place where the normals intersect, so the [shape of the] mirror's surface always plays a role in the shape [of the image] of the visible object that is perceived in the mirror. However, the compound misperceptions [arising] in this [sort of] mirror are identical to the [compound] misperceptions [occurring] in the previously discussed mirrors [i.e., convex spherical and convex cylindrical].[92]

CHAPTER 7
Concerning the Misperceptions That Occur in Concave Spherical Mirrors

[7.1] In these [mirrors], in fact, more [misperceptions] occur than in all the convex and plane mirrors,[93] for what occurs in the latter occurs in these as well—i.e., a weakening of light and color and a variation in orientation and distance—for it is reflection alone, not the shape of the mirror, that causes this [sort of variation]. [But] in addition, there is more variation in [image] size in these mirrors than in convex mirrors, for in convex [mirrors] an object will generally be perceived as smaller [than it actually is], whereas in concave [mirrors] it will sometimes be perceived as larger, sometimes as smaller, [and] sometimes as it actually is, and this happens according to how it changes position with respect to the mirror as well as to the center of sight, as we will demonstrate in this chapter.

[7.2] It also happens in these mirrors that a single visible object may appear as two, or three, or four, and this is not the case in plane and convex mirrors, for in those [kinds] a single visible object is perceived only singly, whereas in concave [mirrors, such is] not [the case].

[7.3] Furthermore, the arrangement of the visible object's parts is perceived in convex and plane mirrors as it actually is, whereas in spherical concave [mirrors it is perceived] otherwise in several situations,[94] and this in two ways: namely, in convex spherical mirrors there is no deception in the perception that a single thing is single and the perception of the arrangement of its parts according to how it actually is, and since there is deception in regard to these aspects in spherical concave mirrors, it is clear that nothing is perceived in these mirrors without deception, either invariably or at some time according to variation in the position [of the object vis-à-vis the mirror as well as the center of sight].

[7.4] However, weakening of light and color as well as change in position and distance occur in these mirrors just as [they] invariably [occur] in the others, and they do so in every situation. But size, shape, and number are subject to deception in these mirrors in some situations, as we will demonstrate.

[7.5] Concerning number it has been shown in chapter [2, book 5] on image [formation] that in concave spherical mirrors one object has one, two, three, or four images, and that the form of a visible object is always perceived at its [appropriate] image-location. However, one object perceived in concave spherical mirrors may be perceived as one, perhaps as two, perhaps as three, and perhaps as four, which does not happen in convex and plane mirrors.

[7.6] As to the arrangement of the visible object's parts, it has also been claimed in chapter [2, book 5] on image [formation] that the form of a single [object-]point is reflected from the circumference of a [great] circle [on the mirror's surface] and that visible objects whose images lie beyond or behind the center of sight, in front of it, or at the center of sight [itself] appear blurred and not clear, and anything of this sort does not have the arrangement of parts that the visible object itself has. And here, as well, what obtains in these mirrors is other than what obtains in convex and plane mirrors. But the reasons for this phenomenon have been discussed in the chapter on image [formation].[95]

[7.7] It thus remains [for us] to show that what is perceived in these mirrors may be perceived larger, smaller, or the same size [as the object itself], and that in certain situations it may be perceived inverted and in others erect, and that a straight object is perceived as concave, convex, or straight in mirrors of this sort, and that convex and concave objects are also perceived other than they [actually] are [in this sort of mirror]. And these [misperceptions] also arise from a variation in the arrangement of the visible object's parts, and we will demonstrate this in the following way.

[7.8] **[PROPOSITION 23]** Accordingly, let there be a concave spherical mirror centered on A [in figure 6.7.23, p. 132], let it be bisected by a plane passing through its center, let it form [great] circle BG, let a line [AU] be extended within it at random, and let it be bisected at O.

[7.9] Take A as a centerpoint, and at the distance of AO [as radius] let us form a circle, and let it be EZ. Choose some point T at random on line OU, and from T extend lines TN and TM perpendicular to line AU. Then from T extend lines TE and TZ tangent to circle EZ, and let us extend AE and AZ, and let them continue to B and G. Let us extend TB and TG, and let us draw BM parallel to AT and also GN parallel to AT, and let us connect AM and AN and extend them in straight lines.[96]

[7.10] Therefore since AO = OU [by construction], AE = EB, and AZ = ZG, and because TE is tangent to circle EZ, TE will be perpendicular to BA, and likewise TZ [will be] perpendicular to AG. Hence, line BT = [line]

TA [by Euclid, I.4], [line] TG = [line] TA, angle TBA = angle TAB [within isosceles triangle TBA], and angle TGA = angle TAG [within isosceles triangle TGA]. And since BM is parallel to AT, angle MBA = [alternate] angle BAT. Therefore, angle MBA = angle ABT, and likewise angle TGA = angle AGN.

[7.11] So when the center of sight is at T, and when M and N lie on some visible object, the form of M will be extended along line MB and will be reflected along BT, and the form of N will be extended along NG and will be reflected along GT. The center of sight at T will therefore perceive points M and N [at locations] beyond points [of reflection] B and G, and [so it will perceive the entire image of] line MN beyond arc BG.[97]

[7.12] Also, since TE is perpendicular to AB, angle ABT will be acute. But angle MBA = angle ABT. Thus, TB > BM, so AT > BM, and they [i.e., lines AT and BM] are parallel. Consequently, [line of reflection] TB will intersect [cathetus] AM. Let them intersect at F, then. F is thus the image of M, and it will be demonstrated equivalently that [line of reflection] TG will intersect [cathetus] AN. Let it intersect at Q, then. Q will thus be the image of N.

[7.13] Let us then connect FQ, which is the cross-section of the image of MN, and since TE and TZ are equal, angles TA[E]B and TAZ[G] will be equal, lines TB and TG will be equal, lines BM and GN [will be] equal, and lines AM and AN [will be] equal. Moreover [given the similarity of triangles AFT and MFB], AF:FM = AT:BM, and AF:FM = AT:GN = AT:BM [because GN = BM], so AF:FM [= AT:GN] = AQ:QN, and AM = AN. Hence, AF = AQ, so FQ is parallel to MN. Thus, FQ > MN. But FQ is the cross-section of the image of MN. Accordingly, if the center of sight is at T and MN lies on some visible object, the eye will perceive its form as larger than [object-line MN] is.

[7.14] **[PROPOSITION 24]** Now [in figure 6.7.24, p. 132] let us duplicate circle BG, line AT[U], and lines AB, AG, and TB [as given in figure 6.7.23]. Let TK be perpendicular to the plane of circle BG at point T, and let us draw KA, KB, and KG. Thus, planes KBA and KGA intersect the sphere [of the mirror] at its center perpendicular to [the appropriate] planes tangent to its surface.[98] Within these [planes], then, the form [of any given visible object] is reflected, and the two common sections between these two planes and [the surface of] the sphere form great circles from whose circumference the forms are reflected.

[7.15] Let us then draw BM parallel to AK in plane BKA, and let it be shorter than AK. Let us draw AM, and let it be extended in a straight line, and extend KB until they intersect at F. Then draw NG in plane KGA, let it be parallel to AK, and assume it is equal to BM. Let us connect AN, let it be extended in a straight line, and extend KG in a straight line until they intersect at Q. Then let us connect MN and FQ.

[7.16] Accordingly, since BT = TA [as concluded in the previous proposition], BK = KA [by Euclid, I.4], and GK = KA [by Euclid, I.4]. Hence, BK = GK, angle KBA = angle KGA, and angle KAB = angle KBA. Likewise, angle KGA = angle KAG, so angle ABM [which is alternate to angle KAB] = angle ABK [which = angle KAB], angle AGN [which is alternate to angle KAG] = angle AGK [which = angle KAG], and angle ABM = angle AGN [since both are equal to equal angles KBA and KGA]. In addition, line BM = line GN [by construction]. Thus, line AM = line AN; so [line] AF = line AQ. The two lines FQ and MN will therefore be parallel, so [line] FQ > line MN.

[7.17] Hence, when the center of sight lies at point K, and when line MN lies on some visible object, the form of M will be extended along line MB and will be reflected along line BK in the plane of the circle passing through points B, A, and K, whereas the form of point N will be extended along line NG and will be reflected along line GK within the plane of the circle passing through points G, A, and K.

[7.18] And [so] point F will be the image of point M, while point Q will be the image of point N, and line FQ will be the cross-section of the image of [the entire line] MN. But we have already demonstrated [in paragraph 7.16] that line FQ > line MN, so when the center of sight is at point K, and when line MN lies on some visible object, the eye will apprehend the form of line MN on line FQ. Therefore, it will perceive the form [of the visible object] as larger than the visible object [itself].

[7.19] Accordingly, if we rotate the entire figure around line AU, while keeping [AU] itself stationary [to form the axis of rotation], point K will produce a circle that is perpendicular to line AU, and so every point beyond that point on that circle will be situated with respect to a line equivalent to line MN as K is situated with respect to MN.

[7.20] Consequently, if the center of sight lies at any point on the circumference of this circle, and if a line equivalent to line MN lies on the surface of some visible object [that is similarly disposed], the eye will perceive the form of that line [as] larger [than the line itself]. Likewise, moreover, if we extend TK in a straight line and take some point on it other than K [as a center of sight], and if we extrapolate at every stage from that point, which is equivalent to point K, the case will be like the case for point K.

[7.21] On the basis of these two propositions [i.e., 23 and 24], therefore, it is evident that in concave spherical mirrors many objects are perceived [as] larger [than they actually are] in many situations.

[7.22] **[PROPOSITION 25]** To continue, let AB [in figure 6.7.25, p.133] be a concave spherical mirror centered on E, and let us produce a plane passing through E, and let it form [great] circle AB [on the sphere]. Let us extend line EZ randomly from E to G, and from G let us drop GD perpendicular to the plane of circle AB, and let us mark point D on it at random. Then let us connect DE and extend it to O, let us produce EB so that it forms an obtuse angle [DEB] with ED, and let us produce EA so that it forms an angle [AED] with ED equal to angle DEB. Let us then connect DA and DB. Accordingly, the planes of the two triangles DAE and DBE intersect one another along line DE, and the two acute angles DBE and DAE will be equal.

[7.23] Now from B in the plane of triangle DBE let us produce a line [BO] forming an angle [EBO] with EB equal to angle DBE. Hence, this line intersects line DE, since angle BEO is acute, and the angle [EBO] at B is acute. So let it intersect at O.

[7.24] From A let us also produce a line [AO] in the plane of triangle DAE that forms an angle [EAO] with AE equal to angle DAE. So let it intersect DE at O because the two angles AEO and BEO are equal [by construction], and because the angles at the two points A and B [i.e., EAO and EBO] are equal [by construction].

[7.25] Let us then produce ET so that it forms a right angle with EB, and let us extend TE in the direction of E and BO in the direction of O, and let them intersect at H, and [so] TE = EH [insofar as triangles TEB and HEB are equal, by Euclid, I.26]. Let us likewise produce EK so that it forms a right angle with EA, let us extend it in the direction of E, and let us extend AO, and let them intersect at L. Therefore, KE = EL.

[7.26] Let us then connect TK and LH. They will thus be equal [because they lie within triangles KTE and HLE that are equal, insofar as KE = EL, TE = EH, and angle KET = vertical angle HEB]. Hence, if the center of sight lies at D, and if LH lies on some visible object, then D will perceive LH in mirror AB, and T will be the image of H [whose form is reflected from point B], K the image of L [whose form is reflected from point A], and so TK will be the cross-section of the image of LH, and it is equal to it.

[7.27] Consequently, if we rotate the entire figure, leaving HL stationary [as the axis of rotation], D will produce a circle, and if the center of sight lies at any given point on its circumference, it can perceive some visible object equivalent to line LH, and the image will be equal [in size] to it. And likewise, if the center of sight lies at O and TK is the visible object, the image will be the same size as the visible object.

[7.28] But yet, if the visible object is LH, if the eye is at D, and if TK is the image, the image will be inverted; [for] if H lies on the right [of object

HL from D's point of view its image] T will lie on the left [of image TK from that same point of view], whereas if H lies to the left, T will lie to the right, and if H lies above the line, T will lie below the line, and the same for L.

[7.29] Moreover, if the visible object is TK, if the center of sight is at O, and if the image is LH, the form will be erect, for image LH will lie beyond the center of sight, and it will be perceived ahead of the visible object, as was shown in chapter [2] on image [formation] in the fifth book, and the eye will perceive H, which is the image of T, along line BO, and L, which is the image of K, along LO.[99]

[7.30] It is therefore clear that an object is sometimes perceived in concave [spherical] mirrors the same size as it [actually] is.

[7.31] **[PROPOSITION 26]** Now let us continue BH [in figure 6.7.26, p. 133] in a straight line and mark R on it[s extension], and let us connect RE. Angle REB will therefore be obtuse [since it is larger than HEB, which is a right angle, by construction].

[7.32] Let us then extend RE to N. Hence, RB > BN [because BT = HB in isosceles triangle HBT, and BN < HT, while RB > HB]. Moreover, RB:BN = RE:EN [by Euclid, VI.3, since EB bisects angle NBR], so line RE > [line] EN.

[7.33] Let us also extend AL in a straight line [to M], and let AM = BR. Let us connect ME, and let it continue to U. Thus, ME > EU. Then let us connect MR and NU. MR will therefore be longer than NU.

[7.34] Accordingly, if MR lies on some visible object, and if the center of sight is at D, NU will be the cross-section of the image of MR, and NU < MR. On the other hand, if the center of sight is at O, and if NU lies on some visible object, MR will be the image of NU, and it is longer than NU.

[7.35] But if MR is the visible object, and if NU is its image [as viewed from D], then the image will be inverted, whereas if NU is the visible object and MR is its image [as viewed from O], the image will be correctly oriented, for if it lies beyond the center of sight, that image will appear ahead [of the object], and every point on the image will appear along a specific line among the [corresponding] radial lines.[100]

[7.36] **[PROPOSITION 27]** To continue, let us mark point Q on line OH [in figure 6.7.27, p. 134]. Let us connect QE, and let it continue to C. Let OF = OQ, and let us connect EF and let it continue to I. The two lines CE and EI will thus be longer than the two lines EF and QE, and [so] line CI > line FQ [in similar triangles EIC and QEF].

[7.37] Hence, if the center of sight is at O, and if CI lies on some visible object, FQ will be the image of CI, and FQ < CI. Moreover, FQ will appear along the two lines AO and OB. Therefore, the form [of CI] will lie in front of the center of sight and will be smaller than the visible object [itself], and it will be properly oriented.

[7.38] But if the center of sight is at D, and if FQ lies on some visible object, CI will be the image of FQ. It is longer than FQ, and [so its] form will be inverted in front of the center of sight.

[7.39] So it is evident that in concave [spherical] mirrors the form of a visible object is perceived as smaller [than the object itself], larger [than that object], or the same size [as the object].

[7.40] **[PROPOSITION 28]** Now let AB [in figure 6.7.28, p. 135] lie [on] a concave [spherical] mirror [with] G its center, let that mirror be bisected by a plane passing through its center, and let it form [segment] AB [of a great] circle. Let us extend line GD at random, let it pass to E on the side of G, let the center of sight be at E, and let T lie on the surface of the eye.

[7.41] Then let us draw TH perpendicular to line ED, let ZT = TH, and let [the center of sight at] E perceive [the form of] H [by reflection] from A.[101] Consequently, the two points A and H will lie on opposite sides of point G, for if they lay on the same side, the line extending from the mirror to A would not cut the angle that the two radial lines [of incidence and reflection] form.[102]

[7.42] Let us then draw lines EA, AH, GA, and GH, and let GH extend in a straight line to K. Hence, the two angles at A [i.e., HAG and GAE] will be equal [by construction], and K [where cathetus HGK and line of reflection EA intersect] will be the image of H.

[7.43] Let arc BD = arc DA, let us draw lines EB, BZ, and BG, and let us extend ZG to L. Thus, the two angles at B [i.e., ZBG and GBE] will be equal, and [the form of] Z will be perceived by the center of sight [according to reflection] from B, and L will be the image of Z.

[7.44] Let us connect KL. KL will therefore be the cross-section of the image of ZH, and since [object-line] ZTH is perpendicular to DE [by construction], and since ZT = TH [also by construction], the two lines EA and AH will be equal [respectively] to the two lines EB and BZ, the two angles [HAG and GAE] at A are equal to the two angles [ZBG and GBE] at B, and line GH = line ZG.

[7.45] Therefore, the two lines AG and GH are equal [respectively] to the two lines BG and GZ, and base AH [in triangle AGH] = base BZ [in triangle BGZ]. Consequently, angle AHG = angle BZL, and angle HAK = angle ZBL. Hence, HK = ZL, and line HG = [line] ZG, so [remainder] GK [of line HK] = [remainder] GL [of line ZL, from which it follows that the two triangles HGZ and GLK are similar and isosceles]. KL is thus parallel to ZH.

[7.46] Moreover, angle HGA is obtuse, and the two angles [HAG and GAE] at A are equal [by construction], so line GH > line GK, and likewise ZG > GL.[103] Hence, line KL < [line] ZH [because of the similarity of isosceles triangles HGZ and GLK]. But KL is the cross-section of the image of ZH. Therefore, line ZH will appear shorter than it actually is. Moreover, line ZH is [on] the viewer's face [insofar as it is a cross-section of the eye that faces the mirror].

[7.47] Therefore, if we rotate the circle at B [i.e., arc BDA] around ED, leaving EG[D] stationary [as the axis of rotation], it will produce a circle, and it will produce a circle on the mirror's surface from the two points A and B. In addition, the position of center of sight E with respect to any line equivalent to ZH on that circle marked off by ZH and with respect to any arc equivalent to arc AB on the segment [of the circle] that the two points A and B mark off on that circle will be equivalent to the position that center of sight E has with respect to line ZH and arc AB. And the proof will be the same whether we suppose the [object-]line to be longer or shorter than ZH.

[7.48] From all of these conclusions, it is clear that the cross-section of the surface of the viewer's face is perceived [to be] smaller than it [actually] is in the concave [spherical] mirror. So it follows that, if the center of sight lies at E, the viewer will perceive his face in such a mirror as smaller than it is, and since K is the image of H, while L is the image of Z, the image will be inverted.

[7.49] Accordingly, the center of sight at E will perceive the viewer's form as such, i.e., it will perceive what lies to the right to the left, and [what lies] below above, and vice-versa. By the same token, if the center of sight lies at any point such that the center of [the mirror's] curvature lies between it and the mirror's surface, it will perceive its [viewer's] form inverted, and this is what we wanted [to demonstrate].[104]

[7.50] It is therefore evident from these four propositions [i.e., 25-28] that in a concave [spherical] mirror [an object] is sometimes perceived as larger, sometimes smaller, and sometimes the same size [as the object itself], and [it] sometimes [appears] properly oriented, sometimes inverted.

[7.51] Moreover, in chapter [2, book 5] on image [formation], we explained that in a concave [spherical] mirror the image will sometimes be single, sometimes double, sometimes triple, and sometime quadruple, and this same phenomenon occurs in the situations just discussed.

[7.52] Hence, whatever yields an image that is larger than itself may yield others that are smaller or the same size, whereas whatever yields a smaller image may yield others larger or the same size, and whatever yields an image the same size [as itself may yield] a larger or smaller [one], and whatever appears upright [according to one image] may appear inverted according to another image, and vice-versa. So it remains to analyze the forms of those things that are perceived in these sorts of mirrors.

[7.53] **[PROPOSITION 29]** Accordingly, let AB [in figure 6.7.29, p. 136] be a [concave] spherical mirror, let us produce a plane bisecting that mirror through the center, and let it form [great] circle AB centered on E. In this circle let us draw two intersecting diameters, AEO and BED, and let the mirror not extend past arc BADO. Let us then select point Z at random on BE, let us select point K on line AE, and let AK > KE. Then let us connect ZK, and let it continue to F. Let us also draw EF, and let angle EFG = angle EFZ.

[7.54] Thus, since FK > KA, and since KA > KE [by construction], FK > KE.[105] Angle FEK is therefore greater than angle EFK [by Euclid, I.19], so it is greater than angle EFG. Hence, line FG will intersect line KE.[106] Let them intersect at G, then. Consequently, the two lines ZF and FG are reflected at equal angles [ZFE and GFE], so K [where cathetus GEK and line of reflection ZKF intersect] is the image of G if the center of sight is at Z.

[7.55] Now let us draw line ZLH at random, let us connect EH, HG, and ZG, and let us extend FE to M [on GZ]. Accordingly, ZM:MG = ZF:FG [by Euclid, VI.3, since FM bisects angle ZFG]. Furthermore, ZH > ZF [by Euclid, III.7], and GH < GF. Hence, ZH:GH > ZF:FG, so [ZH:GH] > ZM:MG [which = ZF:FG]. Consequently, the line that bisects angle ZHG intersects line MG, so it [also] intersects line EG. Therefore, angle GHE > angle EHZ.

[7.56] Let us take angle EHR = angle EHZ. Line HR therefore intersects line GF, and it [also] intersects line EG, so let it intersect line EG at R. Hence, the two lines ZH and HR are reflected at equal angles [ZHE and RHE], and L [where cathetus REL intersects line of reflection ZLH] will be the image of R [for center of sight Z]. I say, then, that the form of any point on line GR is reflected to the center of sight Z from a point on arc FH, and from no other [arc].

[7.57] The proof of this is [based on] both figures 27 and 28 in chapter [2] on image [formation] in book 5, where it has been shown that the two arcs AB and DO cannot be such that anything on line EO will be reflected from them to [center of sight] Z, and the mirror does not extend to arc BO.[107] Consequently, only arc AD is left [for the reflection].

[7.58] However, in the thirty-fifth proposition [of book 5] it has been shown that the form of any point on diameter EO is reflected at some point on arc AD, and in the thirty-sixth [proposition] of chapter [2, book 5] on image [formation] it was demonstrated that the form of point R is reflected to Z from only one point on arc AD.[108] Therefore, the form of any point on line GR is reflected to Z from one point only on arc AD.

[7.59] Let us take point C on line GR [in figure 6.7.29a, p. 136]. The form of C is therefore reflected to Z from one point on arc AD. I say, then, that that point will lie only on arc FH. For if such is not the case, let the form of C be reflected to Z from U, which lies on arc AF, and let us connect lines ZU, CU, GU, and EU.

[7.60] Therefore, line GU > line GF, and ZU < ZF, so GU:ZU > GF:FZ. Hence, [GU:ZU] > GM:MZ [which = GF:FZ, by previous conclusions]. The line that bisects angle GUZ therefore intersects line ZM, so it [also] intersects ZE. Consequently, angle GUE < angle EUZ; so *a fortiore* angle CUE < angle EUZ, and the same holds for any [other] point on arc AU[F]. The form of C is therefore reflected to Z from arc [D]HF only.

[7.61] I say, furthermore, that it cannot be reflected [to Z] from arc HD. For if that were possible, let it be reflected from Q, which lies on arc HD [in figure 6.7.29b, p. 137], and let us connect lines ZQ, CQ, RQ, ZR, and EQ, and let us extend EH to N. Therefore, line ZQ > [line] ZH, and line QR < [line] HR, so ZQ:QR > ZH:HR, which = ZN:NR [by Euclid, VI.3, since angles ZHE and RHE were constructed equal]. The line that bisects angle ZQR therefore intersects line NR, so it intersects line ER. Consequently angle RQE > angle EQZ, so *a fortiore* angle EQC > angle EQZ. The same result follows for any [other] point on arc HD, so the form of C is not reflected to Z from arc HD or from arc AF.

[7.62] However, it has already been shown that it absolutely must be reflected from arc AD. Consequently, the form of C is only reflected to Z from some point on arc FH. Accordingly, let it be reflected from T [in figure 6.7.29c, p. 137], and let us connect lines CT, ET, and ZT. Therefore, since T lies between the two points F and H, line ZT will lie between the two lines Z[K]F and Z[L]H. Line ZT therefore intersects line KL. Let it intersect it at I, then. I is therefore the image of C, and C has no image other than I.

[7.63] And it will be demonstrated in this way that the image of any point on line GR will be a point on line KL. Thus, KL is the image of [the entire line] GR, and KL is a straight line because it is a segment of the circle's diameter. GR is also a straight line because it too is a segment of the circle's diameter. Thus, in [concave] spherical mirror AB, Z perceives the form of GR according to its proper [left-to-right] orientation, and this is what we wanted [to prove].

[7.64] **[PROPOSITION 30]** Now let us copy the [previous] figure, and let us circumscribe two random arcs on both sides of line GR, namely, GNR and GQR [in figure 6.7.30, p. 138], and let arc GNR not intersect line GH. Let us select point M at random on line GR. The form of M is therefore reflected to Z from [some] point on arc FH. Let it be reflected accordingly from T, and let us connect lines ZT and MT.

[7.65] Hence, the two angles ZTE and ETM are equal, so line MT will intersect arc GNR. Let it intersect that arc at N, then, and let us extend line TM on the side of M. It will therefore intersect arc GQR, so let it intersect at point Q. Next let us connect NE and extend it in a straight line. It will therefore intersect ZT below line KL. So let it intersect that [line] at I. Then let us connect QE and extend it in a straight line. Accordingly, it will intersect ZT above KL. Let it intersect that line at C, then.

[7.66] Consequently, since the two angles [ZITE and NTE] at T are equal, I will be the image of N, and the two points K and L are the images of the two points G and R. The image of arc GNR is therefore a line passing through points K, I, and L, i.e., line KIL. But line KIL is convex with respect to the eye, and arc GNR is convex with respect to the mirror. Z will therefore perceive the form of convex line GNR [as] a convex line.

[7.67] Moreover, since the two angles [ZCTE and QTE] at T are equal, C will also be the image of Q, and line LCK, which is concave with respect to the eye, will be the image of arc GQR, which is concave with respect to the mirror's surface. Z will therefore perceive the form of concave arc GQR [as] a concave line.

[7.68] Thus, in concave [spherical] mirrors a convex line is perceived [as] convex, and a concave [line as] concave in various situations.

[7.69] **[PROPOSITION 31]** Now let there be a concave [spherical] mirror containing great circle ABD [figure 6.7.31, p. 138], let G be the center, let us draw line BG at random, and let us cut from it line GT longer than its half. Let us then draw line ETZ from T orthogonal [to BG], and let both ET and TZ be equal to TG. Let us connect ET, EG, and GZ.

[7.70] Let us then circumscribe a circle around triangle EGZ. It will therefore intersect circle AB at two points, for point T is the center of this [new] circle, and TG > TB [by construction]. So let this circle intersect circle AB at the two points A and D, and let us connect lines GA, GD, EA, EB, ED, ZA, ZB, and ZD.

[7.71] Accordingly, since the two lines ET and TZ are equal [because they are both equal to TG, by construction], the two lines EB and BZ will be reflected at equal angles [i.e., EBG and GBZ subtended by equal arcs]. Also, since the two arcs EG and GZ are equal, the two lines EA and AZ are reflected at equal angles [EAG and GAZ subtended by those equal arcs], and the two lines ED and DZ will [also] be reflected at equal angles [EDG and GDZ subtended by equal arcs].

[7.72] Since GT > TB [by construction], GE > EB. Hence, angle EBG > angle EGB, and angle EGB is half a right angle. The two angles EGB and EBG thus sum up to more than a right angle. Consequently, angle BEG < a right angle, whereas angle EGZ is a right angle. The two lines EB and GZ will therefore intersect outside the circle [EGZ] on the side of BZ. So let them intersect at M.

[7.73] In addition, since ED lies within angle MEG, it will intersect line GM. Let them intersect at L. And since GB passes through the center of circle EGZ, segment AEG < a semicircle. Therefore, angle AEG is obtuse, whereas angle EGZ is right. The two lines AE and ZG will thus intersect on the side of EG. Let them intersect at F, then. Accordingly, if the center of sight is at E, and if Z lies on some visible object, points M, L, and F will be images of Z. Z is thus perceived at three places.

[7.74] To continue, let us draw a line at random from E to arc DZ, and let it be EK [see inset to figure 6.7.31]. Let us connect GK, let it intersect arc DZ at K, and let us connect lines KZ and GK. Therefore, since arcs EG and GZ are equal, the two angles EKG and GKZ will be equal. [Let us extend GK to point K′ on the mirror, and let us connect EK′ and ZK′]. Consequently, angle EK′G > angle GK′Z.[109] Let angle GK′N = angle EK′G, then. Consequently, the two lines EK′ and K′N will be reflected at equal angles. Let us then extend EK′ to Q. Q will therefore be the image of N with respect to [center of sight] E.

[7.75] Let us now imagine a plane passing along line MGF perpendicular to circle ABD [as represented in figure 6.7.31a, p. 139], let us draw a line from Z in this plane perpendicular to GZ, and let it extend on both sides [of Z]. Accordingly, let it be CZR. Let us then take G as a centerpoint, and let us produce arc CNR of a circle with radius GN. It will therefore intersect line CR at two points, and let them be C and R. Let us connect lines GC and GR. They will thus lie in a plane perpendicular to plane ABG [of the mirror]. Let us then extend GC and GR in a straight line, and at point G let us produce the arc of a circle with radius GQ. It will thus intersect the two lines GC and GR. Let it intersect [them] at S and O.

[7.76] Hence, since the plane of circle ABD [on the mirror] is perpendicular to the plane of the two lines GC and GR, the two angles EGS and EGO will be right. Both planes EGS and EGO will thus be perpendicular to plane SGO, and both of those planes cut a great circle on the mirror that is equivalent to circle ABD. From the counterpart of point K′ in the [great] circle formed by plane EGC, then, two lines between points E and C are reflected at equal angles.[110]

[7.77] Moreover, lines GC and GR are equal, lines GS, GQ, and GO are equal, and Q is the image of N, S is the image of C, and O is the image of R. Therefore, the image of arc CNR, which is convex with respect to the mirror, is arc SQO, which is concave with respect to the center of sight.

[7.78] Meantime, L is the image of Z, and the two points S and O are the images of C and R. Consequently, the image of straight line CZR is a line passing through points S, L, and O, and such a line is concave with respect to the center of sight.

[7.79] Let us now draw the line passing through points S, L, and O, and let us extend line EG to H. Accordingly, if the mirror does not reach the two points B and H, but one of its two limits lies between the two points B and D, while the other lies inside of H [i.e., between H and D], and if the center of sight lies at E, while the two lines RZC and RNC lie on some visible object, the form of straight line RZC will be concave, i.e., SLO, and the form of convex arc RNC will also be a concave line, i.e., SQO. Furthermore, straight line RCZ will have a single image, and arc RNC will [also] have a single image.

[7.80] Now let us extend BG to I, and let us connect lines EI and IZ. Those two lines will therefore be reflected at equal angles, and EI will intersect FG, so let it intersect at T'. Hence, T' will be the image of Z. Points M, L, T', and F will therefore [all] be images of Z. And if the mirror extends beyond the two points A and I, while the center of sight lies at E, and if the viewer faces the mirror on the side of arc AI, he will perceive the entire arc IDA.

[7.81] Consequently, Z will appear at four places, i.e., at L, M, T', and F, and he will see the two points R and C at the two points S and O, so line RZC will have four concave images. One passes through points S, M, and O, i.e., line SMO; a second will pass through points S, L, and O, i.e., line SLO; a third will pass through points S, T', and O, i.e., line ST'O; and a fourth will pass through points S, F, and O, i.e., line SFO.

[7.82] From this proposition it is therefore clear that in concave [spherical] mirrors a straight line is perceived as concave, a convex [one] is also perceived as concave, and a straight [line] has several concave forms.[111]

[7.83] **[PROPOSITION 32]** Now let there be a concave [spherical] mirror through whose center a plane passes, let it produce [great] circle ABG [in figure 6.7.32, p. 140], and let D be its center. From D let us draw a line at random, let it be DG, and let it extend beyond the circle. From point D let us extend a line in the plane of the circle perpendicular to line DG, and let it be DA. Let us then cut from right angle ADG a small sub-angle GDE at random such that the difference between right angle [ADG] and angle ADE consists of several [increments of] angle EDG,[112] and let us bisect angle ADE with line DB. Let us also cut [from angle ADG] a sub-angle [ADZ] equal to angle EDG, and let us extend a line from D that forms a right angle with DB [i.e., BDT], and let it be DT.

[7.84] Now let us extend AD on the side of D, let it form DK, and let us extend a line [ZH] from Z that forms with ZD an angle [DZH] equal to angle KDT. This line will therefore intersect DA because the two angles KDT [which = DZH, by construction] and ADZ sum up to less than two right angles. So let them intersect at H. Angle ZHD is thus equal to angle ZDT.[113]

[7.85] Then from Z let us extend line ZL to form an angle [HZL] with ZH equal to obtuse angle BDK. Accordingly, the two angles LZD and BDZ sum up to less than two right angles, so line ZL will intersect [line] DB.[114] Let it therefore intersect at L.

[7.86] Let us then connect LH, and let us form circle DHL around triangle HLD. It will therefore pass through Z because the two angles LZH and LDH sum up to two right [angles, by Euclid, III.22]. Consequently angles LHZ and LDZ are equal because they are subtended by the same arc [LZ]. But angle ZHD = angle ZDT [by previous conclusions], so it follows that angle LHD = angle LDT. But angle LDT is right [by construction], so angle LHD is right.

[7.87] Now from line DE let us cut line DM equal to [line] DH, and let us connect LM. Angle LMD is therefore right, so circle LHD passes through M and cuts arc HE at a point equivalent to Z.[115] Accordingly, let it intersect at F, and let us draw DF. Angle LDF will therefore be equal to angle LDZ because arc LM = arc LH [by construction according to the bisection of angle ADE by DB], and arc MF = arc ZH [so arc FL subtending angle FDL = arc LZ subtending angle LDZ]. Consequently, arc FMD = arc ZHD.

[7.88] Let us now draw lines HB, HF, FM, FZ, and FB. Angle BHD will thus be acute, while angle GDH will be right [by construction]. Therefore, line HB will intersect line DG outside the circle. Let them intersect at Q, then. HF will therefore also intersect DG outside the circle, so let them intersect at N.

[7.89] Let us then extend FB until it intersects arc LZ. Accordingly, let it intersect at R, and let us connect RM. Angle FRM, which lies on the circumference [of circle ZDF], is thus subtended by arc FM, and angle FBM > angle FRM, but angle FBM lies on the circumference of [circle] ABG. Therefore, if line BM is extended on the side of M, it will cut a larger arc on circle ABG than the counterpart FM [it cuts on circle ZDF].

[7.90] But arc FM [in circle ZDF] is twice its counterpart FE [in circle ABG].[116] Moreover, arc FE = arc ZA [because arcs FM and ZH are equal], whereas arc ZA = arc EG [by construction], and [so] arc FE = arc EG. Consequently, arc GF is twice arc GE, so arc GF [in circle ABG] is the equivalent [in degrees of arc] of arc FM [in circle FDZ].

[7.91] Hence, if BM is extended in a straight line on the side of M, it will cut an arc on circle ABG beyond point F that is greater than arc FG. Line BM will therefore intersect line DG between the two points G and D. Accordingly, let it intersect at O. Let us then extend line FM, and let it intersect DO at U; let us also extend BM on the side of B, and let it intersect arc LR at C. Let us then connect CD.

[7.92] Accordingly, because angle BFZ lies on the circumference of [circle] ABG, angle BFZ will be half of angle BDZ [by Euclid, VI.33]. But angle BDZ is several times larger than angle ZDA [by construction], so angle R[B]FZ is several times larger than angle ZDH[A]. Consequently, arc RZ is several times the size of arc ZH, and arc CZ > arc RZ, so arc CZ is several times the size of arc ZH.

[7.93] Let us now connect CH. Accordingly, angle CHD + angle C[B]MD = two right angles [by Euclid, III.22], so angle CHD = angle BME [adjacent to BMD]. Therefore, angle ZHD = angle CHD + angle CHZ, which = angle CDZ [since they are both subtended by arc CZ in circle ZDF], and angle CDZ is several times larger than angle ZDA. Therefore, angle CHZ is several times larger than angle EDG [which = angle ZDA, by construction], so angle ZHD exceeds angle CHD by several [increments of] angle EDG. Hence, angle ZHD = angle FMD because arc FMD = arc ZHD [by previous conclusions].

[7.94] Moreover, as we demonstrated [just above], angle CHD = angle BME. Angle FMD thus exceeds angle BME by several [increments of] angle EDG. Therefore, angle FMD exceeds angle OMD by several [increments of] angle EDG [because angle OMD = vertical angle BME]. But angle MOG exceeds angle OMD by [one increment of] angle EDG [by Euclid, I.32], so angle FMD exceeds angle MOG by several [increments of] angle EDG.

[7.95] In addition, angle FMD exceeds angle MUD by only [one increment of] angle EDG [by Euclid, I.32]. Therefore, angle MUD > angle MOG, so angle MOU [adjacent to MOG] > angle MUO [adjacent to MUD]. Hence, line MU > line MO [since MU subtends the larger angle]. And since arc ZHD = arc FMD [by previous conclusions], the two angles HFD and MFD will be equal. The two lines HF and FU will therefore reflect at equal [angles], and likewise HB and BO will reflect at equal [angles]. Consequently, Q is the image of O, and N is the image of U [from the perspective of H, as the center of sight].

[7.96] From M let us then extend a line parallel to line HQ, let it be MS, and let us also extend a line from M parallel to line HN, and let it be MP. Thus, since angle HND > angle HQD, angle MPO > angle MSO. P therefore lies between the two points S and U, and because angle HDN is right [by construction], angle HND will be acute. Accordingly, angle MPD is acute, so angle MPS [adjacent to it] is obtuse. Line MS is therefore longer than [line] MP.

[7.97] But MU > MO, as we [just] established, so SM:MO > PM:MU, and SM:MO = QB:BO because MS is parallel to BQ. Likewise, PM:MU =

NF:FU, so QB:BO > NF:FU. Moreover, QB:BO = QD:DO, whereas NF:FU = ND:DU, as we showed in the twenty-fifth proposition of chapter [2, book 5] on image [formation].[117] Therefore, QD:DO > ND:DU.

[7.98] Now that these points have been established, let us redraw the circle and finish the proof so as to avoid adding lines and confusing letter-designations. Accordingly, let ABG [in figure 6.7.32b, p. 142] be the circle in the second version [of figure 6.7.32, p. 140], let D be its center, and let us draw line DQ. Let DU [in the second version] be equivalent to DU in the original version, and let DO [in the second version] be equivalent to DO in the original version. Also, let DQ [in the second version] be equivalent to its counterpart in the original version, and likewise for DN.

[7.99] Let us then draw DH' perpendicular to DQ [as well as] to the plane of the circle, and let DH' be equivalent to its counterpart [DH] in the original version. Angle H'DQ will therefore be right, and the [great] circle that [the plane containing] H'DQ produces in the mirror will be among the circles [like ABG] within which a form is reflected. Furthermore, the arc that lines H'D and DQ measure off [in the great circle produced on the mirror by plane H'DQ] will be equal to arc AG in the original circle. And from the two points on this [arc] that are equivalent to the two points B and F [on arc AB in the original circle] lines from the two points U and O will be reflected at equal angles [to point H']. Q will therefore be the image of O [for center of sight H'], and N [will be] the image of U.[118]

[7.100] From U let us produce a line perpendicular to line DU within the plane of circle ABG, and let it be ZUE. Let D be the center, and [from it] let us produce the arc of a circle with a radius of DO. It will therefore intersect line ZUE at two points. Accordingly, let it intersect at Z and E, and let it form arc ZOE. Let us then draw DZ and DE, and extend them beyond the circle. At a radius of DQ, let us produce arc TQK around D. It will therefore intersect the [extension of the] two lines DZ and DE at T and K. Let us then draw TK. Accordingly, it will intersect line DQ at L.

[7.101] Consequently, since H'D is perpendicular to the plane of the circle, both angles H'DT and H'DK will be right. Moreover, both planes H'DT and H'DK produce a [great] circle on the mirror's surface, and the arc on it that lies between the two lines H'D and DT will be equal to the arc lying between HD and DQ [in the original figure—i.e., 6.7.32], and the same holds for the arc between the two lines H'D and DK. In addition, both lines DZ and DE are equal to line DO. These two arcs [cut on the mirror's surface by planes H'DZ and H'DE] are therefore of the kind from which lines will be reflected at equal angles to the two points Z and E.[119] Furthermore, the two lines DT and DK are equal to line DQ, so point T is the image of Z, and [point] K is the image of E.

[7.102] Since, moreover, lines DT, DQ, and DK are equal, and since lines DZ, DO, and DE are equal, DT:DZ = QD:DO = KD:DE. But, as we showed in the previous theorem [i.e., in paragraph 7.97 keyed to figure 6.7.32], QD: DO > ND:DU. Therefore, DT:DZ > ND:DU, and the same holds for KD: DE.

[7.103] In addition, since the two lines ZD and DE are equal, and since the two lines DT and DK are equal, line TK will be parallel to line ZE. Therefore, DT:DZ and KD:DE will [both] be as LD:DU. Hence, LD:DU > ND:DU, so line LD > line ND. N therefore lies between L and U. But N is the image of U, and the two points T and K are the images of Z and E. As a result, the image of straight line ZUE is the line that passes through points T, N, and K. But the line that passes through these points is convex, from which it is clear that in concave [spherical] mirrors a straight line sometimes appears convex in certain situations.

[7.104] Now let us take some point M at random on line ZU [in figure 6.7.32d, p. 143], and around M as center let us produce arc RUF with radius MU. This arc will therefore intersect arc ZOE[120] in two points. Accordingly, let it intersect at R and F, let us draw lines DR and DF, and let them pass in a straight line until they intersect arc TQK at C and I. The plane containing the two lines H'D and DC will therefore produce a [great] circle on the mirror from whose circumference lines from R will be reflected at equal angles, and by the same token the plane containing the two lines H'D and DI will produce a [great] circle on the mirror from whose circumference lines will be reflected to F [at equal angles]. C is therefore the image of R, I is the image of F, and N is the image of U.[121]

[7.105] Consequently, the image of arc RUF is the line passing through C, N, and I, but this line will be convex, whereas arc RUF is concave with respect to the mirror's surface. Therefore, when the center of sight is at H', and when any of the lines ZUE, ZOE, or RUF lies on some visible object, straight line ZUE will be perceived [as] convex, convex line ZOE will be perceived [as] concave, and concave [line RUF is perceived as] convex. Consequently, if each of the lines ZUE, ZOE, and RUF has [only] one image, the form of those lines will be just as we showed. But if they have additional images, they may be similar to the other images [i.e., the ones just discussed], or they may be different.

[7.106] From these propositions [i.e., 29-32] it is therefore evident that straight lines are sometimes perceived [as] straight in concave [spherical] mirrors, sometimes [as] convex, and sometimes [as] concave. In addition convex lines are sometimes perceived [as] convex [and] sometimes [as] concave, and concave [lines] are sometimes perceived [as] convex [and] sometimes [as] concave.

[7.107] So the forms of visible surfaces are perceived [as] other than they [actually] are in these sorts of mirrors, for straight lines exist only in flat surfaces, and when a straight line that exists in a plane surface is perceived [as] convex or concave, the surface in which it lies will be perceived [as] convex or concave. Accordingly, when the eye perceives convex, concave, and straight lines other than they [actually] are, it will perceive the surfaces in which they lie other than they are.

[7.108] From the foregoing, then, it is clear that in everything that is perceived in concave [spherical] mirrors, a misperception occurs, but in certain cases it occurs in every situation, without exception, whereas in certain [cases] it occurs in a specific situation. Moreover, compound misperceptions arise in these mirrors just as in the case of compound illusions [in the other mirrors],[122] and this [is what] what we wanted to demonstrate.

CHAPTER EIGHT
Concerning Misperceptions That Arise in Concave Cylindrical Mirrors

[8.1] In these [sorts of mirrors] the same things happen as happen in concave spherical [mirrors], for the misperceptions that arise from reflection [by itself] occur [in these mirrors], i.e., the weakening of light and color and the variation in situation and distance that occur in all mirrors. Moreover, variation in size occurs in these mirrors in the same way as it happens in concave spherical mirrors. Also, one visible object appears [as] one, or [as] two, or [as] three, or [as] four, and [it appears] properly oriented or reversed in various circumstances, and a flat object appears concave or convex. So let us show how the size and number of a visible object may vary in these mirrors, as well as how it appears properly oriented or reversed in the way that we demonstrated [these phenomena] in spherical concave mirrors.

[8.2] **[PROPOSITION 33]** Let us recapitulate the first of the two diagrams provided in [the analysis of] misperceptions [occurring in] convex cylindrical mirrors [i.e., figure 6.5.18, p. 127, redrawn as figure 6.8.33, p. 144], and [let us use] the same letters. Now in that proposition [i.e., proposition 18, in conjunction with proposition 17, pp. 190-193 above] it was shown: that lines EG and GT, EB and QB, and EA and AH are reflected at equal angles; that lines EO, HA, BQ, and TG intersect at O;[123] that line ABG is a straight line extended along the longitude of the mirror; that lines GZ, BL, and AD are perpendicular to the plane tangent to the [mirror's] surface and passing along line ABG; that line ABG is perpendicular to the plane containing triangle EBO; that line TQ = [line] QH, and [line] AB = [line] BG; that S, C, and I are the images of H, Q, and T; that C lies nearer point E than [straight] line SI; that [straight] line SI lies in the plane of triangle UHT; that the two lines UH and UT are equal; that the two lines US and UI are equal; and that the two lines ES and EI are equal.

[8.3] Let us draw CU, and let it intersect SI at F. It will therefore bisect this line [i.e., SI] because HT is bisected at Q [by construction], and CU will lie in the plane of triangle CUE, which lies in the plane of the circle [passing through] B parallel to the base of the mirror.[124] Q will therefore lie in the plane of triangle CUE, and C lies in triangle CEI. Hence, C lies on the common section of these two planes. But this [common] section is line EB; so C lies on a straight line [with] EB.[125]

[8.4] Moreover, the two lines HU and TU [in figure 6.8.33] lie outside the two points D and Z [on the axis], for the two lines HU and TU are the normals passing from H and T to the two lines that are tangent to two sections [on the cylinder's surface] on whose periphery points A and G lie [i.e., the two elliptical sections formed on the mirror's surfaces by planes of reflection TGE and HAE]. Consequently, the plane of triangle UHT lies outside axis DLZ.

[8.5] Even if the axis is extended to infinity, however, no point on it will lie in the plane of triangle UHT, for if it did, then if it were to be connected in a straight line with some point on line HT, the plane in which that straight line and line HT lay would be the plane of triangle UHT, and that plane would be the one in which the two parallel lines HT and DZ lie. Hence, the plane containing the two lines HT and DZ [supposedly] forms the plane of triangle HUT, so the axis will lie in the plane of triangle HUT.

[8.6] But the axis is parallel to line HT by construction, and the axis [supposedly] intersects the two lines HU and TU. Moreover, line TH lies in [i.e., passes through] the plane of triangle UEH, which is the plane of reflection [for object-point H and reflection-point A], and the common section of this plane and the surface of the mirror is some [elliptical] section. Therefore, plane EUH intersects the axis of the cylinder in one point, that is, D, as we showed before [in proposition 18]. And if the axis intersects line HU, the point of intersection with line HU will lie in the plane of triangle UEH. But there is no point in this plane other than D through which the axis might pass. Therefore, line HU intersects the axis at D. But it has already been shown that HU intersects it at a point beyond D, which is impossible.[126]

[8.7] Consequently, axis DZ lies outside the plane of UHT and nearer to point E than plane UHT [i.e., between E and plane UHT]. The plane in which lines HT and DZ lie is therefore nearer to point E than plane UHT.

Moreover, C lies in the same plane as HT and DZ because it lies on line QL, and QL lies in the same plane as HT and DZ. Therefore, C lies nearer to point E than [do] S and I. But C lies in a straight line with EB. If, therefore, EB is extended toward B, it will reach C, so let it reach C.

[8.8] Now that these points are established, I say that if line SI, which is parallel to the mirror's axis, lies on some visible object, if the center of sight lies at O on the concave side of the cylinder, and if the reflecting surface is a concave surface, then SI will be perceived by O in concave mirror ABG, and its images will vary according to how its distance from the axis varies.

[8.9] The proof of this [claim] lies in the fact that angle EBM is acute, so [vertical] angle LBC is acute. Moreover, line EBC lies in the plane of the circle [passing through] B, and LB is [on] the diameter of this circle. Hence, EB intersects the circle, so CB lies inside the mirror's concavity.

[8.10] By the same token, OB lies inside the mirror's concavity because angle OBL is acute, and the two angles OBL and CBL are equal, since they are equal to the two angles EBM and QBM, while LB is perpendicular to the plane that passes through B tangent to the cylinder. The form of C thus passes along CB and reaches B, and it is reflected along BO and perceived by the center of sight at O.

[8.11] Furthermore, when we discussed convex cylindrical mirrors in chapter 5 [of book 4, paragraph 5.18, in Smith, *Alhacen on the Principles*, 332], we showed that the plane tangent to the cylinder at G will lie below E. Therefore, EG intersects the tangent plane, so it intersects the line tangent to G on the periphery of the [elliptical] section [formed on the mirror by the plane of reflection]. As a result, it intersects the [elliptical] section and falls inside it; so it will fall inside the concavity of the mirror. The two lines OG and GI thus lie inside the concavity of the mirror, whereas ZG is perpendicular to the plane tangent to the cylinder at G, and the two angles OGZ and IGZ are equal. Hence, the form of I passes along IG, reaches G, is reflected along GO, and is perceived at O along line GO. So too, [the form of] S passes along SA and is reflected along AO.

[8.12] But when we dealt with misperceptions [arising] from convex cylindrical mirrors, we demonstrated [in proposition 16, lemma 5] that the two lines HU and TU are normal to the two planes tangent to the [elliptical] sections passing through the two points A and G. Therefore, the image of S lies on line HU. Moreover, OA is the radial line extending from the center of sight to the point of reflection, so the image of S lies on OA. H [where the two lines intersect] is therefore the image of S, and it is shown in this way that T is the image of I.

[8.13] Let us then connect CL. Accordingly, since [the form of] C is reflected to O from the periphery [of the circle passing through] B, [its] image Q will lie on line CL. And OB is the radial line extending between the center of sight and the point of reflection, so the image of C lies on line OB. Consequently, the image of C lies at the intersection of [cathetus] QL and [line of reflection] OB [i.e., at Q].

[8.14] When we dealt with images in concave spherical mirrors in chapter [2, book 5] on image [formation], however, it was shown [in proposition 32, in Smith, *Alhacen on the Principles*, 446-448] that the image of a point whose form is reflected from the concavity of a [great] circle [on the mirror's surface] may intersect the radial line linking the center of sight and the point of reflection beyond the circle, or between the center of sight and the circle, or at the center of sight [itself], or behind the center of sight, or CL may be parallel to OB.

[8.15] In that [same] chapter [on image formation], moreover, it was shown that the image might consist of a single point, or of two, or of three, or of four.[127] So the image of C might lie [at some point between B and Q] on BQ, or perhaps beyond Q, or perhaps on BO, or perhaps at O, or perhaps behind O.[128] Moreover, the image of C might consist of a single point, or of two, or of three, or [of] four.

[8.16] Accordingly, if the image of C lies at Q, then HT will be the cross-section of SI's image. So, if all the images of [points on] SI lie on line HT, its form will be a straight line. If not, however, it will be nearly straight because its midpoint lies on a straight line between two endpoints.[129] Nevertheless, if the image of C lies beyond Q, the image of SI will be somewhat concave with respect to the center of sight. And if the image of the visible [point C] lies on line BO [i.e., in front of Q], then the image of SI will be convex with respect to the center of sight.

[8.17] Moreover, if the image of C consists of several points, the image of C will lie on several lines, all of whose endpoints converge at the two points H and T, and their midpoints are distinct and separate. In addition, HT forms the cross-section of image SI, no matter how that image is formed, and [this] cross-section is common to all of its images if it has several images, and line HT [on the image] is longer than [line] SI [on the object] by some amount.[130]

[8.18] It is therefore evident that, when straight lines parallel to the axis of a concave cylindrical mirror lie on some visible object, the image of any [one of them] may be straight or concave, and it may consist of a single [line] or [of] several.

[8.19] **[PROPOSITION 34]** Now let us recapitulate the second diagram [provided in the analysis] of misperceptions in convex cylindrical mirrors [i.e., figure 6.5.19, p. 128, which accompanies proposition 19]. In this proposition [represented by figure 6.8.34, p. 147, abstracted from figure 6.5.19], it has been shown: that the two lines EB and BH are reflected at equal angles; that the two lines EG and GT are reflected at equal angles; that HB and TG converge at L; and [that] HB forms an acute angle with BO. Consequently, HB intersects the plane tangent to the surface of the cylinder at B, so BL lies inside the concavity of the cylinder. And the same holds for GL, as well as for the two lines BR and GY.

[8.20] Moreover, the two angles LBD and DBR are equal, and the two angles LGD and DGY are equal. Hence, if RY lies on some visible object, if the center of sight lies at L, and if the concave surface of the cylinder is polished [and therefore reflective], the form of R is extended along RB and reaches B. It will then be reflected along BL and will reach L, and it will be perceived by L. Moreover, line HU [i.e., the cathetus dropped from R through the mirror's surface] is perpendicular to a line tangent to the [elliptical] section from whose periphery the two lines RB and BL will be reflected. Therefore, H is the image of R. Likewise, it will be proven that the form of Y is extended along YG and is reflected along GL, and its image is T.

[8.21] Let us now draw KU [in figure 6.8.34a, p. 148].[131] Accordingly, it will intersect RY at M. M therefore lies in the plane passing through the axis and through L, for L and K lie in that plane, so KU lies in that plane. Moreover, since the two points M and L lie in a plane passing through the cylinder's axis, the form of M will be reflected to L within that plane. Line AZ is the common section of the cylinder's surface and the plane passing through its axis and through L, so the form of M will be reflected to L from [a point on] AZ.

[8.22] Let us then connect EM, which lies in this plane. EL also lies in this plane, and E lies above the plane tangent to the surface of the cylinder along line AZ. Hence, if AZ is extended in a straight line on the side of Z, it will intersect the two lines EM and EL. Accordingly, let it intersect EM at I and EL at N. N therefore lies between the two points E and L because L lies inside the concavity of the cylinder, whereas N lies on the cylinder's surface, and E lies above the [surface of the] cylinder.

[8.23] Furthermore, in the proof based on this diagram, it was shown that circle BZG lies halfway between [the plane passing through] line HT [parallel to the base of the cylinder] and the plane passing through E parallel to the base of the cylinder. In addition, the perpendicular [EX'] that passes from E through AZ lies in the plane passing through E parallel to the cylinder's base. Therefore, the perpendicular passing through E to line AZN falls outside triangle EIN and on the side of N. Consequently, angle EIN is acute; so [vertical] angle MIA is [also] acute.[132]

[8.24] So let us extend MQ [in figure 6.8.34b, p. 149] from M perpendicular to AI. Q will therefore lie beyond I with respect to N [i.e., below I on AZA']. And let us extend MQ on the side of Q, and let us cut off QS equal to QM. S will therefore lie beyond the surface of the mirror and outside its concavity, while L will lie inside its concavity.

[8.25] Let us then draw LS. Accordingly, it will intersect NQ at F, and from F let us extend FX parallel to QM. It is therefore perpendicular to AN and [lies] in the plane passing through the axis and through L, so it is a diameter of the circle [produced on the mirror's surface by the plane] passing through F parallel to the cylinder's base. Therefore, line XF is perpendicular to the plane tangent to the cylinder and passing along AZ.

[8.26] Let us connect MF. Accordingly, it will be equal to FS, and [so] the two angles [FMQ and FSQ] at M and S will be equal [because triangles FMQ and FSQ are equal, by Euclid, I.4]. Moreover, because XF is parallel to MS, the two angles [LFX and MFX] at F will be equal to the two angles [FSQ and FMQ] that are at S and M.[133] The two lines MF and FL are therefore reflected at equal angles, and XF is perpendicular to the plane tangent to the mirror's surface at F. So the form of M is extended along MF and is reflected along FL, and its image will be S.

[8.27] Moreover, since the two lines RY and HT are parallel and [therefore] perpendicular to the plane passing through the axis and through L (because HT was posited as such [in proposition 19]), the two planes passing through the two lines HT and RY [perpendicular to the axis] will be parallel. Since, moreover, RY is perpendicular to the plane passing through the axis and through L, the plane [consisting] of the two lines RM and MS will be perpendicular to the plane passing through the axis and through L. Furthermore, MS will be the common section of these two planes [i.e., RMS and ELDS in figure 6.8.34b], and since AQ lies in the plane [ELDS] passing through the axis and is perpendicular to MS, which is the common section of the plane [ELDS] passing through the axis and the plane [consisting] of the two lines RM and MS, AN will be perpendicular to the plane [consisting] of the two lines RM and MS.

[8.28] But line AN is parallel to the axis of the cylinder, so the axis of the cylinder is perpendicular to the plane containing the two lines RM and MS. Therefore, this plane is perpendicular to the axis of the cylinder. S therefore lies in the plane passing through line RY perpendicular to the axis of the cylinder.

[8.29] But line HT lies in a plane perpendicular to the axis of the cylinder and parallel to the plane passing through line RY. Hence, S lies outside [and above] HT and nearer to L than HT. In addition, the two points H and T are the images of R and Y, and point S is the image of M, so the image of line RMY is the line passing through H, S, and T.

[8.30] Such a line is curved, however, because S lies outside HT, and a curved line HST must pass through points H, S, and T. And since HT lay beyond the convex [side of the] cylinder, according to construction, HT will lie beyond the surface of the mirror with respect to L. In addition, we have already shown that S lies beyond the concavity of the mirror with respect to L, so the entire line HST lies beyond the concavity of the mirror's surface. Moreover, L lies inside the concavity of the mirror, so L lies outside the plane containing line HST. Therefore, the curvature of line HST will appear clearly to the eye at L.

[8.31] Furthermore, since F lies on the surface of the cylinder, while TH lies beyond the cylinder, and since TH lies in the plane of triangle LHT, line LFS will be higher than the plane of triangle LHT. Line LS will therefore be higher than the two lines LH and HT with respect to the center of sight at L. Consequently, S is higher than the two points H and T, so line HST will appear concave to the center of sight at L.

[8.32] **[PROPOSITION 35]** To continue, let us cut the cylinder with a plane slanted to its axis, but do not let it pass through the entire axis [so as to cut the cylinder along a line of longitude]. Accordingly, it will form an [elliptical] section. Let it therefore be ABG [in figure 6.8.35, p. 150].[134] But in the first of the propositions concerning concave cylindrical [mirrors] it was demonstrated that in the plane of any [elliptical] section on a cylinder there will be a normal to the plane tangent to the cylinder from whose endpoints forms are reflected.[135] So let that normal be GZ[A], let BE[K] be perpendicular to the line tangent to the periphery of the [elliptical] section at B, and let B lie near G. BK will therefore intersect normal GZ, and it will form an acute angle with it. Accordingly, let it intersect at E. Angle BEG will therefore be acute [by proposition 16, lemma 5].

[8.33] From G let us extend line GD parallel to line BK. Hence, angle DGE will be acute, so GD will lie inside the concavity of the cylinder. Let us then take angle EGL equal to angle EGD. GL will thus intersect BE at L. And let us mark M on line LE [inside the elliptical section]. Angle MAG will therefore be acute because AM lies inside the [elliptical] section.

[8.34] Let us then take angle GAD equal to angle GAM. Therefore, AD will intersect GD, since the two angles [GAD and AGD] that are at A and G are acute. So let them intersect at D. AD will therefore intersect BK. Let it then intersect at T.

[8.35] Consequently, if BK lies on some visible object, and if the center of sight lies at D, the form of L will be seen at G because the form of L will be reflected to D from G, and DG is parallel to normal LB [by construction].[136] Meantime, the form of M is seen at T because the form of M is reflected to G from A, and T is the image of M.

[8.36] Now let a plane pass through D parallel to the base of the cylinder [to form the circle represented at the bottom of figure 6.8.35]. Accordingly, it will intersect the plane of ABG and will form circle COR on the surface of the cylinder. The plane of this circle will therefore intersect BK, for it intersects GD, which is parallel to it [by construction]. Let it therefore intersect BK at K, and let point H be the center of circle CR. Let us then draw DH, and let it pass to R. Let us also draw KH, and let it pass to C.

[8.37] Hence, the form of K is reflected to D from the periphery [of the circled centered on H] within arc RC, as was shown in [the analysis of] images [formed] in [great] circles [within concave spherical mirrors].[137] So let it be reflected from O, and let us draw KO, DO, and HO. The angles [DOH and KOH] at O are therefore equal, and [reflected ray] DO will intersect [cathetus K]HC at N. So N is the image of K.

[8.38] Let us then connect KD. Accordingly, KD will be the common section of circle RC and [elliptical] section ABG, since the two points K and D lie in both planes, for there is nothing except line KD in plane ABG of the [elliptical] section that is [also] in the plane of circle RC. G therefore lies outside the circle, and likewise T, and both lie in the plane of the [elliptical] section.

[8.39] N, meanwhile, lies in the plane of the circle, and the form of LMK passes through points G, T, and N, and [so] the line that passes through these points is curved. However, the plane of the [elliptical] section is slanted with respect to the cylinder's surface, so the [major] axis of the [elliptical] section does not pass along the entire axis of the cylinder, nor is it parallel to the cylinder's base.

[8.40] From this and the previous two propositions it is therefore evident that straight lines parallel to the axis of the cylinder, as well as those parallel to its base, and also those that are slanted with respect to its surface may appear curved, or straight, or reversed. Furthermore, since T is the image of M and N the image of K, the form of MK will be reversed.

[8.41] In addition, if the line also lies in the plane of a circle parallel to the cylinder's base, and if the plane of that circle passes through the center of sight, the image may be the same size [as its object] and properly oriented, or it may be reversed, as was claimed in [propositions 25-27 of] the seventh chapter of this book [dealing] with images in [great] circles [on concave spherical mirrors].

[8.42] It is thus evident that the forms of objects perceived in concave cylindrical mirrors may be properly oriented, or they may be reversed.

[8.43] **[PROPOSITION 36]** Now let us recapitulate the diagram for the third proposition [dealing] with misperceptions in concave spherical mirrors, leaving the letters as they are [in figure 6.8.36, p. 151, which combines figures 6.7.26 and 6.7.27]. Let BZA be a circle on the surface of a concave cylindrical mirror, and let the center of sight be at D [on DG, which is perpendicular to the plane of BZA]. It will therefore lie outside the circle's plane, and the two lines EA and EB will [each] be perpendicular to a plane tangent to the cylinder's surface [at points A and B]. In addition, the plane of triangle DGE will be perpendicular to the plane of the circle because DG is perpendicular to the plane of the circle.

[8.44] Hence, the plane of triangle DGE passes through the entire axis as well as through D, whereas neither plane DBO nor plane DAO, which intersect along line DO, passes through the entire axis. Moreover, there is nothing but E on the cylinder's axis in either plane, E being the circle's center. And each of the planes DBO and DAO forms an [elliptical] section on the cylinder's surface, and forms are reflected from these [elliptical] sections at the two points A and B.

[8.45] The form of R is therefore reflected to D from B, whereas the form of M is reflected to D from A, and NU will be the cross-section of the image of MR, and it is shorter than MR. Likewise, the [forms of the] two points H and L are reflected to D from the two points A and B, and TK will be the cross-section of LH's image, and it is the same size as TK. Finally, CI will be the cross-section of FQ's image, and it is longer than FQ. All of these images, moreover, will be reversed.

[8.46] But if the center of sight lies at O, and if lines CI, TK, and NU are the visible objects, the opposite will obtain, for in that case the cross-section of the image [FQ] of CI will be shorter than CI, whereas the cross-section of the image [MR] of NU will be longer than NU, and the cross-section TK [of LH's image] will be the same size as it, and these images will all be properly oriented. All these points were shown in the preceding chapter.

[8.47] Furthermore, when either endpoint of any of these [lines] has a single image, and when any intermediate point [on that line] has several images, that line will have as many images as the intermediate point has. If, moreover, one endpoint or the other [of the line] has several images, and if the intermediate point has one, then the line will have as many images as the endpoint has. And if one endpoint or the other has several images, and if the intermediate point has several images, the line will yield images according to the greatest number [as pointed out in note 130, p. 256]. This will be shown as was shown for images in concave spherical mirrors.

[8.48] Hence, in concave cylindrical mirrors misperception occurs in all respects as it occurs in concave spherical mirrors, that is, concerning the shapes of visible forms, concerning the sizes and number of their images, and concerning their proper orientation or reversal, along with the misperceptions that apply to reflection [itself]. And the misperceptions in these cases will be as they are in the aforementioned mirrors, and these are the points we wanted to demonstrate in this chapter.

CHAPTER NINE
Concerning the Misperceptions That Occur in Concave Conical Mirrors

[9.1] In these [sorts of mirrors] the misperceptions that occur are those that occur in concave cylindrical mirrors. Indeed, the weakening of color and light, as well as variation in location and distance, occur in these mirrors as in all [other kinds of] mirrors, for the cause of this is reflection [itself]. In addition, a multitude of images arises in these mirrors, just as in concave cylindrical and spherical mirrors, as was claimed in chapter [2, book 5] on image [formation]. What happens in these mirrors is also like what happens in concave cylindrical [mirrors], i.e., what is straight appears convex, or it appears concave.

[9.2] The demonstration of this is that straight lines that extend along the length of the mirror and pass through the cone's vertex, as well as those that are near these [lines in orientation], appear convex, or they appear concave, or perhaps [they appear] straight.

[9.3] **[PROPOSITION 37]** The demonstration of this point is like the demonstration [given] for concave cylindrical mirrors [in proposition 33, pp. 221-224 above], for if we recapitulate the second diagram concerning misperceptions in convex conical mirrors [i.e., the top diagram of figure 6.6.22a, p. 131, from which figure 6.9.37, p. 152, has been abstracted], we will find the cross-section of the image of straight line [AN] placed toward that mirror, which, in that case, is [curved line] A[P]Y inside the concavity of the conical mirror, and we will find the point that is below the plane tangent to the cone and passing along the line [AZE] from which the form of the straight line [AN] is reflected to the center of sight, which is F in that case.

[9.4] If [that] point is the center of sight, all the points that are on the cross-section of the image will be reflected to point F, and the images of the two endpoints A and Y [on object-line APY] will be the endpoints of straight line AN, and the image-location of intermediate [point P] on AY will vary. And this will be demonstrated by the same train [of logic] we followed in the proof [provided] in the first proposition [dealing] with concave cylindrical mirrors [i.e., proposition 33].[138]

[9.5] From this it is clear that, if AY lies on some visible object, and if F is the center of sight, the image may appear convex, or it may appear concave.[139] And it is also evident from the second proposition concerning misperceptions in concave cylindrical mirrors [i.e., proposition 34] that lines placed along the width of a [concave conical] mirror will appear concave with a remarkable curvature and that images of straight lines that lie in planes passing through the axis and the center of sight will be straight.

[9.6] **[PROPOSITION 38]** Now with the same letters, let us recapitulate the third figure concerning misperceptions in concave spherical mirrors [as given in figure 6.8.36, p. 151, which combines figures 6.7.26 and 6.7.27]. If, therefore, some point [i.e., E] lies on the axis of the cone, and if the two lines EA and EB [passing through that point] are perpendicular to planes tangent to the cone (and this is possible because they are equal, since they can form two equal acute angles with the axis), then when [each of] these two lines is perpendicular [to a plane tangent to the mirror], and when the center of sight is at D, the plane containing lines GE and ED will pass through the entire axis as well as through the center of sight.

[9.7] Furthermore, both planes [containing] DAM and DBR will be inclined to the axis of the cone, and [so] the common sections of those two [planes and the mirror's surface] will be conic sections. The form[s] of points R, H, and Q will be reflected to D from B, and the forms of points L, M, and F are reflected to D from A. Hence, if lines MR, LH, and FQ lie on some visible surface, and if the eye is at D, then NU will be the image of MR, TK will be the image of LH, and CI will be the image of FQ.

[9.8] Thus, the image [NU] of MR will be shorter than [MR] itself, the image [CI] of FQ will be longer than [FQ] itself, and the image [TK] of LH will be the same size as [LH] itself, and all the images will be reversed.

[9.9] If, moreover, the center of sight is at O, and NU, TK, and CI are on the surfaces of visible objects, their images will be MR, LH, and FQ. Accordingly, the image [FQ] of CI will be shorter than [CI] itself, the image [MR] of NU [will be] longer [than NU itself], and the image [LH] of TK will be the same size [as TK itself].

[9.10] And these images will be properly oriented, for these images will lie behind the center of sight and will be perceived facing the center of sight along [direct] radial lines.[140] Consequently, points M, L, and F are perceived along line [of reflection] AO, whereas points R, H, and Q are perceived along [line of reflection] OB, and so their form will be reflected with proper orientation.

[9.11] From what we have claimed in this chapter, therefore, it is clear that straight lines sometimes appear convex in these mirrors, sometimes concave, and sometimes straight, and [they] sometimes [appear] longer, [sometimes] shorter, and [sometimes] the same size [as they actually are], and [they sometimes appear] properly oriented, and [sometimes] reversed.

[9.12] In chapter [2, book 5] on image [formation], moreover, we showed that in mirrors of this kind every visible point sometimes has one image, sometimes two, or three, or four. Therefore, misperception occurs in everything that is perceived in this sort of mirror, just as in concave cylindrical [mirrors], and compound misperceptions also occur in these as in the rest of the mirrors. Examples and proof of these [kinds of compound misperceptions] are as [they can be found] in plane mirrors. And we intended to explain this in this chapter. Now, however, let us end the sixth book.

NOTES TO BOOK SIX

[1]In other words, the object may appear to lie farther behind the mirror than it should because of its diminished brightness or because the ambient light is diminished enough to cause a misjudgment of the distance.

[2]Throughout his analysis of mirror imaging in book 6, Alhacen focuses on facing images, the paradigm case being that in which the viewer sees his own image in a directly facing mirror. Thus, in the case of plane mirrors, the right-hand side of the viewer's face will be seen on the right-hand side of the facing image, and vice-versa for the left-hand side. A more general and "objective" way of understanding image-reversal according to Alhacen's analysis is as follows. Assume that both ZH and its image DG in figure 6.3.1 are facing objects, and suppose that viewpoint E is posed between them. When facing ZH, E will see point H on the right-hand side of the object and point Z on its left-hand side. When facing DG, however, E will see point G—which corresponds to right-hand point H on ZH—on the left-hand side of the object and point D—which corresponds to left-hand point Z on ZH—on the right-hand side of the object. Consequently, the left-right orientation of the image is opposite to that of the object. On the other hand, the up-down orientation remains the same, so there is no accompanying inversion, as there is in the case of concave spherical mirrors.

[3]What Alhacen is getting at here can be illustrated by figure 6.3.1. If ZH is the object, DG its image, and E the center of sight, then, because image DG lies below the mirror, its apparent distance from E will be greater than the actual distance between E and the object ZH. Image DG will therefore appear commensurately smaller than object ZH. In addition, image DG is dimmed by the weakening of its light and color by reflection itself. It will therefore appear even farther away from E than it would if it were an actual object whose light and color were not dimmed by reflection.

[4]This is a misperception of separation or disjunction, and it is akin to the misperception described in 3, 7.77-78 (Smith, *Alhacen's Theory*, 611).

[5]That AD:DT = AM:MT and BD:DQ = BL:LQ follows from the fact that M is the endpoint of tangency for reflection-point H and L the endpoint of tangency for reflection-point N.

[6]FB = FM – BM, and KT = MT – MK. Since it has been established that FM:MT = BM:MK, then, by Euclid V.19, it follows that (FM – BM):(MT – MK) = FM:MT. But FM:MT = BL:LQ by construction. Thus, FB [which = FM – BM]:KT [which = MT – MK] = BL:LQ.

[7]If angle EDH is right, then of course line EH is the hypotenuse, which is the longest of the three sides in the triangle. By the same token, if angle EDH is obtuse, then EH will be the longest side because it will subtend the largest angle in the triangle. Hence, since EH < AB, then *a fortiori* ED < AB.

[8]Note the use of "form" rather than "image" here, a conflation that occurs throughout book 6 and that appears to have no significance beyond a relaxing of

terminological distinctions. The case in which image EH > object AB is dealt with in the very next proposition.

[9]As will be evident later (see note 24 below), this restriction on the length of ZD is arbitrary and pertains only to the construction and analyis from this point to the end of paragraph 4.52.

[10]This is tantamount to finding a third proportional, HT, such that AH:HD = HD:HT, which can be done by Euclid, VI.11.

[11]That CH,HA = $3HD^2$ follows from the fact established earlier that AH:HN = HQ^2 = one-fourth AH,HT. But it was also established earlier that HQ^2 = one-fourth HD^2, so one-fourth AH,HT = one-fourth HD^2. Accordingly, since CH = 3HT, then CH,HT = 3AH,HT = $3HD^2$. From this, of course, it follows that AH,HT = HD^2, so, by Euclid VI.17, AH:HD = HD:HT. Accordingly, the rectangles AH,HD and HD,HT are similar, and, by Euclid VI.20, AH,HD:HD,HT = HD^2:HT^2. But AH,HD: HD:HT = AH:HT, so AH:HT = HD^2:HT^2, or, conversely, HT:AH = HT^2:HD^2.

[12]It has been established above that ($AI^2 - AH^2$):AH^2 = HT,TN:QH^2 and that HT,TN = three-fourths HT^2. It has also been established that HQ^2 is one-fourth HD^2. Therefore, HT,TN:QH^2 = three-fourths HT^2:one-fourth HD^2 = $3HT^2$:HD^2, so it follows that ($AI^2 - AH^2$):AH^2 = $3HT^2$:HD^2.

[13]See the end of note 10 above. According to Euclid, V.1, CA:AH = rectangle CA,AI:rectangle AI,AH. Accordingly, since CA:AH = AI^2:AH^2, then rectangle CA,IA: rectangle IA,AH = AI^2:AH^2. Now, according to Euclid, VI.20, polygons (including rectangles) are to one another in the duplicate ratio of their corresponding sides (i.e., as the squares on their corresponding sides). Hence, rectangles CA,AI and AI,AH are the relevant polygons, and AI and AH are the corresponding sides. If we then divide both sides by the common factor AI:AH, we get CA:AI = AI:AH, which leaves AI the mean proportional between CA and AH.

[14]It is clear from the figure that IA (which = AH + IH) + AH = 2AH + IH. If we divide both sides by three, we get one-third (IA + AH) = two-thirds AH + one-third IH. Since we just concluded that TH:IH = one-third (IA + AH):AH, it must also be as (two-thirds AH + one-third IH):AH.

[15]What Alhacen has accomplished to this point of the construction is to establish that M and L constitute endpoints of tangency for reflection-points G and B, respectively, on the mirror. In book 5, proposition 7 (Smith, *Alhacen on the Principles*, 404), it has been established that the ratio of the distance from the object-point to the center of curvature (designated as *a*) and the distance from the image-point to the center of curvature (designated as *b*) is the same as the ratio of the distance from the object-point to the endpoint of tangency (designated as *c*) and the distance from the endpoint of tangency to the image-point (designated as *d*), which translates to *a*:*b* = *c*:*d*. In the case of figure 6.4.3a, I is the object-point for both reflections, so IA (i.e., *a*) in both reflections is the distance from the object-point to the center of curvature. In the case of reflection from B, L is the endpoint of tangency, so IL (i.e., *c*) is the distance from the object-point to the endpoint of tangency. By construction, however, we know that IL:LH = IA:AH, so we have *c*: LH = *a*:AH, which reversed is *a*:AH = *c*:LH. H will thus be the image-point, and AH will be the distance from the image-point to the center of curvature (i.e., *b*), and LH will be the distance from the endpoint of tangency to the image-point (i.e.,

d). According to our initial proportion, then, *a*:*b* = *c*:*d* = IA:AH = IL:LH. The same holds *mutatis mutandis* for M as the endpoint of tangency for reflection from point G on the mirror, so it follows from the proportion IM:MT = IA:AT, which is given by construction, that T is the image-point in that reflection (i.e., IA:AT = IM:MT = *a*:*b* = *c*:*d*).

The purpose of the long and rather involved line of reasoning preceding this paragraph is to locate endpoint of tangency M between I and H, which follows from the fact that IH:HT > IA:AT, which = IM:MT, by construction. It therefore follows that IH:HT > IM:MT, which means that IM < IH and MT is equal to or greater than HT. However, if MT = HT, then M must either coincide with H or lie at some point between A and T. If the latter, then IM > IH, which contravenes the initial condition that IH:HT > IM:MT. Therefore, MT > HT, and so it follows that M must lie between I and T. By the same token, IA:AT > IA:AH, so it follows that IM:MT > IA:AH, which = IL:LH, by construction. Therefore IM:MT > IL:LH, from which it follows that IL < IM and MT > LH. Hence, L must lie between I and M and thus also between I and H.

16 The relevant theorem in this case is actually proposition 7 cited in the previous note.

17 In other words, if IA:AH = AB:AP, then IA:AB = AH:AP. Likewise if IA:AT = AB:AR, then IA:AB = AT:AR, so AH:AP = AT:AR, from which it follows that AH: AT = AP:AR.

18 That triangles OAY and GBS are similar follows from the fact that angle OAY = angle GAY + angle OAG, whereas angle GBS = angle XBS + angle GBX. But it has already been concluded that angle GAY = angle XBS and that angle OAG = angle GBX. Therefore, since they are composed of equal elements, angle OAY = angle GBS. Moreover, angles OYA and GSB are both right. Therefore, by Euclid, I. 32, the remaining angles BGS and AOY must be equal. By Euclid, VI.4, then, the two triangles are similar, and their corresponding sides are proportional.

19 We have already established that $AH^2 + HB^2 = DA^2 + 2AH,HD + 2AH,DF$, so $AH^2 = DA^2 + 2AH,HD + 2AH,DF – HB^2$. By Euclid II.7, moreover, $AH^2 + HD^2 = 2AH,HD + AD^2$, or, conversely, $2AH,HD + AD^2 = AH^2 + HD^2$. Therefore, if we substitute the value for AH^2 derived earlier—i.e., $DA^2 + 2AH,HD + 2AH,DF – HB^2$—for AH^2 in this relationship, we get $2AH,HD + DA^2 = DA^2 + 2AH,HD + 2AH,DF – HB^2 + HD^2$. Adding HB^2 to both sides, we get $HB^2 + 2AH,HD + DA^2 = DA^2 + 2AH,HD + 2AH,DF + HD^2$. Dropping the common term (AD^2 + 2AH,HD)—which is equivalent to (AB^2 + 2AH,HD)—from both sides, we end up with $HB^2 = 2AH,DF + HD^2$.

20 We established earlier that $AT^2 + TD^2 = AD^2 + 2AT,TD$, so $AT^2 = AD^2 + 2AT,TD – TD^2$; and we just established that $AT^2 + TG^2 = AD^2 + 2AT,TD + 2AT,DK$, so $AT^2 = AD^2 + 2AT,TD + 2AT,DK – TG^2$. Therefore, $AD^2 + 2AT,TD – TD^2 = AD^2 + 2AT,TD + 2AT,DK – TG^2$. Adding $TD^2 + TG^2$ to both sides, and subtracting common term (AD^2 + 2AT,TD) from both sides, we end up with $TG^2 = TD^2 + 2AT,DK$.

21 From above, we take AH as divided into 25 parts. We know from previous conclusions that HT < one of those parts and that HD > 5 HT, so AH:HD = 25:>5 HT. On the other hand, AT, which = AH – HT, is less than 25 and more than 24 of those parts. Finally, TD = HD – HT, i.e., > 5HT – HT, so TD > 4HT. Therefore AT:

TD [i.e., > 24: > 4HT] > AH:HD [i.e., 25: > 5HT]. Since, therefore, AT:TE′ = AT:TD, it follows that AT:TE′ > AH:HD.

[22]The reason for establishing that arc Q′D > arc GB will become clear in fairly short order. The proof that arc Q′D is in fact greater than arc GB, which occupies paragraphs 4.34-44, pp. 169-171, entails an enormous number of steps, but the logic behind it can be reduced to the following, in reverse order. Ultimately, what needs to be demonstrated is that Q′D:DA > BG:BA and, therefore, that Q′D > GB in view of the equality of DA and BA, which are radii of the mirror. This depends on proving that QH:AH = Q′D:DA and, moreover, that QH:AH > FK:OY. This, in turn, depends on proving that BG:GA = FK:OY, which, given the equality of GA and BA, is tantamount to BG:BA = FK:OY. Hence, if Q′D:DA > FK:OY, which is proportional to BG:BA, then Q′D:DA > BG:BA, from which it follows that Q′D > BG.

[23]In other words, since angle QMA is acute, QM must intersect circle FDB at some point to the left of M before reaching M itself. Hence, extension MZ of QM cannot intersect the circle to the right of point M, which is a roundabout way of saying that MZ can be a line of reflection. In essence, what we have done to this point is rotate mirror DGB in figure 6.4.3b to the left according to angle IAN while leaving line AGR fixed so as to carry line ABC the same distance in the same direction to coincide with line AMU. Points I, H, D, C, and B will thus sweep out arcs IN, HQ, FD, CU and BM. Hence, arc DF = arc BM, arc IN = arc CU, and line AFQN will be cut so that NA = IA, QA = HA, and FA = DA. Under those conditions, the form of point N will reflect along MZ according to the equality of angles NMU and UMZ, which are the respective angles of incidence and reflection. For any center of sight located on MZ, then, the image of N will appear at point Q, where the extension of line of reflection MZ intersects cathetus NA dropped from object-point N to the mirror′s center of curvature at point A. It should be noted, however, that the construction and proof will apply whether M is located to the left or to the right of B, just as long as the resulting arc MB is equal to arc FD.

[24]It is at this point that the restriction on the length of ZD posed in paragraph 4.12 at the beginning of the proposition must be lifted. To understand this point and its implications, let us summarize the method of construction based on ZD, as represented in figure 6.4.3a, p. 100. Add it to radius AD of the mirror, bisect it at point H, and through that point form a circle centered on A. Bisect HD, form chord QH equal to it, and extend line AQ through Q. Find HT such that AH:HD = HD:HT, and connect QT. Extend line AQ well beyond Q, and find point I on ZD such that line IS dropped from it parallel to chord QH and touching the extension of AQ is equal to line QT. IS will thus be the chord of a circle with radius AI. Locate point M on line IT according to the proportion AI:AT = IM:MT, and locate point L on line IH according to the proportion AI:AH = IL:LH. Draw tangents MG and LB to the mirror from endpoints of tangency M and L. Hence, G will be the point of reflection according to which IG forms the line of incidence and TG the line of reflection along which the form of I is seen at T. B, on the other hand, will be the point of reflection according to which IB forms the line of incidence and HB the line of reflection along which the form of I is seen at Q. Then, in figure 6.4.3b, p. 101, find point M to the left of G such that arc BM = arc FD (= Q′D in figure

6.4.3a, with N replacing S in that same figure). According to symmetry, it follows that, just as IB and HB form the respective lines of incidence and reflection for point B of reflection, lines NM and QM will form the respective lines of incidence and reflection for point M of reflection. So point I will appear at point T for any center of sight posed to the right of G on line of reflection TG, whereas point N will appear at point Q for any center of sight posed to the right of M on line of reflection QM.

Let us extend ZD (= Z'D in figure 6.4.3b). As it increases in length, its half, HD, will increase commensurately, and likewise HD's half, HQ, will increase along with FD, since both HQ and FD subtend the same angle. Moreover, as HD increases, so does HT, which is tied to it according to the proportionality AH:HD = HD: HT. Along with these increases, AI also increases, and with it NI, which subtends the same angle as HQ, FD, and BM (which equals FD by construction). Since the lengths of lines AI, AH, and AT determine the location of endpoints of tangency M and L and thus the location and size of angle GAB, any change in those lines will cause a change in that angle. The only constant throughout these changes is the equality of NI (= SI in figure 6.4.3a) and QT.

Even a moderate increase in the length of Z'D in figure 6.4.3b (= ZD in figure 6.4.3a), will cause a relatively significant increase in the size of angles BAG and GAM (and angle BAM composed of them) because of a significant increase in the length of lines AI, AH, and HT, as well as in the sizes of NI, QH, and FD subtending the same angle. However, as we increase HD and all its accompanying parameters, angle BAG increases faster than angle GAM so that a point is eventually reached at which angle GAB = angle GAM, as represented in figure 6.4.3c, p. 102. In order for that to happen, though, HD has had to increase dramatically enough to be almost equal to AH, which means that radius AD of the mirror has dwindled to virtual insignificance in relation to HD. Meantime, arcs NI, QH, FD, MB, and GB have increased in size, as have lines AQ, AN, AH, AI, NI, QH, HT, and QT. In fact, HT has become nearly as long as HD. But over the course of these changes the increase in the size of angles NAH and MAB has decelerated drastically because, as HD approaches equality with AH, chord QH, which equals half HD, approaches equality with half AH. Accordingly, the closer HD approaches AH in length, the more angle NQAHI, approaches constancy and the more NQ, AQ, IH, AH, and QT approach equality. One other crucial point emerges from this analysis: as angle BAM increases in size, line ABC moves clockwise toward the horizontal, which is to say that angle DAB approaches 90°. As that happens, lines AG and AM also move clockwise such that line AM moves somewhat faster than line AG, until finally the two reach a point at which angles BAG and GAM are equal. It therefore follows that, when ZD, and thus HD, is increased beyond the point at which it yields equality between angles GAB and GAM, angle GAB will begin to overtake angle GAM in size, but by tiny increments according to the ever-diminishing increase in the overall size of angle GAM.

[25]This is a special case in which ZD in the original construction is just the right length that arc NU = arc UZ, and arc IR = arc RZ. Thus, incident rays NM and IG are equal, respectively, to reflected rays MZ and GZ. Accordingly, as seen from center of sight Z, where the two reflected rays converge, image TQ will be the same length as its object-line NI, as demonstrated earlier.

[26]The explanation that line KG intersects line MZ because points K and M are "lower" than points Z and G, respectively, indicates the analysis was conceived with the original diagram (whether actually drawn or merely imagined) rotated 90° counterclockwise. In order to get BAG even minimally larger than GAM according to the construction outlined in note 25, it is necessary to lengthen ZD by so much that, within the scale of a page, the mirror dwindles almost to a point in relation to AH and QH. As represented in figure 6.4.3d, line of reflection QMZ barely grazes the mirror as it continues on to intersect the outer circle containing object-line NI. Accordingly, angle IAGR < angle ZAGR by a small amount. Meantime, reflected ray TGK continues to point K on the outer circle such that angle TAG = angle KAG. Thus, lines QMZ and TGK will intersect to the right of points M and G—at point L in this case. If a center of sight is placed at L, then, the entire image QT of object-line NI will be seen, and the two lines are equal by construction.

[27]As in the previous case, so in this one, the reflected ray TG for incident ray IG, when it is extended, intersects the outer circle at some point—O in this case—other than Z. Likewise, as angle BAG approaches equality with angle GAM, point O approaches point Z, and as it does, the intersection-point of reflected rays OG and ZM also approaches point Z.

[28]In other words, if the differential between angles BAG and GAM is small enough, the intersection-point of the two rays will lie somewhere to the right of reflection-points M and G, so this point will serve as a center of sight from which image TQ = object IN. As that differential increases, however, the intersection-point migrates toward reflection-points M and G and eventually beyond them to the left, as in fact is illustrated in figure 6.4.3e, where X marks that intersection. In that case, of course, the intersection-point can no longer serve as a center of sight.

[29]According to the conditions previously laid out, BAG < GAM. But BAG can be as small as we please and still fulfill that condition. Let BAG be as small as possible—i.e., virtually 0°. Hence, BAM = GAM, and since BAM = IAN, by construction, it follows that GAM = IAN. Consequently, if angle BAG > 0°, GAM < IAN. But NAZ = IAO + GAM. Hence, if GAM < IAN, NAZ exceeds IAO by less than IAN.

[30]According to Euclid, XI.23, two conditions must be met for a solid angle to be formed from the three angles IAO, NAZ, and IAN. First, any two of the angles taken together must be greater than the remaining angle. This condition is satisfied, because we just established that OAI + ZAN > NAI, OAI + IAN > ZAN, and ZAN + IAN > IAO. The second condition is that the three angles taken together sum up to less than four right angles. That the three angles under consideration fulfill this condition is clear from the following. If a tangent is dropped to a circle from any point on the extension of a diameter, the angle formed by the extended diameter and the radius intersecting the point of tangency will be less than a right angle. LAB in figure 6.4.3a, p. 100, is such an angle, by construction, so it is less than a right angle. But LAB = IAB, so IAB < a right angle. By the same token, MAG in figure 6.4.3a < a right angle, by construction. But MAG = IAG, so IAG is less than a right angle. Since NAZ in figure 6.4.3e = 2IAB, NAZ < two right angles, and since IAO = 2IAG, IAO < two right angles. But IAG = IAM + GAM, and, as we just determined, GAM = IAN when BAG = 0°. It therefore follows in this case that IAG = IAB. Hence, at its very largest IAO = NAZ + IAN. But IAO < two right angles, so NAZ + IAN < two right angles, so IAO + NAZ + IAN < 4 right angles.

[31]The solid angle resulting from this construction will therefore form a pyramid upon base NAI of the circle. Within that pyramid (as represented in figure 6.4.3f) edges AS, AI, and AN are equal, edge IS is equal to the line IO in the base-circle, and edge NS is equal to line NZ in the base-circle.

[32]The construction described to this point can be easily visualized according to figure 6.4.3g, p. 106. Imagine that all the lines in the figure are rigid filaments. Imagine further that arc IURZO lies flat on the ground and that the entire structure IAOGT in black is a flap lying on it and hinged along IA. The entire flap can thus be rotated toward S, and as it is rotated it will carry filament AG along with it. Then imagine that structure NAZMQ represents another flap atop IAOGT and hinged on NA. It too can be rotated toward S and, in the process, will carry filament AM along with it. Start by lifting this latter flap, then lift flap IAOGT, and bring them together until their respective sides OA and ZA come together. When they do, the two flaps will form a pyramid with its vertex at S, and all the angular relationships that obtained within them when they lay flat upon the base will also obtain when they assume that position.

[33]The argument to this point can be readily understood by recourse to figure 6.4.3h, p. 107. It has already been established that triangle SAT = triangle OAT and, therefore, that ST = TO, SA = AO, and AT is common. It has also been established that angle IAG = angle GAO, so AG bisects OT. G, however, is one of the two points at which OT intersects the sphere of the mirror on great circle DG. Hence, Y will be the corresponding point on TS where that line intersects the sphere of the mirror on great circle DYO′. The same analysis holds *mutatis mutandis* for the reflection from point M, so that triangle NAZ = triangle NAS in figure 6.4.3f, p. 105, and from there it follows that line AZ′Z″ bisects angle NAS, line QZ′ in triangle QAZ′ corresponds to line QM in triangle QAM, line SZ′ in triangle SAZ′ corresponds to line MZ in triangle MAZ, and therefore point Z′ on great circle O′Z′F corresponds to point M on great circle FDMB.

[34]Let NI and QH in figure 6.4.3k, p. 108, be the chords subtending arcs NI and QH, and let the sought-after perpendicular meet QN either at point P, between Q and N, or at point P′, beyond N. If P′I were the perpendicular, it would form triangle INP′, to which INQ would be an exterior angle. According to Euclid, I.32, the exterior angle of any triangle equals the sum of the opposite interior angles, which in this case would be IP′N and NIP′. Hence, INQ = IP′N + NIP′. But IP′N is supposedly a right angle, so IP′N + NIP′ > a right angle, from which it would follow that angle INQ, which is acute by construction, is also greater than a right angle. Hence, it follows that point P must lie below point N.

[35]Book 5, proposition 17, in Smith, *Alhacen and the Principles*, 414-415. According to the enunciation of that theorem, for any two object-points facing a convex spherical mirror, “the image-location of the point nearer the [mirror’s] center will lie farther from the center of the [mirror] than the image-location of the point farther from the [mirror’s] center.” Since point N in figure 6.4.3k lies farther from centerpoint A of the mirror than does point P, image-point Q of point N must lie nearer A than image-point Q′ for point P, which is to say that Q′ must lie above Q, between it and P.

[36]While the conclusion of this paragraph is correct, the details of analysis are not. On the one hand, it is true that, if Z in figure 6.4.3m is the center of sight, M the point of reflection, and Q the image, line ZQ will intersect the circle at point M as well as at another point E to the left of M. It is also true that if a tangent is drawn from Z to the circle, it will touch it at some point V on arc ME. On the other hand, it is not true that that this tangent defines the endpoint of tangency. That point lies on the line tangent to the mirror at the point of reflection, XMY in the diagram, which extends below ZM. It is also true that no object-point at or below endpoint of tangency X can yield an image for Z because it cannot reflect to that point.

[37]Later in the theorem it becomes clear that Z must lie the same distance as E from center of curvature G.

[38]As with point Z, so with point H, it must be located precisely the same distance as E from center of curvature G. Alhacen's failure to make this specification until the next paragraph is somewhat puzzling, given the exquisitely systematic approach he normally takes to proof.

[39]As we established earlier, AG:GD = (AG:HM)(HM:GD). But we just established that AG:HM = AZ:ZH and that HM:GD = HE:ED, so, if we substitute the equivalent ratios, it follows that AG:GD = (AZ:ZH)(HE:ED).

[40]That this claim is patently false follows from reversing the construction and proof. It has been established in that proof that, when base line AGBD is cut according to the ratio AB:BD = AG:GD, any line drawn from A that passes above it through BE, DE, and GE will be cut according to that ratio. AZHT is one such line, but there is an infinitude of lines between it and AGBD that fulfill the stated condition. Thus, if we take AZHT as our base line and extend lines ET, EH, and EZ below it, any line (including AGBD) below AZHT that passes from A through those extensions will be cut according to the mandated ratio. The same will therefore hold for any line below AGBD that passes from A through the extensions of BE, DE, and GE.

Whether, in fact, this erroneous claim originates with Alhacen is unclear, because the text is problematic insofar as five of the manuscripts have *in quam* ("in which") instead of *numquam* ("never") in line 13. To choose *in quam* over *numquam* at this point, however, would have required such a tortured reading of the sentence that I felt compelled to plump for *numquam*, despite my conviction that Alhacen was far too good a mathematician to have made such a mistake.

[41]Thus, although the object AEB, the mirror, and the image QML are all concentric according to the construction, image QML will nonetheless be more sharply curved than the surface of the mirror, which in turn is more sharply curved than object AEB. It follows, therefore, that image QML does not take on the curvature of that surface. This point is of course obvious to anyone who looks at himself in a convex spherical mirror and notices that the outlying edges of the image appear to lie farther away than they should with respect to the center of the image.

[42]Alhacen's point here seems to be that the largest angle the perpendicular dropped to the plane from D forms with that plane is smaller than the largest angle formed by all other lines dropped from D to that plane, since those lines

necessarily form an obtuse angle on one side. To illustrate this point, let YZ in figure 6.4.8a, p. 111, represent the sphere from which the mirror is composed, G the center of curvature, and Y'Z' a great circle on the surface of that sphere within the plane of arc AB. Let GD' be perpendicular to that plane at point G. Let center of sight D be displaced to the side of D', and let DX be the perpendicular dropped from D to the plane of arc AB. Accordingly, within the plane formed by DX and DG, which forms common section XGR with the plane of arc AB, angle DGR will be obtuse and thus greater than right angle D'G. No matter where D is displaced with respect to D'G, therefore, line DG will always form an obtuse angle on the plane of arc AB. It is also the case that, since D is displaced to the right of D', it will be closest to point A and farthest from point B. Hence, AD < ED < BD. Moreover, with respect to the plane of arc AB, the obtuse angle DA forms with that plane will be smaller than the obtuse angle DE forms with that plane, which will be smaller yet than the obtuse angle DB forms with that plane.

[43]As will be evident from the analysis that follows, DG in this case must lie to the right of the perpendicular dropped to G on the plane of arc AB, as just illustrated in figure 6.4.8a.

[44]According to construction, BG = CG, and according to proposition 4, lemma 1, since B lies farther from D than C, endpoint of tangency L for point B lies farther from center of curvature G than endpoint of tangency M for point C. Hence, GL > GM, from which it follows that remainder CM > remainder BL.

[45]That QT, LM, and BC will intersect at point O when extended follows from the fact that the images of B and C are seen by a single center of sight, D. Hence, by book 5, proposition 7, GC:CM = GQ:QM and BG:BL = GT:TL. But those proportions are proportional, since they are based on the same center of sight, so it follows that GC:CM = GQ:QM = BG:BL = GT:TL. Thus, line GC is divided at Q and M in proportion as GL is divided at T and L, so, by proposition 6, lemma 3, lines QT, LM, and BC, when extended, will intersect at a single point.

[46]In other words, if center of sight D is displaced above perpendicular D'G so that the perpendicular dropped from it to the plane of arc AB intersects that arc at its midpoint, the images of the endpoints will lie precisely the same distance from center of curvature G, so the curvature of the resulting image will have the same orientation as that of the mirror, although it will be sharper, as will be shown in the following proposition.

[47]Although there is no indication at this point in the construction that object-line AB and normal DG, whose endpoint D represents the center of sight, do not lie in the same plane ABGD, it becomes clear later in the proposition that in fact they are to be imagined not to lie in the same plane. Accordingly, in figure 6.4.12, the plane formed by AG and BG—i.e., AGB—cuts the mirror along great circle SEZ (reading clockwise from S), and AB extended is tangent to the mirror at point E on that great circle. On the other hand, plane AGD formed by AG and DG cuts the mirror along a great circle containing arc PZ, whereas plane BGD formed by BG and DG cuts the mirror along a great circle containing arc PH. These two planes, AGD and BGD, therefore intersect along common section DPG, and they intersect plane AGB containing object-line AB along arc ZH to form triangle PZH on the

mirror's sphere. Although the subsequent analysis is based on this three-plane arrangement, that analysis will also obtain if AG, BG, and DG all lie in the same plane. This case is represented in figure 6.4.12a, p. 115, where the construction and analysis for this proposition is somewhat easier to follow than in figure 6.4.12.

[48]In book 1 of the *Sphaerica*, Menelaus of Alexandria (fl. c. 100 AD) shows the parallels between spherical and plane triangles. This work was well known to medieval Arabic mathematicians and was thus doubtless known to Alhacen. Suffice to say, when DG, AG, and BG lie in the same plane, as represented in figure 6.4.12a, p. 115, triangle HZP is conflated into the single arc PZH, with PZ < PH and HZ taken together.

[49]According to construction, of course, angle AGD = angle AGK, and DG = KG. Thus, the reflection of point A's form to K from point of reflection R' will be perfectly equivalent to the reflection of point A's form to D from point of reflection R.

[50]The fundamental purpose of this rather elaborate construction is to establish M as the endpoint of tangency for B and N as the endpoint of tangency for A vis-à-vis center of sight D. From previous reasoning it follows that the extension of line NM will meet the extension of line IO connecting the images of A and B at a single point Q on AE. The image U of point Q, as seen from D, will necessarily lie on cathetus QG, and that cathetus lies below the two catheti AG and BG for points A and B. Hence, the image of Q will lie below those two images and thus below line IOQ connecting them. Since image IOU must be continuous, it follows that it must be curved in order to include all three points.

[51]Having arc HF' = arc HE is tantamount to sliding line AZG in figure 6.4.12, p. 114, toward B along AE while, at the same time, pulling AE inward until it coincides with line BF. By the construction in paragraph 4.115 of proposition 12, ZP = ZY, but in the construction for figure 6.4.13, arc ZP (which = arc HP) = arc HS (which = arc ZY). So HS and HY coincide, which means that GL and GK coincide. Point Q is therefore the point from which the form of coincident points A and B reflects to coincident points L and K. Hence, points F and R' in figure 6.4.12, which are the respective points from which the forms of B and A reflect to points L and K, coincide at point Q. By symmetry, then, points F' and R in figure 6.4.12, which are the respective points from which the forms of B and A reflect to point D will coincide. Points A and B will therefore coincide, as will points C, M, and N and E, Q, and T. Accordingly, images I and O of A and B, as seen from D, will coincide at point O on cathetus AG (= BG). Let A be the endpoint on that cathetus. If we choose random points X and Y between A and Q in figure 6.4.13a, p. 117, and if we determine their respective images X' and Y', the composite image OX'Y'Q of line AQ will be curved, as predicted by the analysis. Substitute B for any of these points, and it follows that the image of AB will be appropriately curved.

[52]Thus, as represented in figure 6.4.13b, p. 118, HE > HF', and ZP = ZY, according to the construction for figure 6.4.12. As before, M is the endpoint of tangency for B, whose form reflects to D from point F' and to L from Q (which is point F in figure 6.4.12), so HF' = HQ. Point R is the point from which the form of A reflects to D, and R' the point from which the form of A reflects to K, so the two

reflections are perfectly equivalent, from which it follows that ZR = ZR′. Point N on the extension of the tangent to R′ is therefore the endpoint of tangency for point A. Accordingly, the extension of NM will intersect line AE at Q′. Point I is the image of point A, and point O the image of point B. Hence, according to previous reasoning, the extension of IO will intersect line AE at point Q′. When image-point U of point Q′ is determined for center of sight D, the composite image IOU of AB will therefore be appropriately curved.

[53]In figure 6.4.13c, p. 119, HE < HF′. As before ZP = ZY, HF′ = HQ, ZR = ZR′, M is the endpoint of tangency for B, and N is the endpoint of tangency for A. Thus, when extended, NM and IO intersect at Q′. When image U of Q′ is located, the resulting composite image IOU of A, B, and Q will be appropriately curved. The limiting case for this situation occurs when arc HE dwindles to 0° so that A and B lie on the same cathetus AG. In that case, point Q will lie where that cathetus intersects the mirror, so its image U coincides with it. Thus, the entire foreshortened image IOU of ABG will lie directly in line with it on cathetus AG, as was empirically determined in 5, 2.6 (Smith, *Alhacen on the Principles*, 387). IOU will therefore not be curved, as Alhacen points out at the beginning of the very next proposition.

[54]Actually, this stipulation was never made explicitly, although as we saw, the construction in proposition 12 implies that the visible line and the center of sight do not lie in the same plane.

[55]What Alhacen means by "slanting away from the eye" (*declinatio ex parte visus*), is open to interpretation, and the manuscripts are no help, since they have no diagrams to accompany this proposition. According to my interpretation, such a slant is defined by the angle formed on the side of the eye by the visible line and the line bisecting the angle formed by the two normals dropped from the center of sight and the object-point. Thus, in figure 6.4.14, the slant of object-line AB is determined by angle GCA. Since it is obtuse, AB is assumed to slant away from the eye.

[56]In order for the entire line AB to be invisible in the mirror, it would of course have to intersect normal DG so as to block all reflection to D. As Alhacen lays out the analysis, however, it seems necessary to leave an opening between this normal and line AB so that a supposed point of reflection R′ can be chosen on the arc between DG and CG such that it will yield an open line of reflection to D—the proof resting ultimately on the fact that this cannot be a legitimate point of reflection if R is a legitimate point of reflection. Despite the rather infelicitous articulation of this proof, Alhacen's basic point seems to be as clear as it is banal: if an object gets in the way of its reflection, it will not be seen in a mirror.

[57]What Alhacen seems to have in mind here is that, if it were not blocked by the remainder of line AB, the form of A would pass through X after reflection from B, whereas if it were able to reflect from point R′, the form of A would also pass through X on its way to D. Therefore, two distinct forms of A would reach D, which is impossible.

[58]The points made in these two paragraphs are illustrated in figure 6.4.14b, p. 120. First, if line AB from the previous figure is shortened to AC, then its form

can be reflected to D from B, but only endpoint C of that line will be seen in the mirror. Since the line is physical, that endpoint will have some dimension, albeit tiny, so its image will have some dimension, however minuscule. Moreover, if that slanted line is transferred to position A'C' between normals FBG and DG, it will be visible in the mirror from D, and the curvature of its image will depend on the angle it makes with FBG—i.e., the closer it approaches FBG, and thus the closer it approaches a perpendicular position with respect to the mirror, the less curved it will appear, which is the point of paragraph 4.127.

[59]What Alhacen is getting at here is not clear because it is difficult to know precisely what is meant by *conterminabilis*. I have chosen to translate it somewhat vaguely as "bordering on," although it could mean "neighboring" or even "sharing the same endpoints." Accordingly, my interpretation of the passage is as follows. Let DB in figure 6.4.14c, p. 121, represent the line of reflection from the previous examples, and let arc EBC be the visible portion of the mirror defined by tangents DE and DCK dropped to the mirror from D. FH is the one line parallel to DB that touches the mirror on the right side in the invisible portion. Anything that borders on line FH below it (including FH itself) will therefore be invisible. Thus, ML is obviously invisible. On the other hand, although not everything that borders on FH above it will be visible, everything that does so and extends above extension CK of the tangent dropped from D will be visible. Accordingly, the segment of M'L above CK will be visible in the mirror, the point where M'L and CK intersect being the endpoint of tangency.

[60]My best guess at the meaning of this passage is illustrated in figure 6.4.14d, p. 121, where the two parallels are taken to be the line of reflection itself and the line parallel to it that is tangent to the circle of the mirror at point F. Accordingly, if the object-line is slanted toward the center of sight, as represented by XY, and if it does not touch either of the parallels, it will be visible in the mirror from center of sight D. On the other hand, if it slants away from the eye, as represented by line X'Y', it will be visible as a line, rather than merely according to its endpoint, so long as it does not coincide with the line of incidence extending from its endpoint X' to point B of reflection. If that is the case, then it is unclear what Alhacen means by stipulating that it will be visible as long as the line dropped from its endpoint parallel to the parallel lies above that parallel.

[61]That a line beyond the center of sight cannot be seen, except perhaps for terminal portions, is presumably due to the inability of its form, or at least the main central part of its form, to reach the mirror because it is blocked by the head. By the same token, any line intersecting the visual axis and thus lying between the center of sight and the mirror will block its own form from reflecting back to the center of sight. However, a line parallel to the visual axis will be visible, as long as it extends above the visible portion of the mirror.

[62]The more completely the visible line is exposed to the mirror vis-à-vis the center of sight, the longer its image will appear and, therefore, the more "clearly" it will appear. This exposure varies with the slant, so the greater the slant, to put it in Alhacen's terms, the more complete the exposure. Wherever the object-line is placed with respect to the mirror's surface and the center of sight, its greatest

exposure occurs when the line from its midpoint to the center of curvature is perpendicular to the mirror's surface. As to the curvature of the resulting image (whether of a straight line or of an arc), in saying that it accords with the curvature of the mirror Alhacen does not mean that it takes on the actual curvature of the mirror. This he has in fact denied in proposition 8 above. What he means, instead, is that the sharper the mirror's curvature, the sharper the curvature of the object seen in it. Thus, for a line of a given size, the smaller the sphere from which the mirror is composed, the more sharply curved the object-line will appear in it.

[63]In other words, when the visual axis of either eye or the visual axes of both eyes lie in the same plane as the object-line, the object-line will appear only lengthwise in the mirror, and it will be commensurately foreshortened. As the center or centers of sight move to face the object-line more directly, however, the image is seen under a larger visual angle, so it appears less foreshortened and therefore clearer.

[64]It is worth noting that when object-line HE is parallel to line DG, as represented in figure 6.4.15, p. 122, or when it slants above that position, lines HC, TK and ZL will intersect to the left of cathetus HG, and this will continue to be the case until object-line HE achieves the slant in figure 6.4.15a, p. 122, at which point HC, TK, and ZL are parallel. Thus, when HE inclines below its position in figure 6.4.15a, as represented in figure 6.4.15b, HC, TK, and ZL will intersect to the right of cathetus HG.

It is also worth noting that Alhacen does not address the case in which the extension of object-line HE intersects the mirror's surface at some point to the right of B. In fact, this case has already been addressed implicitly in propositions 12 and 13, where the proofs hold whether the normals from the center of sight and the endpoints of the object-line lie in different planes or in the same plane (see note 47 above). Hence, the primary difference between this proposition and propositions 12 and 13 is that in this case object-line HE (i.e., AB in propositions 12 and 13) does not touch the circle of the mirror at any point. Otherwise, the proof is essentially the same, point C in proposition 15 serving the same function as points Q and Q' in propositions 12 and 13, respectively.

[65]These errors include all the ones stemming from the fact that reflection weakens light and color, both of them being further affected by the mirror's color. It also includes image-reversal. See paragraphs 3.5-3.12, pp. 163-164, for a full account of these errors.

[66]If the Latin is to be taken literally, line EU should fall below (*sub* in the text) both axis DH and point D on line DB. As represented in figure 6.5.16, however, although line EU does fall below point D on line DB, it falls in front of rather than below the axis. Presumably, then, the diagram Alhacen had in mind was rotated 90° clockwise so that EU would actually lie below the axis. Consequently, in this and succeeding theorems I have rendered *sub* as "outside" rather than "below."

[67]Whether DL intersects the plane of circle ESP inside the circle itself or outside it depends upon the slant of the elliptical section; the more acute CDL becomes, the farther point F migrates toward point S on the circle until, finally, it falls outside that circle. DL is, of course, the major axis of the ellipse, and BO is its minor axis.

[68]According to Euclid, VI.8, TM is the mean proportional between DM and MC. Therefore, DM,MC = TM^2. However, it has just been established that TM = FE, so by substitution it follows that DM,MC = TM,FE.

[69]In other words, if angle LED were right, then according to Euclid VI.8, rectangle DF,FL should be equal to EF^2. But we have just established that DF,FL > EF^2, which means that EF^2 is equal to some rectangle consisting of FL and some length D'F < DF, so D' will fall somewhere between D and F. Hence EF^2 = D'F,FL, which fulfills the condition for Euclid VI.8, which in turn means that angle D'EL is right. If so, then angle DEL must be obtuse, since it is subtended by a longer side (i.e., DL) than angle D'EL. Hence, EU, which forms a right angle with EL, by construction, must pass through point D' and thus outside point D.

[70]The point of this proposition is actually quite simple and can be easily understood in the context of figure 6.5.16a, p. 125, where the problem is viewed within the plane of the elliptical section. Let BD be the minor axis of that section and DFL the corresponding major axis. With N as the object-point, assume that its form reflects from point B at the end of the minor axis and therefore within the plane of circle BO passing through that point. Let V be the center of sight, so that VB is the reflected ray. The problem is to determine image-location I, which will lie at the intersection of the cathetus dropped from point N to the ellipse and the extension of line of reflection VB. The point of this theorem is to demonstrate that the cathetus, NEU, which is perpendicular to tangent QEL at point E, will intersect the normal BD dropped to point of reflection B at some point U beyond point D such that the cathetus will lie below line BD. Furthermore, it should be clear that cathetus NEU bypasses the axis of the cylinder in front of it, since point D' lies on segment DL of the major axis, which diverges away from axis DH in figure 6.5.16. On the other hand, if N were the center of sight and V the object-point, then the relevant tangent would be Q'E'L', and the cathetus VE'U would fall above line BD and would bypass the cylinder's axis behind it.

[71]That is, C lies at the intersection of planes QLHK and EGBA and therefore lies in both planes. QL is, of course, the cathetus, so image-point C lies where it is intersected by the extension of line of reflection EB.

[72]In other words, since points T and H are equidistant from point Q, their reflection within their respective planes of reflection will be perfectly symmetrical, which means that angle of incidence TGN in T's plane of reflection will be equal to angle of incidence HAZ' in H's plane of reflection, and so will respective angles of reflection EGN and EAZ'. This in turn means that image-locations I and S will be equivalently situated within their respective planes of reflection, so that TI = HS, which means that straight line IS will be parallel to line TH, as well as to axis ZK and line of longitude AG. It bears noting, moreover, that the two catheti, TIX and HSP will both lie behind axis ZK insofar as the ellipse formed on the cylinder by plane of reflection TGE will tilt downward toward E, whereas the ellipse formed on the cylinder by plane of reflection HAE will tilt upward toward G, both ellipses being tilted at identical angles. It therefore follows from our analysis in note 70 above that both catheti will bypass axis ZK behind it.

[73]That the Latin text seems to be defective at this point was noted by Risner, who adjusted it to read *quia perpendicularis ducta a puncto C ad punctum sectionis*

lineae IS et superficiei circuli est valde parva ("because the perpendicular dropped from point C to the intersection-point of line IS and the plane of the circle [BF] is extremely small"). Unlike the reading in the Latin text, this adjusted one makes clear and obvious sense.

[74]This claim could be taken in two ways. On the one hand, it might be understood to mean that, if TH is slanted with respect to AGB within plane TOH, its image will become increasingly curved. On the other hand, it might be understood to mean that, if TH is slanted so as to lie in a different plane from ABG, its image will become increasingly curved. Both claims are true, but within the context of the proposition, the second alternative is more likely the one Alhacen had in mind. That image ICS of straight line TH is somewhat curved according to the conditions set in the proposition is empirically false, but Alhacen is forced to that conclusion by the cathetus-rule. According to his analysis, however, the image's curvature is virtually undetectable not only because it is so slight, but also because image ICS faces the eye directly so that it and the center of sight lie in the same plane.

[75]In other words, if the object-line is posed somewhat to the side of the normal dropped to the mirror from the center of sight, then a small portion of the nearer end of that line will be visible in the mirror through the opening between the object's endpoint and the aforementioned normal.

[76]In the hope of making this proposition easier to follow, I have provided three views of the construction. At the top is a three-quarter view from above, at the lower left is a side view from the left, and at the lower right is a bird's eye view.

[77]Angle MBL in triangle MBL = angle BED by construction, but angle BED = angle MEB in triangle MEB, and angle BMLE is common to both triangles. Therefore the third angles BLM and EBM, respectively, are equal, so, by Euclid, VI.4, the two triangles are similar, leaving their respective sides proportional. Accordingly, EM:BM = BM:ML. Since BM is the mean proportional between EM and BM, it follows from Euclid, VI.17 that EM,ML = BM^2.

[78]That angle BDM > angle ZDM can be shown as follows. From point M in the top diagram of figure 6.5.19, drop line M*x* perpendicular to CD, and from point *x* on CD erect perpendicular *xy* to intersect DB at point *y*. M*y* will therefore be the hypotenuse of right triangle M*xy* and will therefore be longer than M*x*. Since M*y* subtends angle BDM, while M*x* subtends angle ZDM, it follows that angle BDM > angle ZDM because it is subtended by a longer line. By the same token, since sides BD and DM of triangle BDM are equal to sides ZD and DM of triangle ZDM (BD and DZ being radii of the circle), it necessarily follows that side BM subtending larger angle BDM is longer than side MZ subtending smaller angle ZDM.

[79]It has just been established that MZD > MDZ (i.e., EDZ), and it was established at the end of 5.36 that MZL > ZED. Therefore, DZL, which = MZD + MZL, is greater than EDZ + ZED.

[80]In other words, if the form of point Q, which lies between F and B on line FNQB, were to reflect to E from some point Z′ on AZ above Z, the straight line connecting Q and that point would necessarily intersect line FZ, so the form of that intersection-point would reflect from both Z and Z′, which is impossible.

[81]In order to conserve space, I have not extended the two normals HU and TU to their actual intersection-point at U. Because of the confusion of lines, moreover, I have not attempted to represent normal TU in the top diagram of figure 6.5.19 and have thus not represented its intersection with reflected ray EG.

[82]Although all the manuscripts, as well as Risner, have *spericis* rather than *columpnaribus*, the comparison that follows is clearly between conical and cylindrical mirrors, not between conical and spherical mirrors. Moreover, in the introductory paragraph to chapter 9 on concave conical mirrors, the comparison is explicitly between those sorts of mirrors and concave cylindrical ones.

[83]As Alhacen explains in 4, 5.43-45 (Smith, *Alhacen on the Principles*, 341-342), if the plane of reflection cuts a conic section on the cone's surface, there can be one or at most two points of reflection on that section. There will be one only if the planar cut is perpendicular to the line of longitude, in which case the point of reflection lies where the axis, or major axis, of the conic section intersects the line of longitude. Otherwise, there will be two points of reflection, and each of them will lie at the intersection of a line of longitude and a line extended orthogonal to it from the section's focus, as determined by the intersection of the cone's axis and the section. In all cases, therefore, the appropriate reflection-point(s) will lie on lines passing through that focal point. As represented in figure 6.6.20, the section is an ellipse, point D is one of its foci, and point E is where line DE extended from that focal point to line of longitude AO is orthogonal to that line of longitude. If it is the only such point, then ED is the major axis of the ellipse. If it is one of two such points, then there will be a complementary point of reflection on arc FB of the ellipse lying the same distance as E from endpoint F of major axis FD.

[84] There is a general consensus among the manuscripts at this point in the text that the relevant proposition (*figura* in the Latin text) in book 5 is the nineteenth; there is no such consensus among them at the relevant spot in the text of book 5. Several, but not all, of the manuscripts give number-designations for the figures that accompany particular propositions. Among those that do (i.e., O, L3, E, and C1), O and L3 list it as 28 (see folios 65r and 74r, respectively), while E and C1 list it as 18 (folios 118r and 102r, respectively). This lack of agreement over the appropriate figure-designation reinforces the point made earlier in Smith, *Alhacen on the Principles*, cx, that such designations are totally unreliable as guides to the specific propositional structure of the *De aspectibus*.

[85]What Alhacen set out to do in this lemma was, of course, to show for convex conical mirrors what he showed for convex cylindrical mirrors in proposition 16, lemma 5: namely, that for a given point of reflection within a given plane of reflection, the normal, or cathetus, dropped from a given object-point will intersect the normal dropped through the point of reflection at some point outside the axis of the mirror. Thus, in figure 6.6.20, if H represents some object-point within the plane of conic section BFZ, and if HZ is normal to the section at point Z, then for some center of sight poised within the same plane on the other side of point E of reflection, H's image will lie at the point on ZX inside the conic section where the reflected ray meets cathetus HX.

[86]In other words, since RF is parallel to TZ, by construction, and since EF is parallel to OH, also by construction, and since OZ is a straight line within the same

plane as those two lines, then angle ZFR = alternate angle OZT, while angle FRZ = alternate angle TZR. But angles OZT and TZR are equal, by construction, so angles ZFR and ZRF are equal.

[87]To this point, proposition 22 is essentially a reprise of proposition 21 with some crucial differences that indicate a change of translators between the two propositions, a change that remains in effect for the remainder of book 6, as well as book 7. At the gross level, of course, there are certain instructions in this portion of proposition 22 that are missing in proposition 21, such as the formation of conic section BEG' on the cone's surface (paragraph 6.25) and the extension of line AU through the point at which HO intersects the circle passing through Z (paragraph 6.22). There is also the change in lettering between the two propositions, translator 2 substituting R and K in proposition 22 for C and Q in proposition 21. Another noticeable difference between the two is that in proposition 22 and the succeeding text of book 6, angles are often referred to by vertex-designation only (i.e., *angulus H*), whereas in the entire text of book 6 up to proposition 21, angles are invariably spelled out completely (i.e., *angulus AHZ*). Beyond these gross variations, moreover, there is a host of stylistic differences that indicate a change of translators. For one thing, translator 2's sentence-structure tends to be choppier than that of translator 1. Accordingly, he often strings together long successions of clauses beginning with "et," "ergo," and "tunc." Unlike translator 1, as well, translator 2 makes frequent and consistent use of the hortatory subjunctive (i.e., *extrahamus lineam AB*—"let us extend line AB") rather than the jussive (i.e., *extrahatur linea AB*—"let line AB be extended" or "extend line AB"). Most telling, however, are the differences in phrasing and vocabulary between the two translations. Whereas translator 1 occasionally uses the phrase *quocumque modo* ("randomly" or "at random"), translator 2 uses it more often, adding *sit* or *fuerit* (*quocumque modo sit/fuerit*). Although translator 1 uses the phrase "est sicut" (or simply "sicut" with the "est" understood) to link two proportions in a ratio (i.e., *AG ad AN sicut GI ad IN*), he almost never uses it to mean *est equalis* ("is equal [to]"). Translator 2, on the other hand, uses *est equalis* and *est sicut* interchangeably throughout the remaining portion of book 6. As far as simple vocabulary shifts are concerned, a few salient examples should suffice. Whereas translator 1 always uses *exterioris* to mean "convex" (e.g., *in speculis spericis exterioribus*—"in convex spherical mirrors"), translator 2 always renders "convex" as *convexus*. Whereas translator 1 uses the term *error* for "misperception," translator 2 uses the terms *deceptio* and *fallacia*. Whereas translator 1 uses the form *columpnaris* for "cylindrical," translator 2 invariably uses *columpnalis*. Whereas translator 1 never uses *nam*, translator 2 uses it frequently. Whereas translator 1 never uses *tunc*, translator 2 often interchanges it with *ergo*. Whereas translator 1 renders "chapter" as *pars*, translator 2 renders it as *capitulum*. Whereas translator 1 almost invariably renders "to reflect" and "reflection" as *referre* and *reflexio*, translator 2 overwhelmingly prefers *convertere* and *conversio*. And finally, whereas translator 1 always renders "section" (as in "conic section") as *sectio*, translator 2 always renders it as *sector*. For further discussion of this issue, see the section on manuscripts and editing, pp. xlv-xlviii above.

[88]Because the construction that follows is so difficult to represent in three dimensions (as in the top diagram of figure 6.6.22a), I have provided a bird's eye view of the construction for the plane of reflection containing R, E, and N in the lower diagram, that view being from directly above the plane of the conic section.

[89]This follows from proposition 20, where it is demonstrated that the normal to point C will fall between CD and CZ', both of which lie in the plane of the conic section. Hence, angle DCZ' must be greater than a right angle.

[90]The Latin text I have translated as "and if line AON lies on some visible object" actually reads *et [si] linea longitudinis fuerit in aliquo visibili* in the majority of the manuscripts. Translated literally (i.e., "and if the line of longitude lies on some visible object"), this phrase makes no sense, so I have adjusted it accordingly. Risner went even further, rephrasing the Latin to read *et [si] forma alicuius visibilis reflectatur a linea longitudinis*, but this adjustment is more extreme than necessary. That P is in fact the image of O follows from the fact that lines ZH and UH are equipoised within circle ZU such that they both intersect that circle where it intersects the conic section formed on the cone by plane of reflection OZR, which contains those two lines. Thus, cathetus OUH is normal to that conic section, just as ZH is normal to it, by construction.

[91]Alhacen thus uses the same reasoning here that he used in the case of convex cylindrical mirrors to explain why the image, although curved, appears essentially straight: i.e., that its convexity faces the eye directly; see note 74 above.

[92]The notion of "compound misperceptions" (*fallacie composite*) crops up later in paragraph 7.108, p. 221. This presumably refers to the full complement of individual misperceptions that occur as a group in reflection. One group of such misperceptions is due to reflection itself, regardless of the shape of the reflecting surface—i.e., misperceptions arising from the intrinsic weakening of reflected light and color as well as from the mingling of the mirror's color with that of the object's form. The other group is specific to the shape of the reflecting surface and therefore involves not only distortions of shape, size, number, and distance, but also image-reversal or inversion. Note, by the way, that the term *fallacia* is used here for "misperception." This term is apt, because, as Alhacen explains it in book 2, chapter 3, the act of perception involves a low-grade syllogistic or deductive process. If the conclusions drawn from that process are false, then a fallacy occurs.

[93]The Latin text here reads *in omnibus speculis convexis et superficialibus*, which is open to interpretation because of the phrase *et superficialibus* ("and to superficial [mirrors]"). Given the context, I have chosen to intepret "superficial" as "plane."

[94]See paragraph 7.6 below for Alhacen's understanding of "the arrangement of parts" and its distortion in concave mirrors.

[95]See book 5, proposition 32, in Smith, *Alhacen on the Principles*, 446-449.

[96]The situation described here is a gross approximation of that in which an eye looks at itself in the mirror when the eye's surface lies between the center of curvature and the mirror's surface. Thus, MN can be taken to represent a cross-section of that surface within the plane of great circle BUG on the mirror.

[97]That the entire image falls outside arc BG follows from the fact that the image of any point on line MN will lie beyond the reflecting surface. Thus, as illustrated in figure 6.7.23, if some point X is chosen randomly on segment TN of that line, its form will reflect to T from point R along line of reflection RT. When that line of reflection is extended, it will meet cathetus AX well outside the mirror, and the closer to T the point that is chosen, the farther beyond the mirror's surface its image will lie.

[98]In other words, the planes containing triangles KGA and GBA will cut great circles on the surface of the sphere from which the mirror is composed.

[99]In other words, if LH and TK were two facing objects, with O a viewer poised between them, then from O's perspective when facing object KT, point K would be seen on the object's left-hand side, and point T would be seen on its right-hand side. By the same token, if the viewer turned to face LH, point L, which is the image of K, would be seen on the left-hand side of LH, and vice-versa for object-point T and its image H. Thus, object and image would correspond perfectly in their respective left-right orientations, and they would both be upright.

[100]I take the sense of this peculiar phrase *in linea in qua est de lineis radialibus* to be that, as before, if NU and MR were to represent facing objects, with center of sight O between them, then if O were to look directly at NU, rays OU and ON along which NU would be seen would correspond in left-right orientation to rays OM and OR along which MR would be seen.

[101]Line HTZ is in fact meant to represent a line on the surface of the eye, T presumably lying at the center of the pupil. Being straight, that line cannot actually be on the surface, so it is to be taken as a virtual rather than a real representation of the situation. As it turns out, point T's only function in the analysis is to anchor points H and Z at equal distances from line DE along a line perpendicular to DE. Suffice to say, this proposition is intimately related to proposition 23, where cross-section MTN in figure 6.7.23, p. 132 is equivalent to cross-section HTZ in the figure for this proposition—the main difference between the two situations being in the placement of that cross-section with respect to the center of curvature.

[102]Although somewhat vaguely put here, the point of this stricture is clear enough: A and H must be disposed in such a way that the angle formed by incident ray AH and reflected ray AE contains centerpoint G of the mirror so that the normal AG will bisect that angle. The crucial thing, in fact, is that the object-point and the center of sight flank the mirror's center of curvature—a point made earlier by Alhacen in book 5, paragraph 315 (in Smith, *Alhacen on the Principles*, 448-449).

[103]That GH > GK follows from the fact that AG bisects angle HAK in triangle HAK and cuts base HK at G. Thus, by Euclid VI.3, HA:AK = GH:GK. But HA > AK, so GH > GK. The same reasoning applies to triangle ZBL and the bisection of angle ZBL by GB, which cuts base ZL at G.

[104]The situation envisioned in this proposition is essentially the same as that in propositions 25-27, where the center of sight is posed behind the visible object (in this case the surface of the eye) so that the resulting image is formed between the center of sight and the reflecting surface and thus appears diminished and inverted.

[105]That FK > KA follows from the fact that triangle EFA is isosceles, because sides EF and EA are radii of the circle, so base angles EFA and EAF are equal. Therefore, since FK intersects EA between E and A, angle KFA < angle EAF. Consequently, in triangle KFA, side FK subtends a larger angle than side KA, so, by Euclid, I.19, FK > KA, and *a fortiore* it is longer than KE, because KA > KE by construction.

[106]In other words, we know that angles FEK, EFK, and FKE = two right angles. But angle FEK < angle EFK, so FKE + 2 EFK < two right angles. Since EFG = EFK, by construction, it follows that EFG + EFK = 2 EKF, so FKE + EFK + EFG < two right angles, from which it follows that EK and FG will intersect on the side of G.

[107]In this case, the citation of "figures 27 and 28" applies to actual figures that accompany the relevant section of book 5, chapter 2, which consists of proposition 34 in Smith, *Alhacen on the Principles*, 450-451. Here again, we encounter some discrepancy between the citation in the text at this point and the denomination of the figures in book 5. In O and L3, the figures are numbered 38 and 39 (folios 69r-v and 78r, respectively), whereas in E and C1 they are numbered appropriately as 27 and 28 (folios 106v-107r and 124r, respectively). Thus, the enumeration in E and C1 accords with the citation at this point in the text, although as we saw in note 84 above, this was not the case earlier. The point of book 5, proposition 34 is to demonstrate that, if the visible point and the center of sight lie on intersecting diameters inside a great circle on the mirror, there can be reflection from arc AD subtended by angle AED and from arc OB subtended by angle OEB on the opposite side of the circle in figure 6.7.29, but not from arcs DO or AB. Since in this case the mirror does not extend beyond O or B, according to specification, arc OB on the opposite side of arc AD is irrelevant, leaving arc AD as the only possible area of reflection in great circle BADO.

[108]The relevant theorems in this case are propositions 39 and 40 in Smith, *Alhacen on the Principles*, 459-60. As to numerical designations in the manuscripts, L3 fails to give any at this point, but O numbers them 47 and 48 (folio 71r). E and C1, on the other hand, label them in accordance with the citation here—i.e., as 35 and 36 (folios 127v and 109r, respectively). The ultimate reason that there can only be one reflection on arc AD in figure 6.7.29 is that object-point R and center of sight Z are poised on their respective diameters such that angle REZ facing arc AD is greater than two right angles, whereas if it were less than two right angles—as would be the case with respect to the missing portion of the mirror on arc BO—there could be as many as three points of reflection.

[109]The text is problematic at this point because "K" evidently designates two different points along the line extended to the mirror from G. In the initial part of the paragraph, it designates the point where that line intersects circle EGZ, whereas later it evidently designates the point where that line intersects the mirror's surface. That being so, the point of the argument here is clear. Since angles EKG and GKZ subtend equal arcs in circle EGZ, they are equal, so it follows that angles EK'G and GK'Z in circle ABD are not equal. Moreover, in the two triangles EK'K and ZK'K, angles EKK' and ZKK' are equal, whereas angle K'EK < angle K'ZK, so it follows that angle EK'K (i.e., EK'G) > angle KK'Z (i.e., GK'Z).

[110]The argument here can be easily understood by recourse to figure 6.7.31b, p. 139, which is abstracted from figure 6.7.31a. If we think of PH as a hinge, and

arc PK'H with constituent lines GNQ and EK'Q as a flap, then if we rotate that flap upward on PH until lines GNQ and EK'Q assume the respective positions GCS and EK"S, all the constituent points on the flap will remain in place. Thus, C corresponds perfectly to N, K" to K', and S to Q, so if the form of N reflects to E from point K' on arc PK'H, the form of C will reflect to E from point K" on arc PK"H. By the same token, if point Q is the image of N within plane of reflection EGQ, point S will be the image of C within plane of reflection EGS. And the same holds by symmetry for point R within plane of reflection EGO.

[111]As presented in the manuscripts, this proposition is marred not just by a welter of conflicting letter-designations, but also by a few complete mis-designations. The gist of the theorem is evident enough, however. In the construction at the beginning, Alhacen determines the image-locations for the three points of reflection A, B, and D in the arc on the mirror subtended by angle EGZ. The construction itself is based on the simplest possible case, in which center of sight E on diameter EG and object-point Z on diameter ZG are equidistant from center of curvature G and lie closer to the mirror's surface than they do to G. On that basis, as we have seen earlier, the points of reflection are easily determined by the intersection of circles EGZ and ABD. Once he has established the three image-locations F, M, and L for points of reflection A, B, and D, respectively, he adds the reflection for N on line MG from point K' and then determines its image-location Q on that same line.

The next phase of the analysis is based on passing a plane perpendicular to circle ABG along line MG, selecting points C and R within that plane, and determining the image-locations of C and R with respect to Z and N within that plane. On that basis, Alhacen shows that, if the viewer confines his view to the portion of the mirror limited to the arc containing points D and K' of reflection, the images SQO and SLO of lines CZR and ZNR, respectively, will appear concave to the center of sight at E.

In the final phase of the analysis, Alhacen determines the fourth point I from which the form of Z will reflect to E from the arc on the opposite side of ABD and locates the resulting image at T'. Then, if the viewer faces the entire portion of the mirror defined by arc ABDI, line CZR will have four images, all of them containing image-points S and O of points C and R, and each of them differentiated by whether the resulting image-line passes through point M, point L, point T', or point F. Whichever the case, that line is concave with respect to the center of sight, hence the conclusion—which is limited to the analysis of line CZR—that straight lines have several concave images.

[112]Contrary to the apparent sense of the Latin wording here, angle EDG is meant to be several times smaller than angle ADE rather than the difference between ADG and ADE, which is, after all, EDG itself.

[113]This follows from the fact that angle ZDK = angle KDT + angle ZDT. But ZDK is exterior to triangle HDZ, so, by Euclid, I.32, it is equal to the sum of the opposite interior angles DZH and DHZ. But DZH = KDT, by construction, so remainder ZDT of external angle KDZ = remainder ZHD of the two opposite interior angles of triangle ZHD.

[114]LZH = BDK, by construction. But BDK = BDT + KDT, and LZH = LZD + DZH. Therefore, since DZH = KDT, by previous conclusions, LZD = BDT. Consequently,

LZD + BDZ = BDT + BDZ = TDZ, which is less than two right angles, so LZ and DB will intersect on the side of BZ.

[115]That angle LMD is right follows from the equality of triangles DML and DHL and therefore the equality of their respective angles. Thus, as we established earlier, angle LHD in triangle DHL is right, so its corresponding angle DML in triangle DML must be right as well. From that fact it follows that the circle must pass through M, since it forms a right angle on diameter DL.

[116]The rationale behind this conclusion is as follows. From Euclid, III.27, we know that angle FRM = angle FDM, since both angles lie on the circumference of circle FDZ and are subtended by the same arc FM. From Euclid,VI.33, moreover, we know that the angle with its vertex at the circle's center and subtended by a given arc on the circle is twice the angle with its vertex on the circle's circumference and subtended by the same arc. Therefore, the angle at the center of circle FDZ subtended by arc FM is twice angle FDM, whose extension to circle ABG is FE. But angle FDE is at the center of circle ABG. Therefore, in relation to circle FDZ, arc FM is double arc FE in relation to circle ABG—or, to put it another way, arc MF in circle FDZ occupies twice as much on the circumference of that circle as arc FE does on the circumference of circle ABG.

[117]The relevant theorem is book 5, proposition 33, in Smith, *Alhacen on the Principles*, 449-450, which is numbered 26 in C1 and E (folios 106v and 123v, respectively) and 36 in O and L3 (folios 69r and 77v, respectively). The logic underlying these proportionalities is as follows. Let U and O in figure 6.7.32a, p. 141, be object-points whose forms are reflected to center of sight H from points F and B, respectively, on the mirror. Q, which lies on the extension of line of reflection HB will be the image of O, and N, which lies on the extension of line of reflection HF will be the image of U. QD is therefore the cathetus dropped from both image-points. Draw tangent BT from B to intersect QD at T, and draw tangent FT′ from F to intersect QD at T′. According to Alhacen's designation in book 5, proposition 6 (Smith, *Alhacen on the Principles*, 403), T is the endpoint of tangency for reflection-point B, and T′ is the endpoint of tangency for reflection-point F. Accordingly, in proposition 33 of the fifth book, Alhacen demonstrates that DO:DQ = OT:TQ and, by extension, that ND:NU = T′N:T′U. Therefore, by reversal of terms, it follows that DQ:DO = TQ:OT and NU:ND = T′U:T′N. From the equal-angles law, however, it follows that tangent BT bisects angle QBO, and tangent T′F bisects angle NFU. By Euclid, VI.3, then, QB:BO = TQ:OT, and NF:FU = T′N:T′U. Consequently, QB:BO = DQ:DO (which = TQ:OT), and NF:FU (which = T′N:T′U) = ND:DU.

[118]The point here is illustrated in figure 6.7.32c, p. 142, for points U and O within the plane of H′DQ. That plane cuts arc A′G on the mirror, and within this arc the form of O will reflect to H′ from point B′. Accordingly, Q will be the image of O within this plane. Likewise, the form of U will reflect to H′ from point F′, and its image will be N.

[119]In other words, since points Z and E lie on arc ZOE, they lie the same distance from the mirror's surface in the plane of circle ABG as does O. Consequently, their forms will reflect from a point on the respective arcs on the circle within their respective planes H′DZ and H′DE that is equivalent to point B′ on arc A′G in figure

6.7.32c, and that point in turn is equivalent to point B on arc AG of the original circle in figure 6.7.32.

120All seven manuscripts, as well as the Risner edition, designate the arc that is cut as "UOE," but the sense of the passage clearly indicates that the arc in question is ZOE.

121The construction of arc RUF according to the circle centered on M with a radius MU is somewhat puzzling. It would have made more sense to locate M between N and U and then draw the arc with radius MU, because in that case the two intersection points R and F on arc ZOE would have flanked U at equal distances, leaving concave arc RUF uniformly disposed with respect to DQ.

122The intent of this final phrase is far from clear. Although all the manuscripts have *eodem modo quo in compositis* (or *incompositis*), Risner alters the reading to *eodem modo quo incompositae*, so that the sentence would translate to: "Moreover, compound misperceptions occur in these mirrors the same way as uncompounded [ones]." I take the intent here to be that, as in the other types of mirrors, so in these types, compound misperceptions arise in the same essential way; see note 92 above for a brief explanation of compound misperceptions.

123The list of lines intersecting at O varies among the manuscripts, but it is quite clear from proposition 17, pp. 190-192 above, that lines HA, BQ, and TG intersect at O. Three of the manuscripts (O, L3, and E) include TK in this list, and three of them (F, P1, and S) include EK, as does Risner. The problem is that point K lies on the cylinder's axis below D, and EO, which forms the base of isosceles triangle EBO, lies in the plane of circle BF, which is intersected by the axis at point L. Therefore, line EO lies outside the circle (and thus the axis), and point K lies below the plane that includes TU, TZ, TGO, and EO. Accordingly, TK cannot possibly intersect EO, and EO cannot possible pass through point K on the axis. For those reasons, I have substituted EO for all the readings at that point in the text.

124The line of reasoning to this point in the paragraph, as well as for much of the analysis that follows, is puzzling, because CU will never intersect line SI, much less bisect it. This point becomes obvious in light of figure 6.8.33a, p. 145, which represents the situation in figure 6.8.33 from a bird's eye perspective in the top diagram and from a directly facing perspective in the bottom diagram. Let the circle in the top diagram be the top one in figure 6.8.33 that passes through G, which is the point from which the form of T reflects to center of sight E. Let NGZ be normal to point G, and let it intersect the axis of the cylinder at centerpoint Z of the circle. According to the cathetus-rule, then, I, where cathetus TU (which bypasses the axis) intersects the extension of reflected ray EG, will be the image of T from the perspective of E. From the bird's eye perspective of figure 6.8.33a, moreover, points Q and H lie directly below and in line with T, point S lies directly below and in line with I, points B and A lie directly below and in line with G, and points L, D, and K on the axis lie directly below and in line with Z. Accordingly, the form of Q will reach the mirror along line QB directly under and in line with TG and will reflect to E along BE directly under and in line with GE. Since the plane of reflection for Q includes the circle passing through B, the cathetus dropped from Q to the mirror will intersect the axis along line QL directly under and in line with TZ. Q's image C will therefore lie at the intersection of this line and line of reflection EB. H's image S, finally, will lie directly below and in line with I. Consequently, straight line SI will lie to the left of C, and the entire image SCI will be oriented with its convexity directly facing center of sight E. SI, in the lower diagram, will therefore be bisected at F by the extension of reflected ray EB. Furthermore, it has been established that both catheti TU and HU will intersect at U, as will EO. CU will therefore intersect reflected ray EBCS obliquely, so it cannot possibly intersect SI.

[125]Again, by recourse to figure 6.8.33a, p. 145, it is true that Q lies in the plane of triangle CUE, because that is the plane in which Q's form reflects to E. It is also true that C lies in the triangle CEI, so it follows, as claimed, that C lies on straight line EB. Yet, as we just saw, the claim at the beginning of the paragraph that CU bisects SI is false, because it does not intersect it anywhere. EC does bisect SI, so bisection-point F will lie on that line. Accordingly, from the bird's eye vantage of figure 6.8.33a, it will lie below and directly in line with I.

[126]In other words, since it has already been established that the plane containing HUT bypasses the axis, then, if we assume that the axis somehow intersects line HU at some point, that point will have to be D, where the plane of reflection containing HU intersects the axis. Hence, within that plane, HU will have to both intersect and bypass the axis at the same point.

[127]See Smith, *Alhacen on the Principles*, xxxix-xliv for a summary account of how the number of possible images will vary from one to four depending on the placement of the object-point and center of sight within a great circle on a concave spherical mirror; that account includes citations of specific relevant propositions in book 5, chapter 2. Moreover, on the basis of this analysis, Alhacen goes on in proposition 52, ibid., 478-481, to show that, when the center of sight and the object-point lie in a plane that cuts an elliptical section on the surface of a concave cylindrical mirror—as is the case for both T and H with respect to O—there can be as few as one and as many as four possible images of the object-point.

[128]The possibility of there being multiple images of C depends on the placement of center of sight O. If it lies outside the great circle of reflection, as in figure 6.8.33, then clearly there can be only one image. As to where the image of C might lie on line of reflection OB or its extension along BQ, that depends on C's placement with respect to the mirror's surface.

[129]Presumably this qualification is meant to account for situations in which C is displaced from its original position in such a way that image TQH of object-line SCI is convex or concave within plane TOH. In that case, of course, the convexity or concavity of the resulting image will face the eye directly, so it will be far less clearly perceptible than it would be if viewed from the side.

[130]As Alhacen points out, if we take SCI as the visible line, then it is possible for the points on it to have up to four images, depending on the placement of those points with respect to the center of sight. Thus, as illustrated in figure 6.8.33a, p. 145, the original case based on convex object-line SCI and center of sight O yielded straight-line image TQH. However, depending on SCI's placement with respect to O within the mirror, the three points S, C, and I could have as many as four images, in which case line SCI could have as many as four images—all of them distributed

in various ways with respect to center of sight O (i.e., behind the reflecting surface, as with image TQH; between the center of sight and the reflecting surface; at the center of sight itself; or behind the center of sight). Moreover, if we displace C from its original position while leaving points S and I unchanged, and if we focus on the original images T and H of I and S, then there can be as many as four separate images. Take the simplest case, in which the center of sight and endpoints S and I of the object-line are equidistant from the center of curvature. Thus, as represented in figure 6.8.33b, let C be shifted to C′ so that C′L = OL and so that the circle passing through C′, L, and O intersects the circle of the mirror at points B_2 and B_3. The form of C′ will therefore reflect from those two points, as well as from B and B_1. Accordingly, the images of C′ will be distributed as follows: the image for reflection from B will be C'_1, where reflected ray BO intersects cathetus C′L behind O; the image for reflection from B_1 will be C'_2, where cathetus C′L intersects reflected ray B_1O; the image for reflection from B_2 will be C'_3, where cathetus C′L intersects reflected ray B_2O; and the image for reflection from B_3 will be C'_4, where cathetus C′L intersects reflected ray B_3O. All of the resulting images are magnified insofar as cross-section TH common to all of them is longer than cross-section SI of object-line SC′I. However, SC′I can have as many as sixteen images when S and I are posed to reflect from four points on their respective great circles, and when the four reflection-points are equivalently situated within those respective great circles.

[131]Point K in this proposition corresponds to point Q in proposition 19. In paragraph 5.34 of that proposition (p. 194 above), Q is located where the plane containing E and the cylinder's axis—i.e., plane EX′D in figure 6.8.34a—intersects TH. Since the remainder of the proof from this point applies to that plane, I have supplied a view of it from the side in the diagram at the bottom of the figure in order to make the steps of the construction and analysis easier to follow.

[132]The reason for establishing that MIA is acute is to establish that any line from M that intersects AN below N, and thus below line ENL, will form an acute angle with AN. Consequently, any line drawn from that point to L will form an acute angle with AN on the opposite side. From this it follows that there must be some point on AN below line ENL from which the form of M will reflect to L at equal angles.

[133]That is, since angles FSQ and FMQ are equal, their respective alternate angles LFX and MFX will be equal.

[134]In order to make the construction and analysis that follows easier to visualize, I have represented the ellipse ABG in the middle diagram of figure 6.8.35, since the relevant lines and angles are all formed within its plane.

[135]The proposition to which Alhacen is referring is proposition 33, pp. 221-224 above. More appropriate to the point Alhacen is making here, however, is the analysis of concave cylindrical mirrors in book 4, chapter 5, paragraph 5.56 (Smith, *Alhacen on the Principles*, 344), where he shows descriptively that for any plane of reflection forming an ellipse on a concave cylindrical mirror there are only two possible points of reflection, and they lie at the endpoints of the ellipse's minor axis, which is the common section of the ellipse and the circle passing through those points.

[136]Alhacen's point here is that the reflected ray DG is parallel to the cathetus BL dropped from the point-object L to the mirror, so there is no intersection of the two lines and, therefore, no definite image-location. In paragraph 2.312 of book 5 (Smith, *Alhacen on the Principles*, 448), Alhacen claims that in such cases the image appears at the point of reflection, which in this case is G.

[137]The reference here is to book 5, propositions 39 and 40 (Smith, *Alhacen on the Principles*, 458-460), where Alhacen demonstrates not only that the form of K will reflect to D from a point on arc RC, and only arc RC, but also that there can be only one such point of reflection.

[138]In other words, just as Alhacen reversed the analysis in proposition 19 for proposition 33, here he is reversing the analysis in proposition 22. In that proposition he establishes that the form of line AON, in figure 6.9.37, p. 152, reflects from line of longitude AZE to center of sight R, which faces the convex surface of the mirror, and yields image APY, which is a curved line inside the mirror. Accordingly, the image of A at the cone's vertex coincides with A itself. The form of O, meanwhile, reaches the mirror along incident ray OZ, reflects to center of sight R along ray ZR, and appears at P, where reflected ray RZP intersects cathetus OPH inside the mirror. And the form of N reaches the mirror along incident ray NE, reflects to center of sight R along ray ER, and appears at Y, where reflected ray REY intersects cathetus DYN inside the mirror. Therefore, if we shift the center of sight from R to F and let curved line APY be the visible object, the form of that line will reflect to F from line of longitude AZE and will yield straight line ANO as its image. Accordingly, the image of A coincides with A itself. The form of P, for its part, reaches the mirror along incident ray PZ, reflects to center of sight F along ray ZF, and appears at O, where reflected ray FZO intersects cathetus HPO. And the form of Y reaches the mirror along incident ray YE and is reflected along ray EF, so its image will lie at N, where line of reflection FEN intersects cathetus DYN.

[139]Whether its image is straight, convex, or concave will of course depend on the curvature of AY, which in turn depends on where P lies within plane APY.

[140]In other words, if the center of sight could see those objects face-on, their right-hand sides would correspond to the right-hand side of the image that faces the eye.

FIGURES FOR INTRODUCTION AND LATIN TEXT

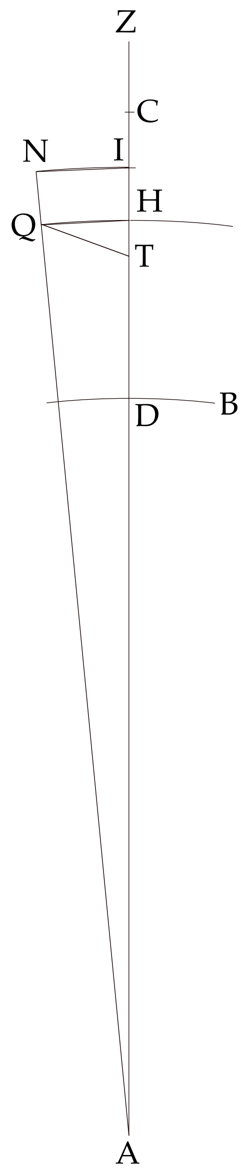


figure 1

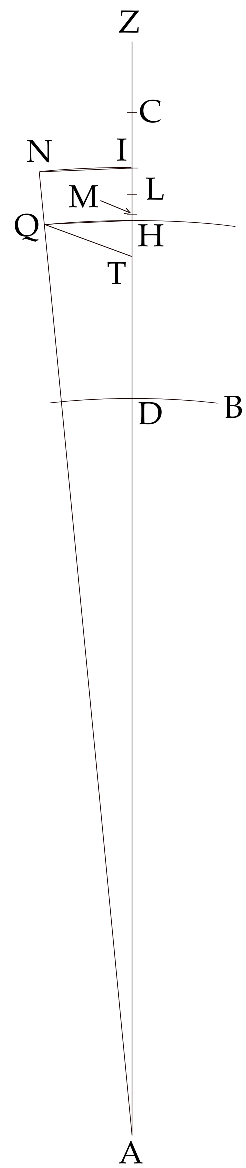
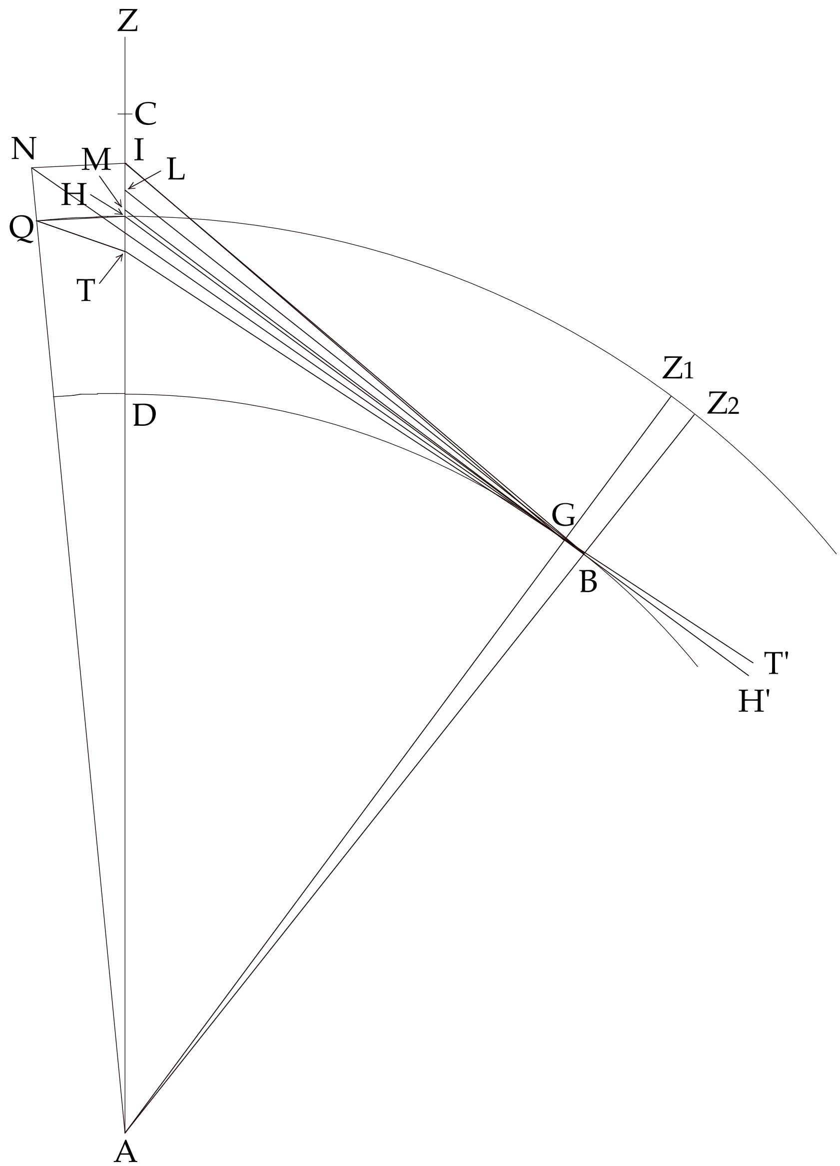


figure 1a



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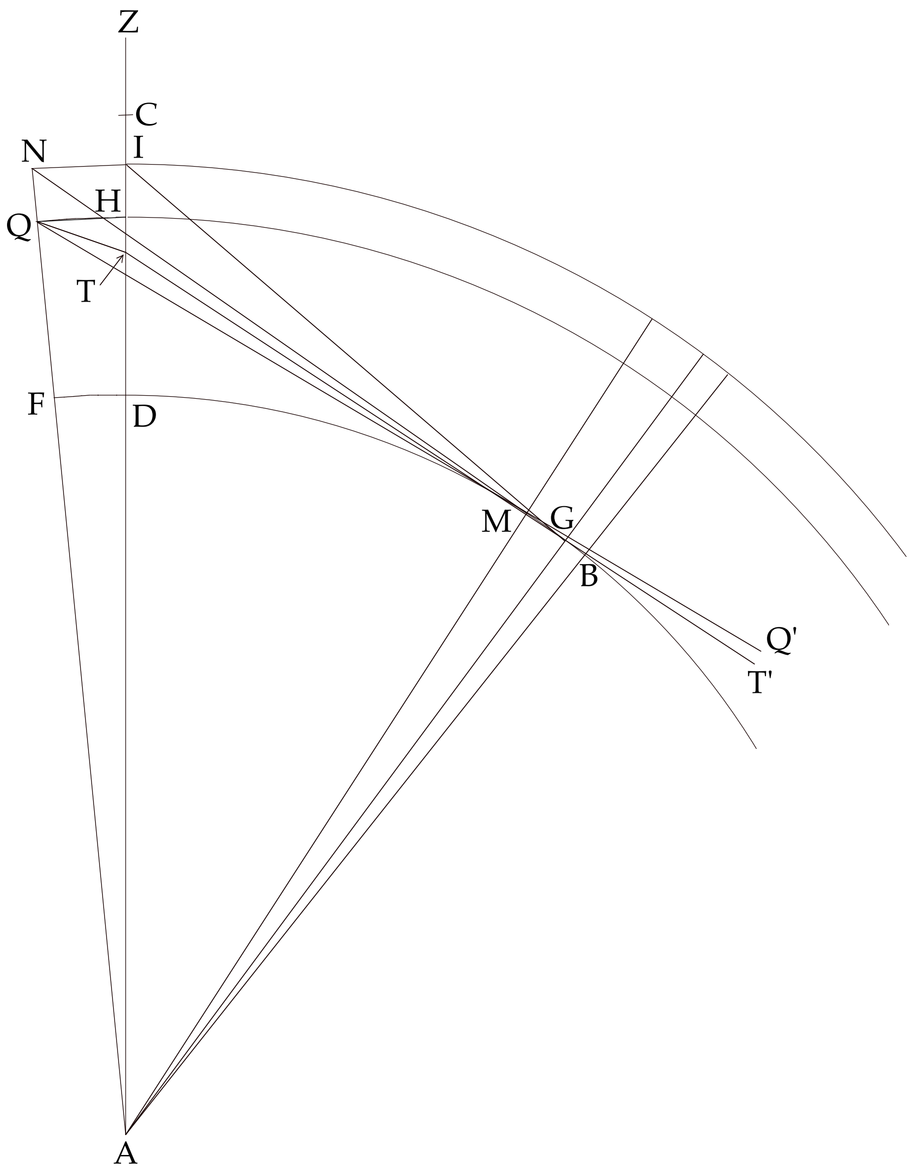

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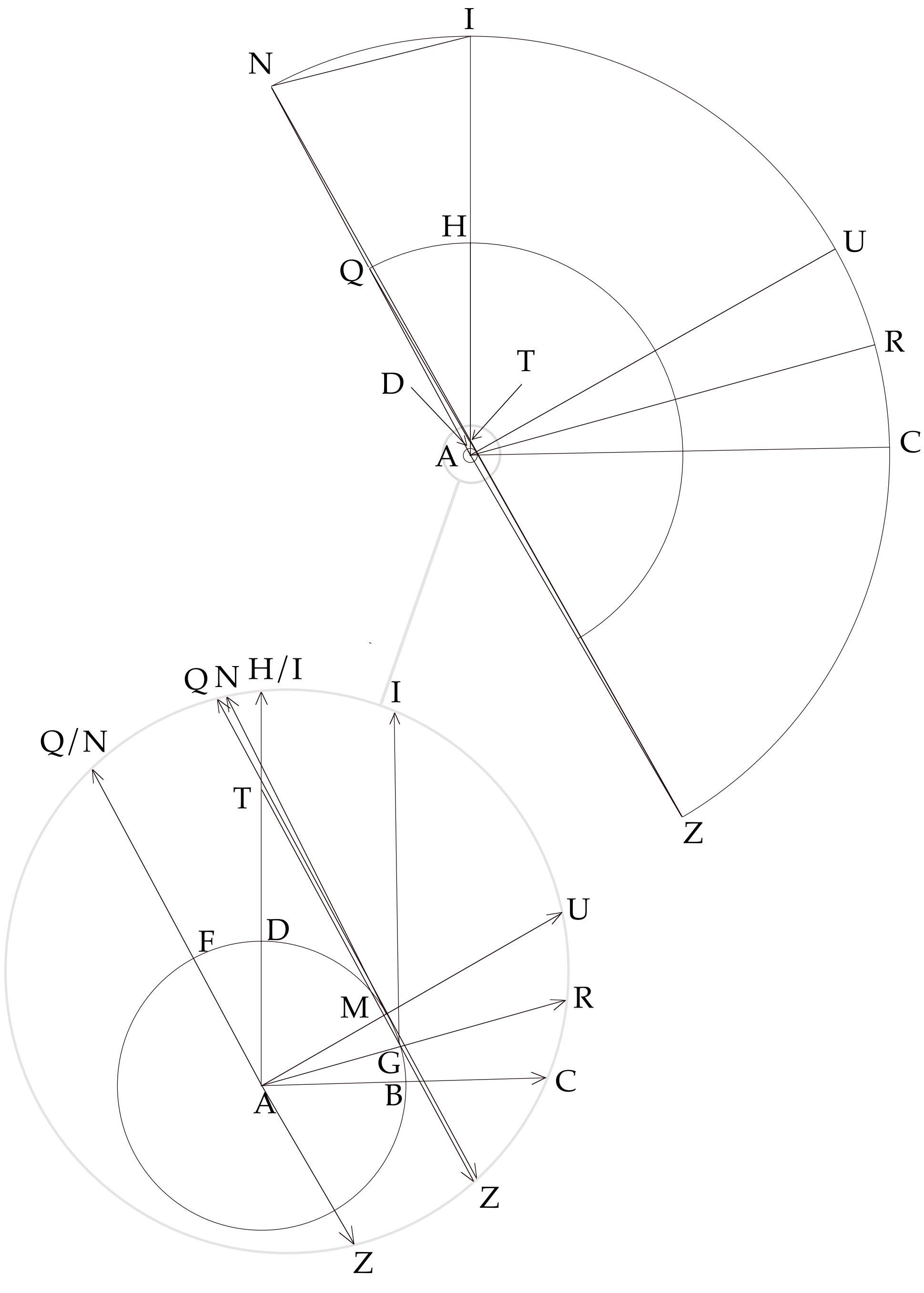


figure 4

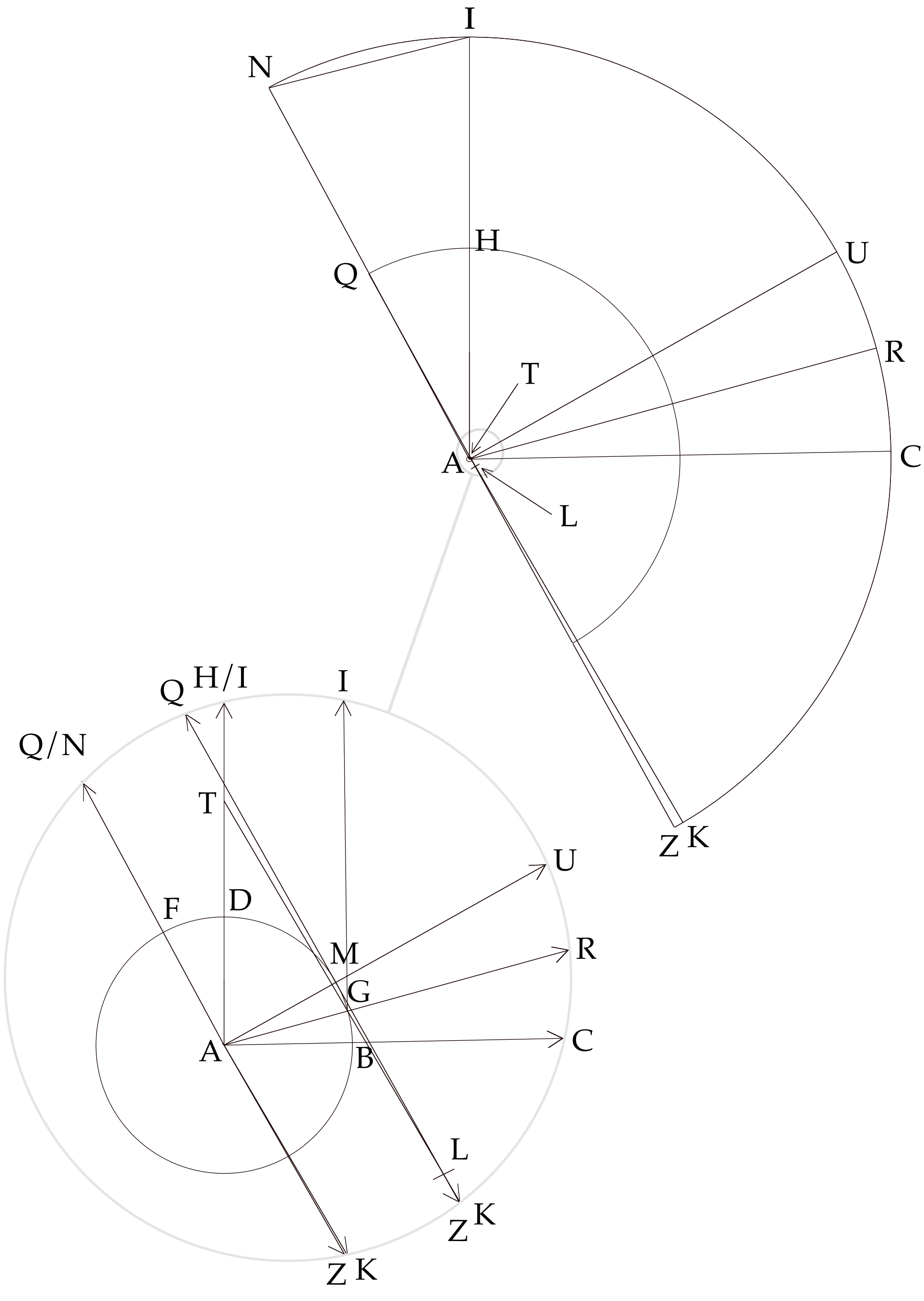


figure 5

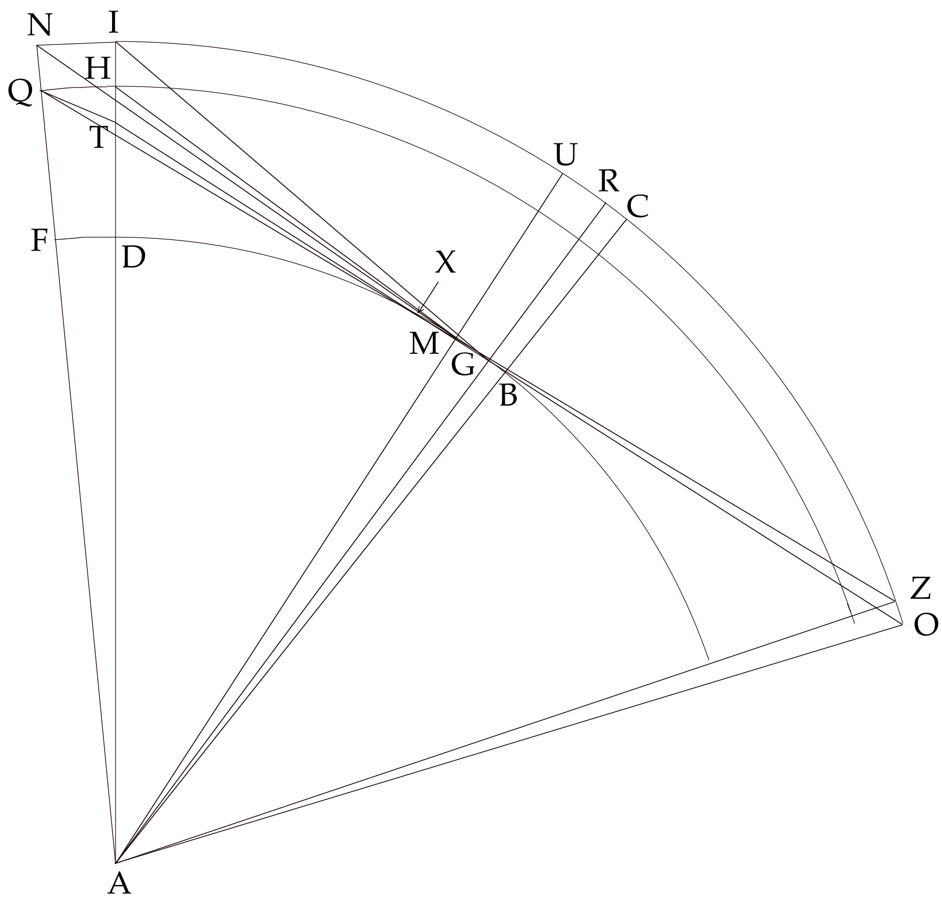
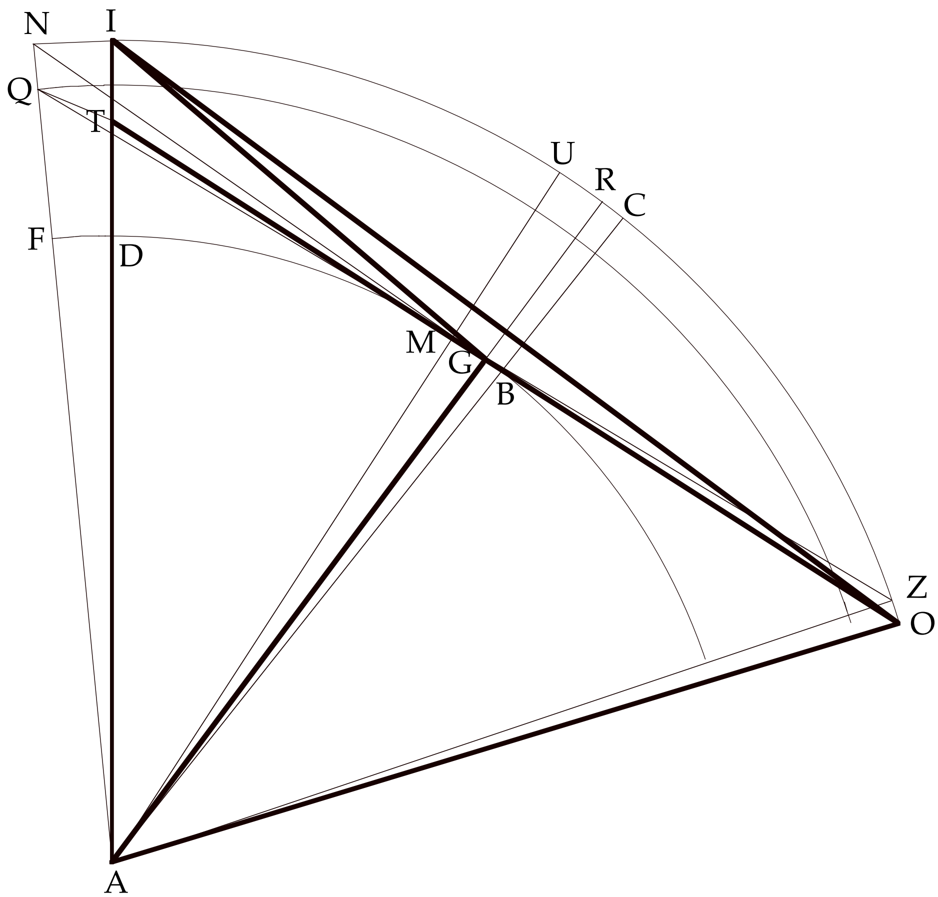


figure 6



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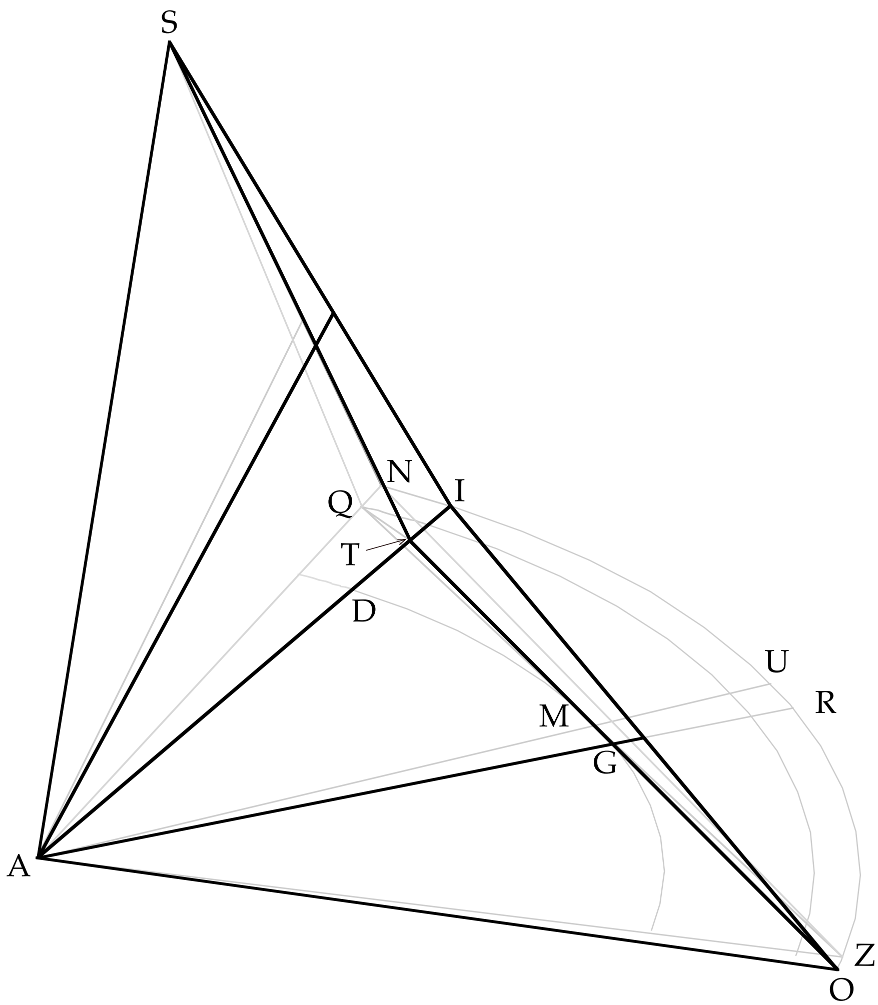

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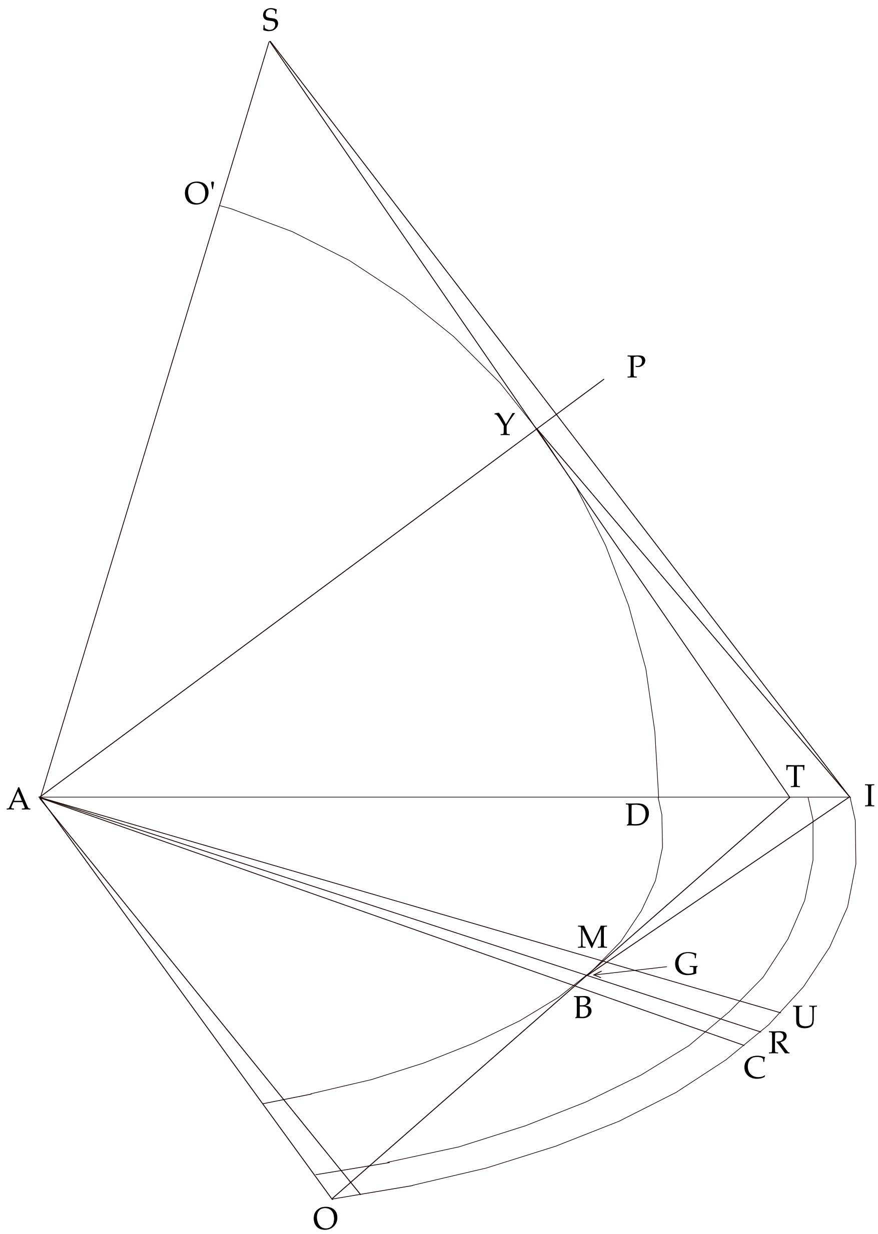


figure 9

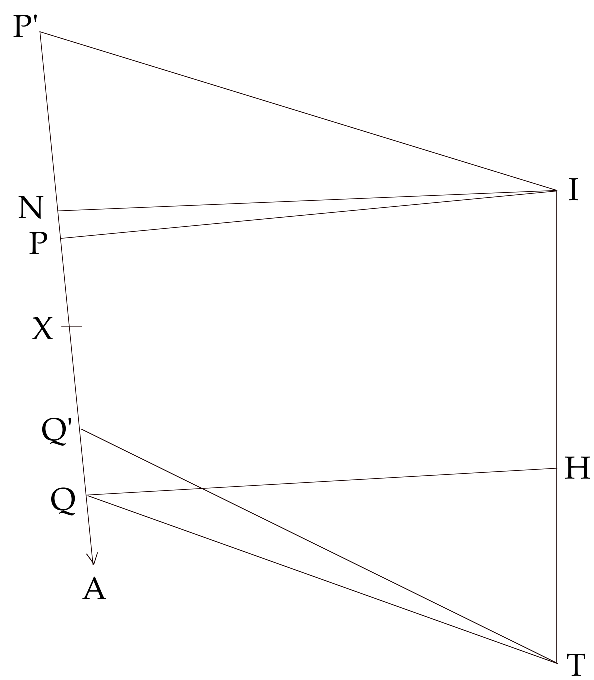


figure 10

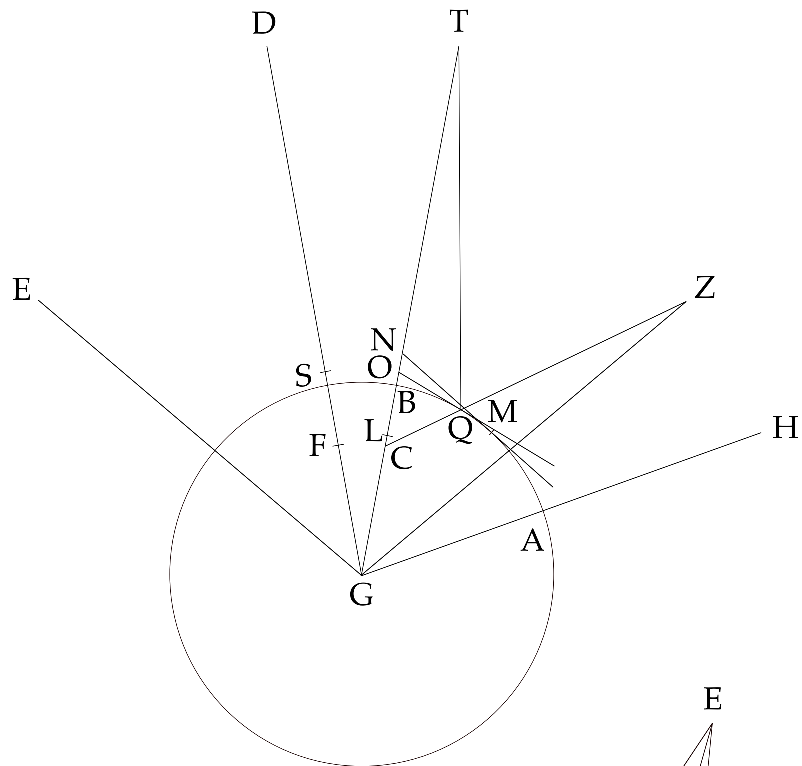
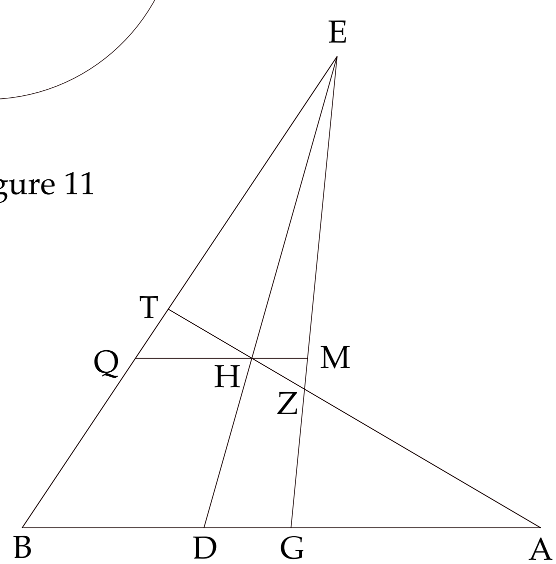


figure 11



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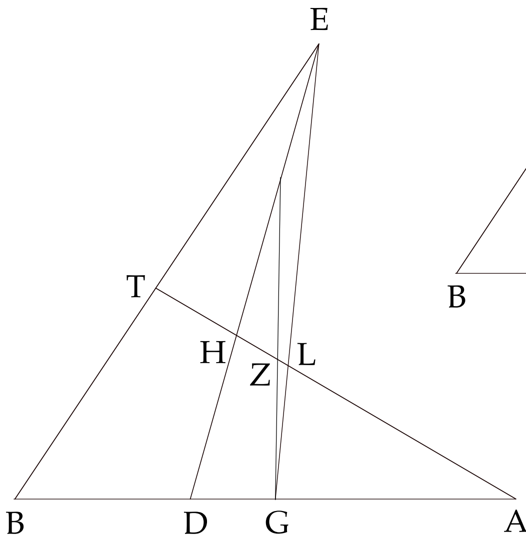


figure 13

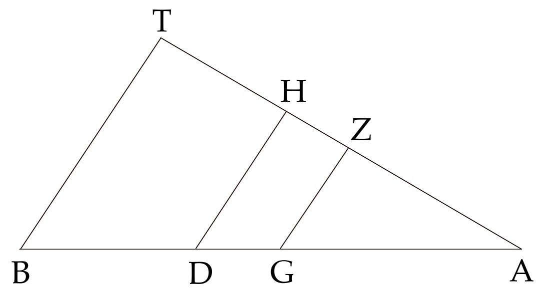


figure 14

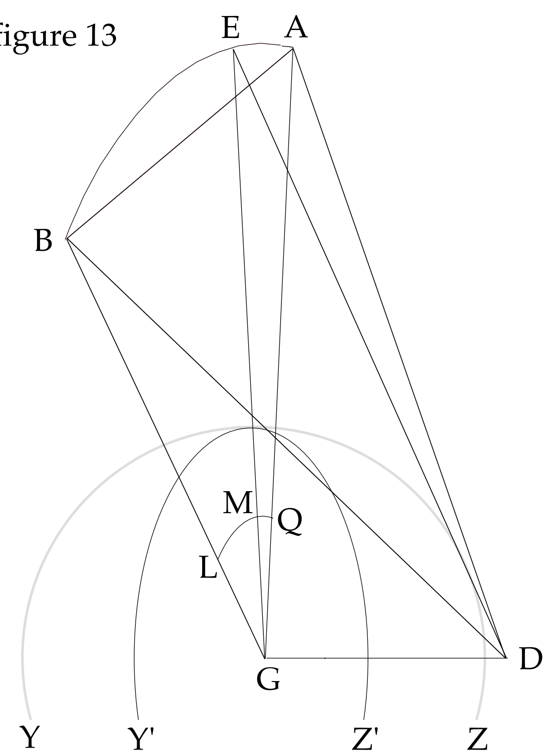


figure 15

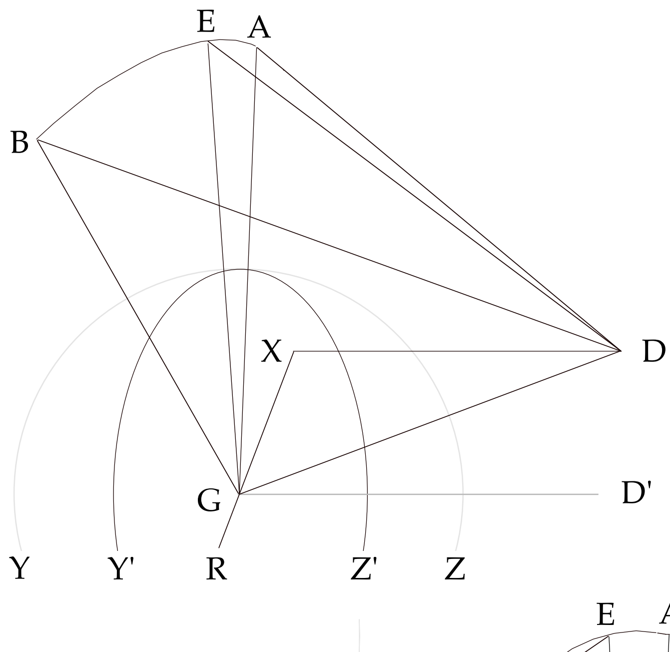


figure 16

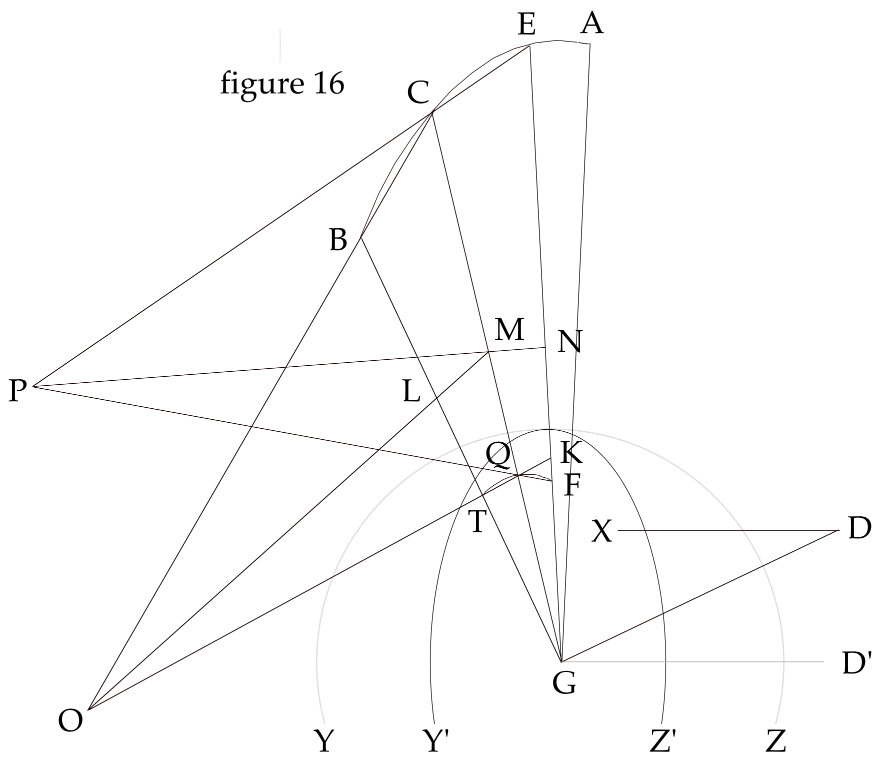


figure 17

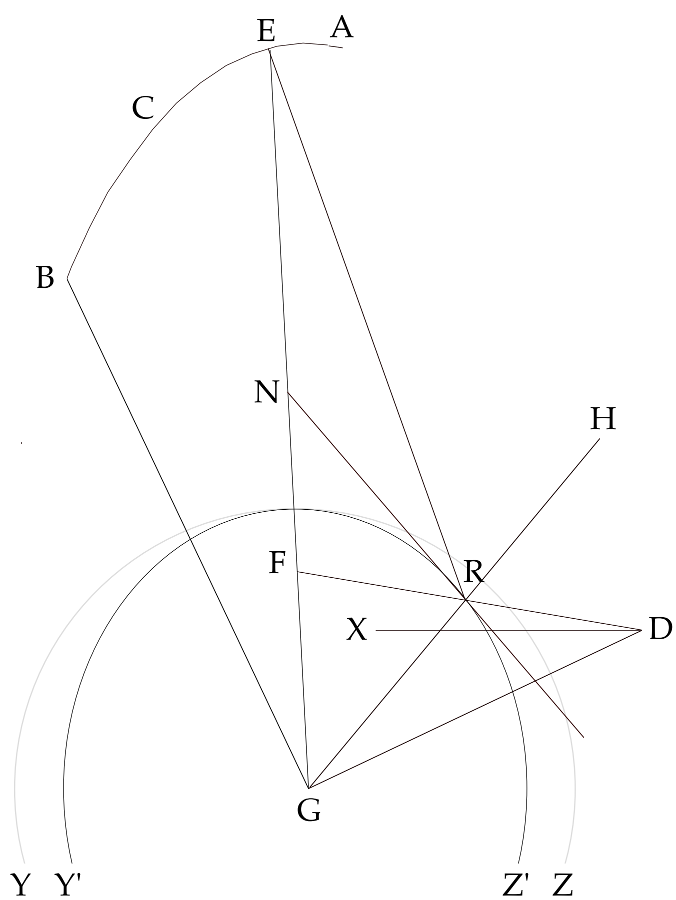
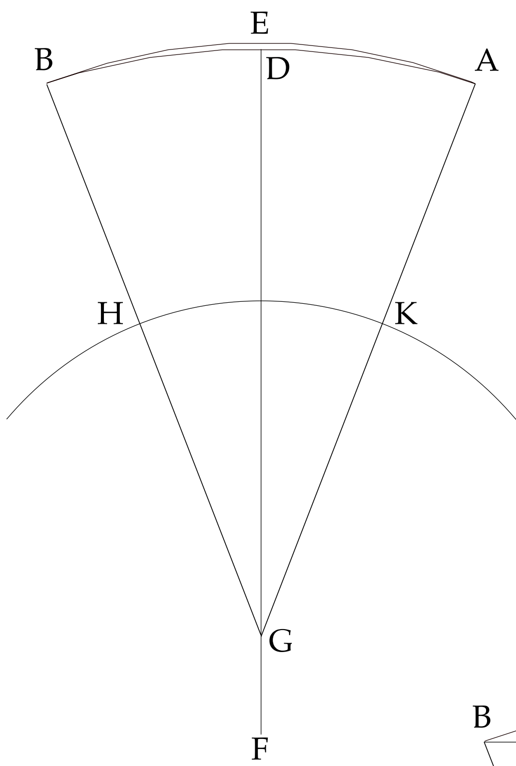


figure 18



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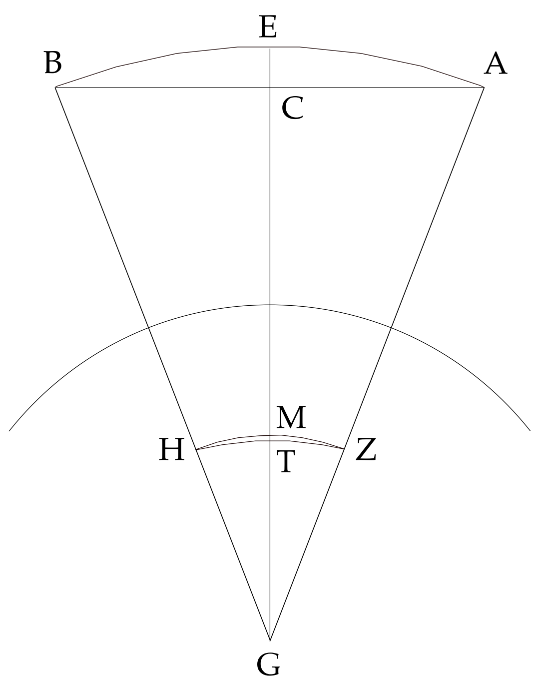

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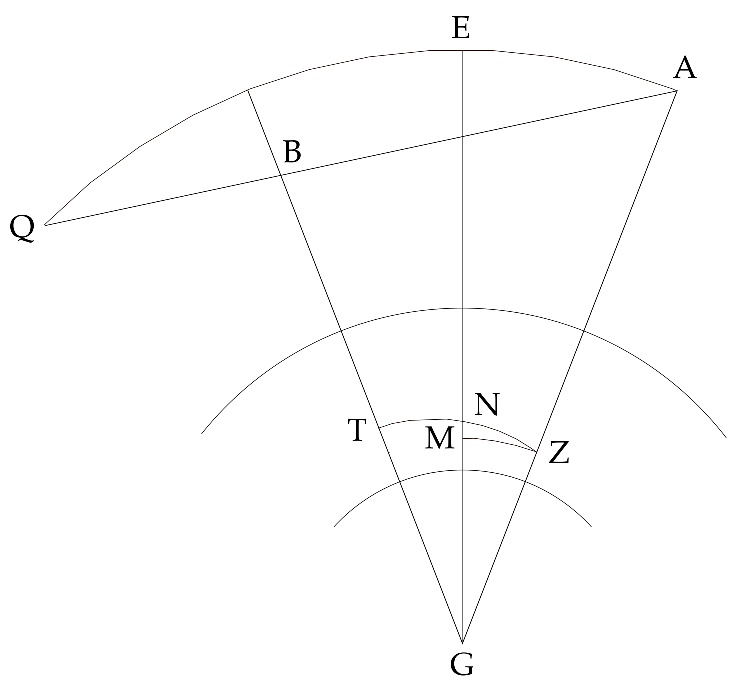

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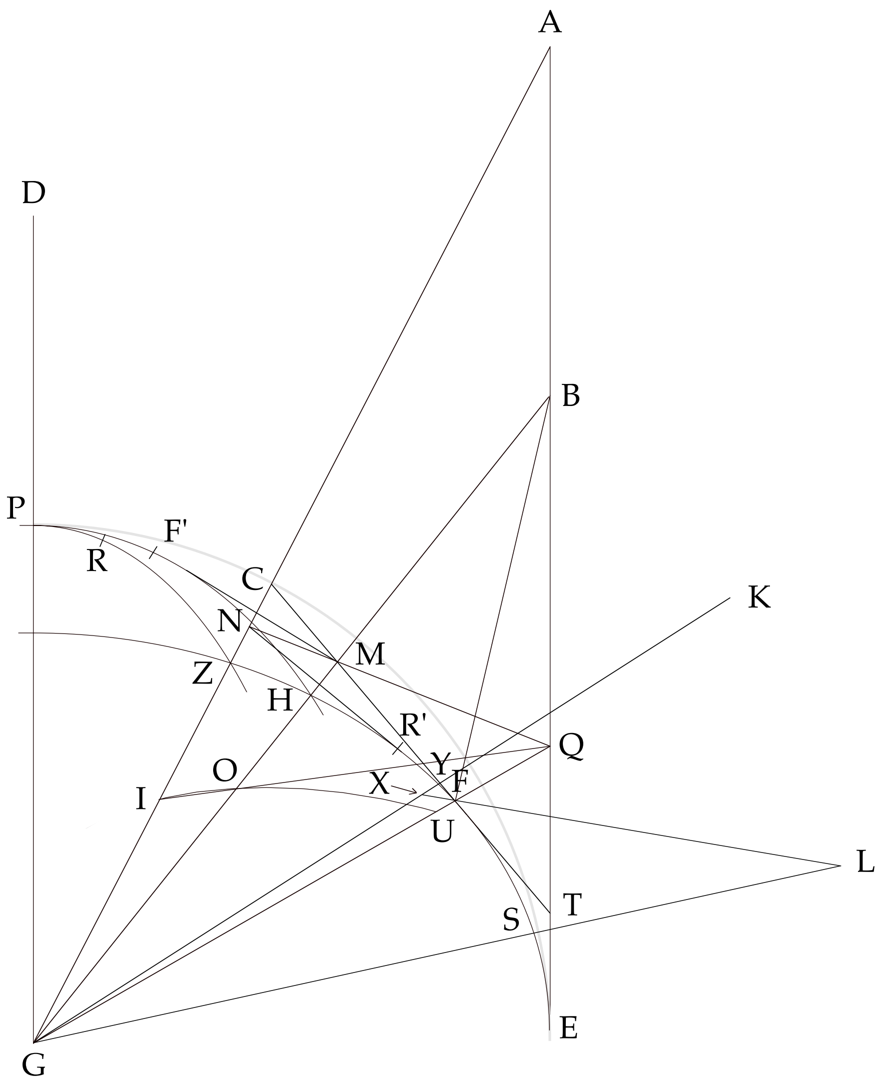

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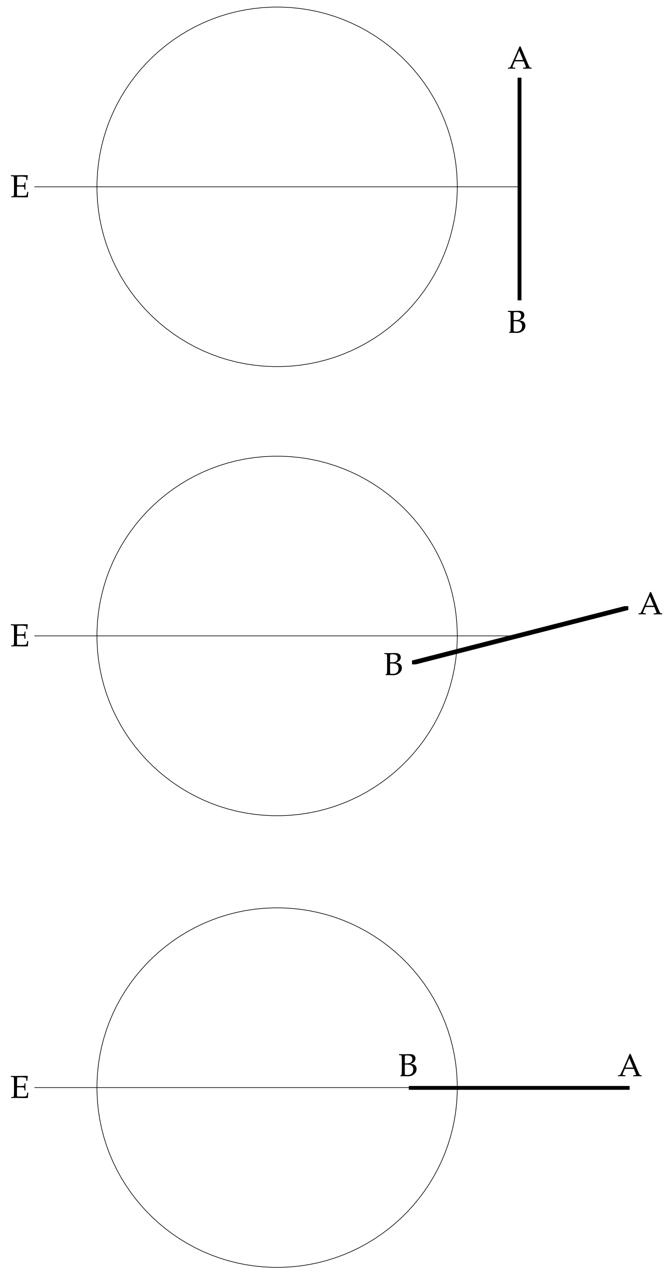

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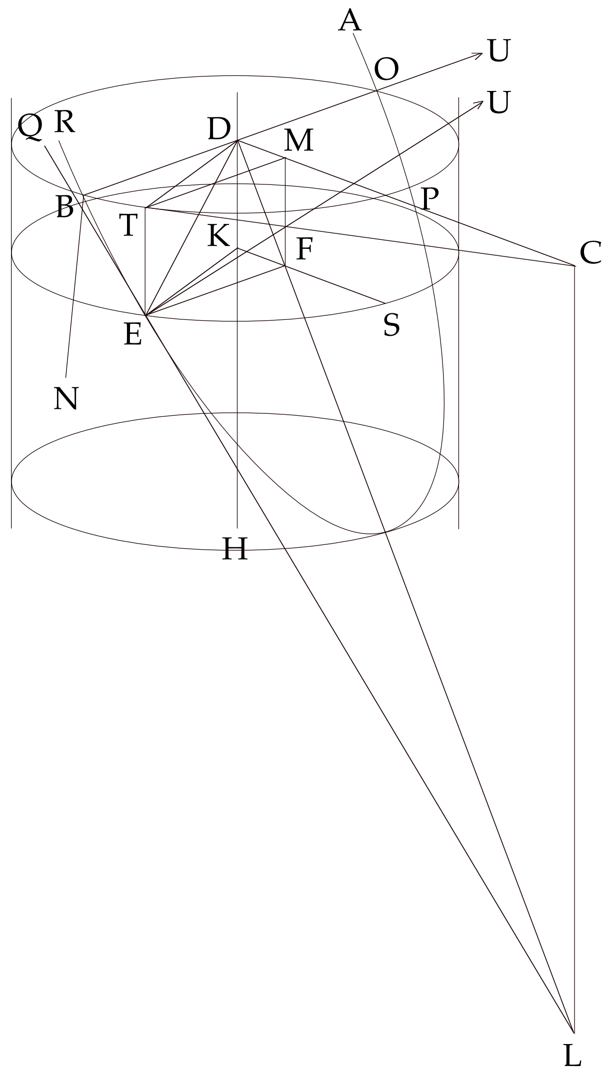

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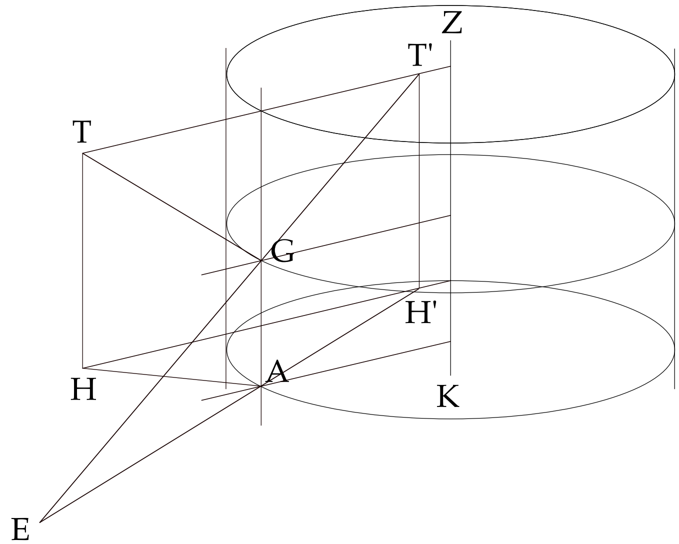

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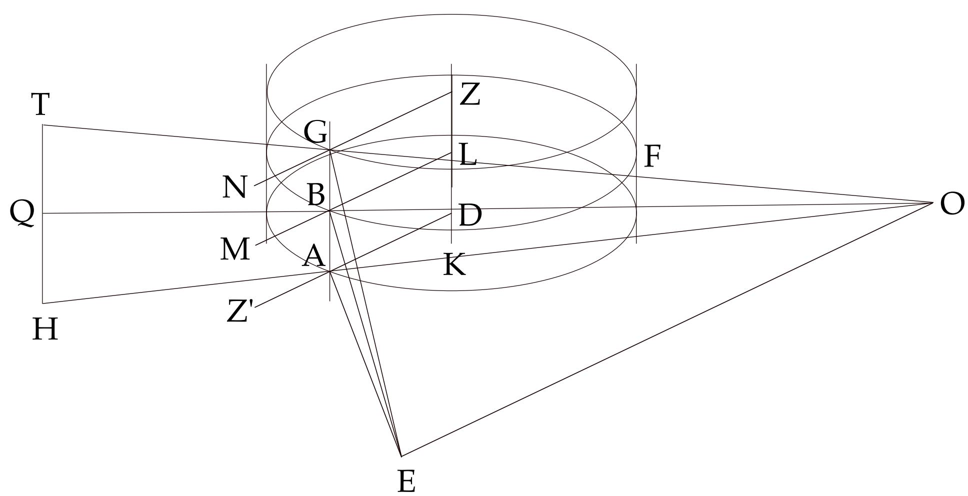
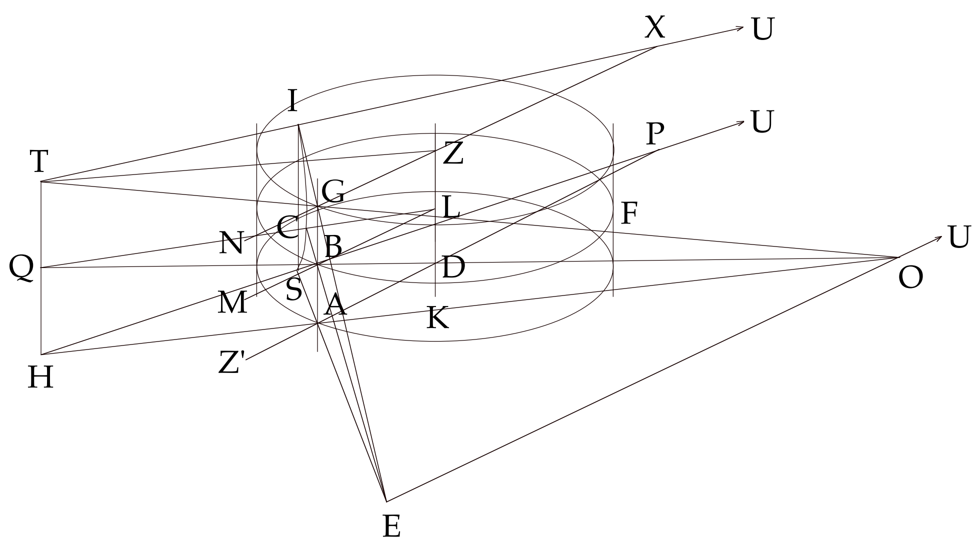


figure 25a



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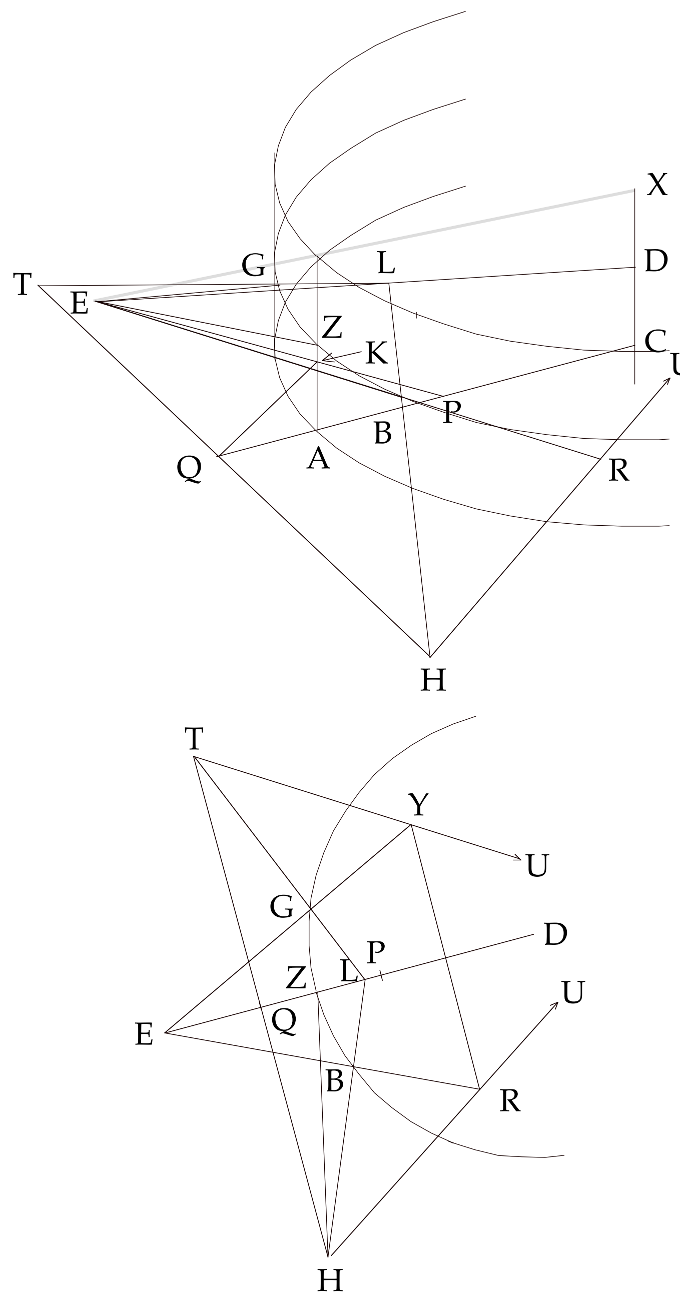

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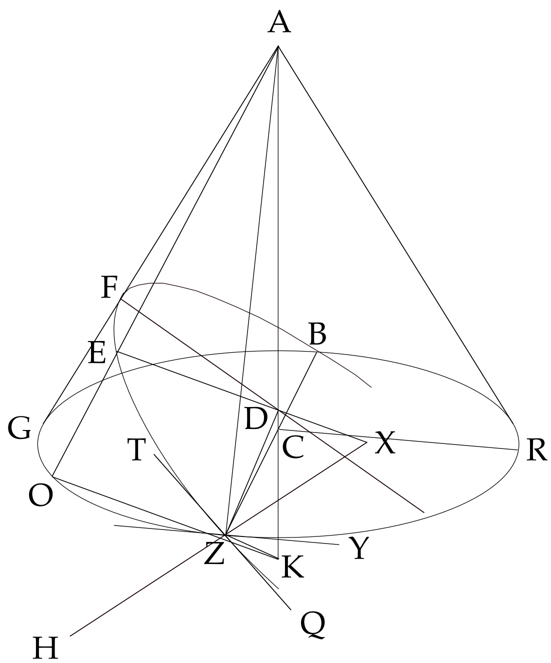

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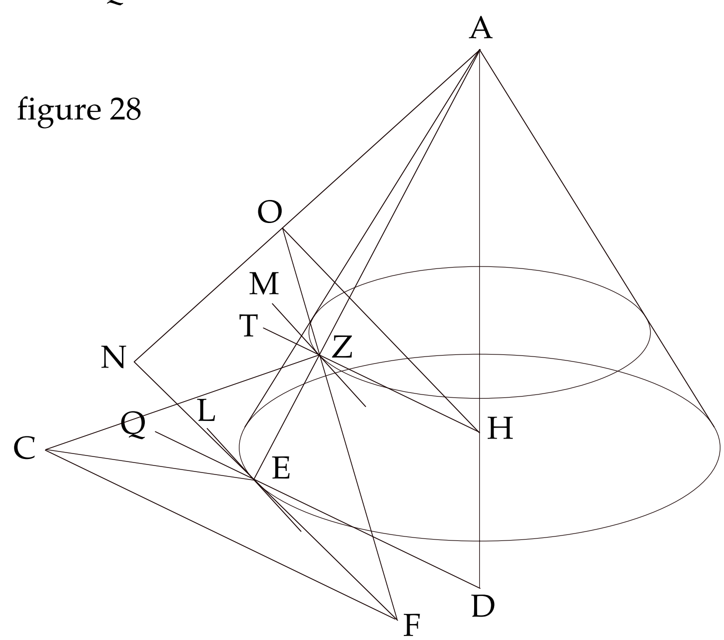

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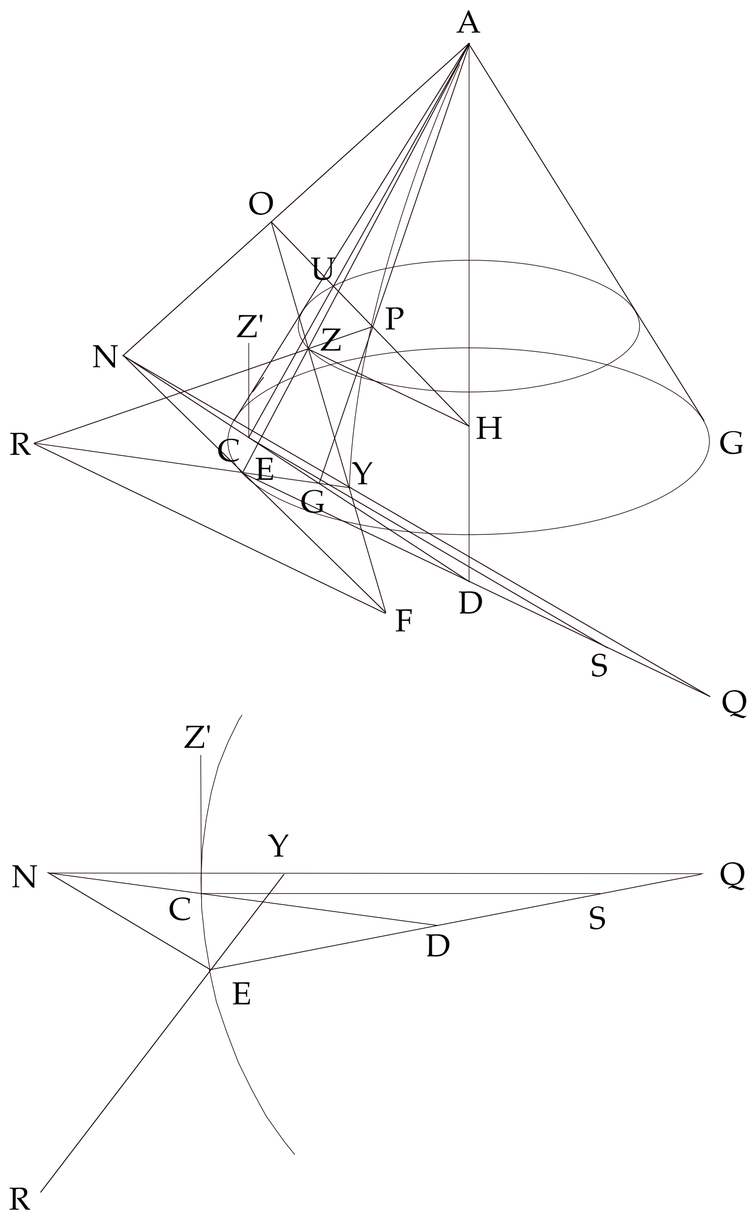

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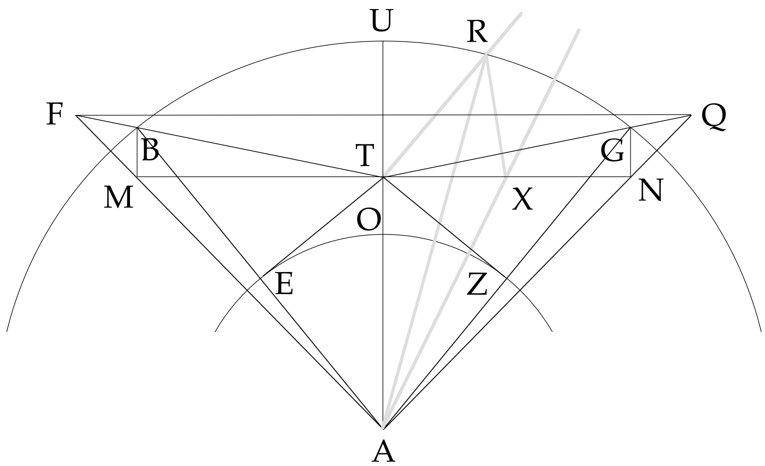

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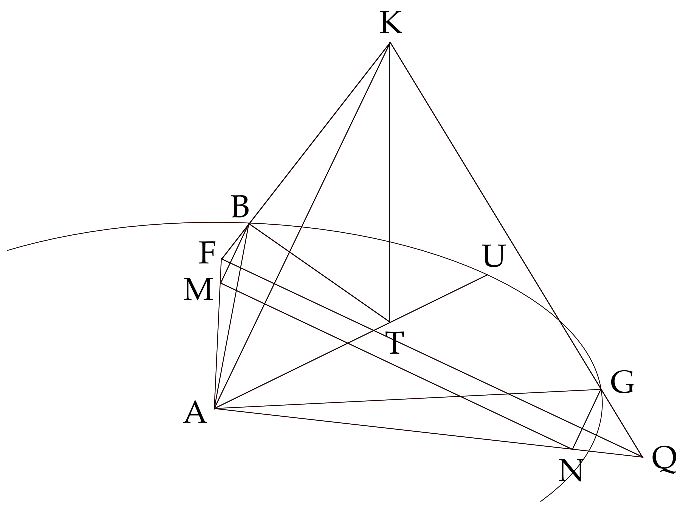

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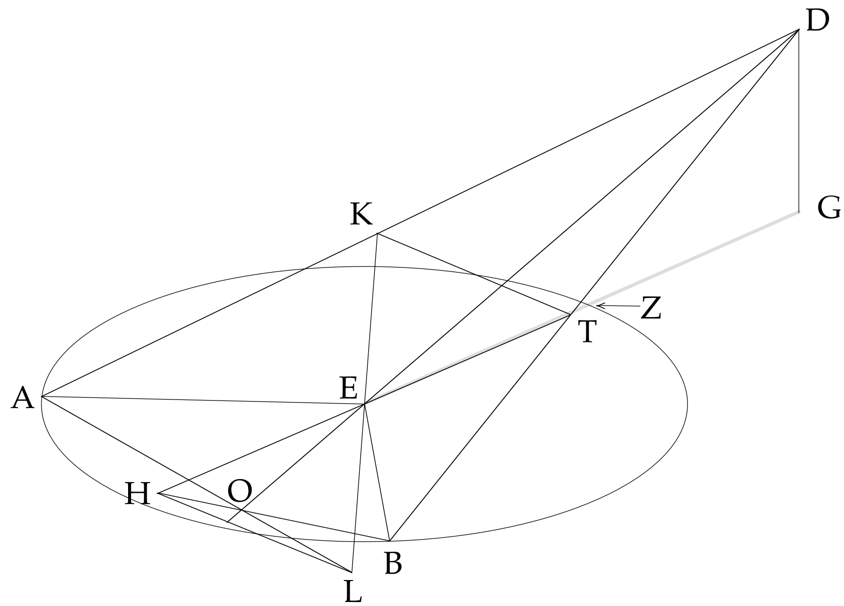

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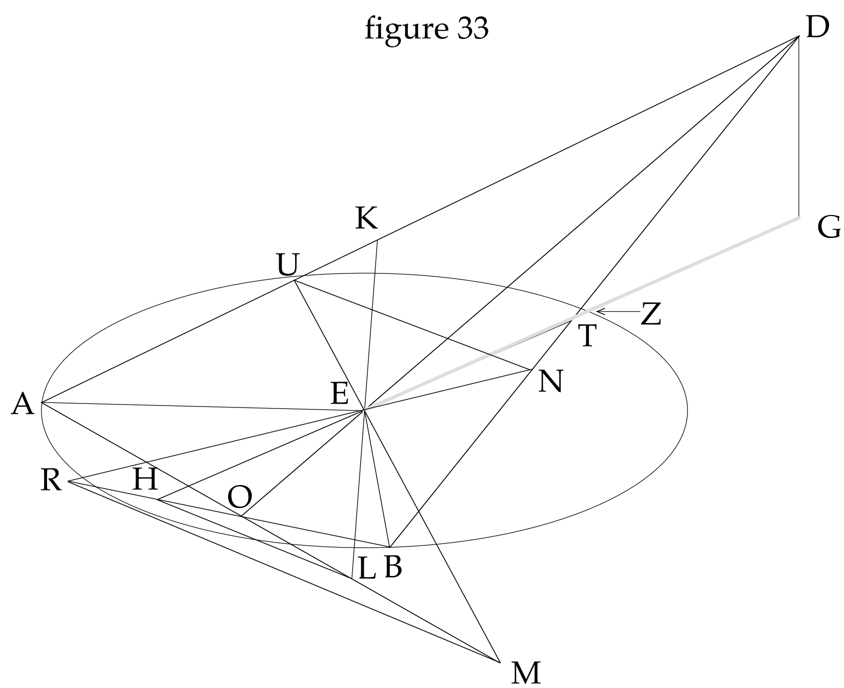

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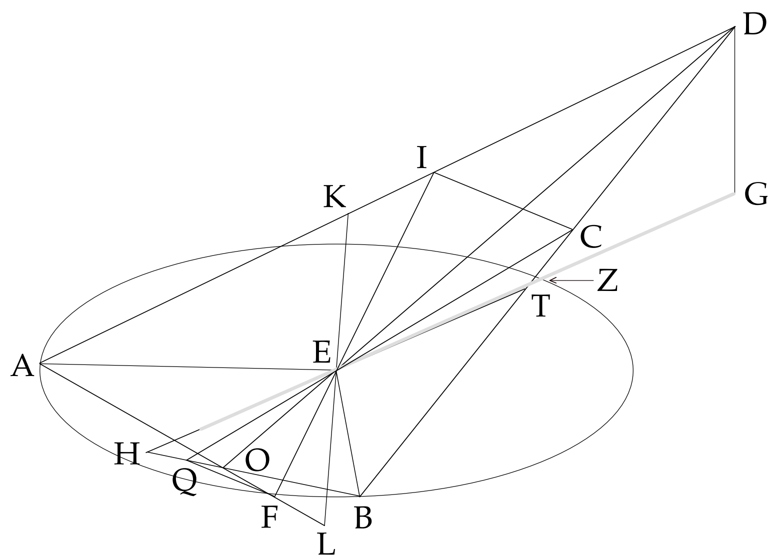

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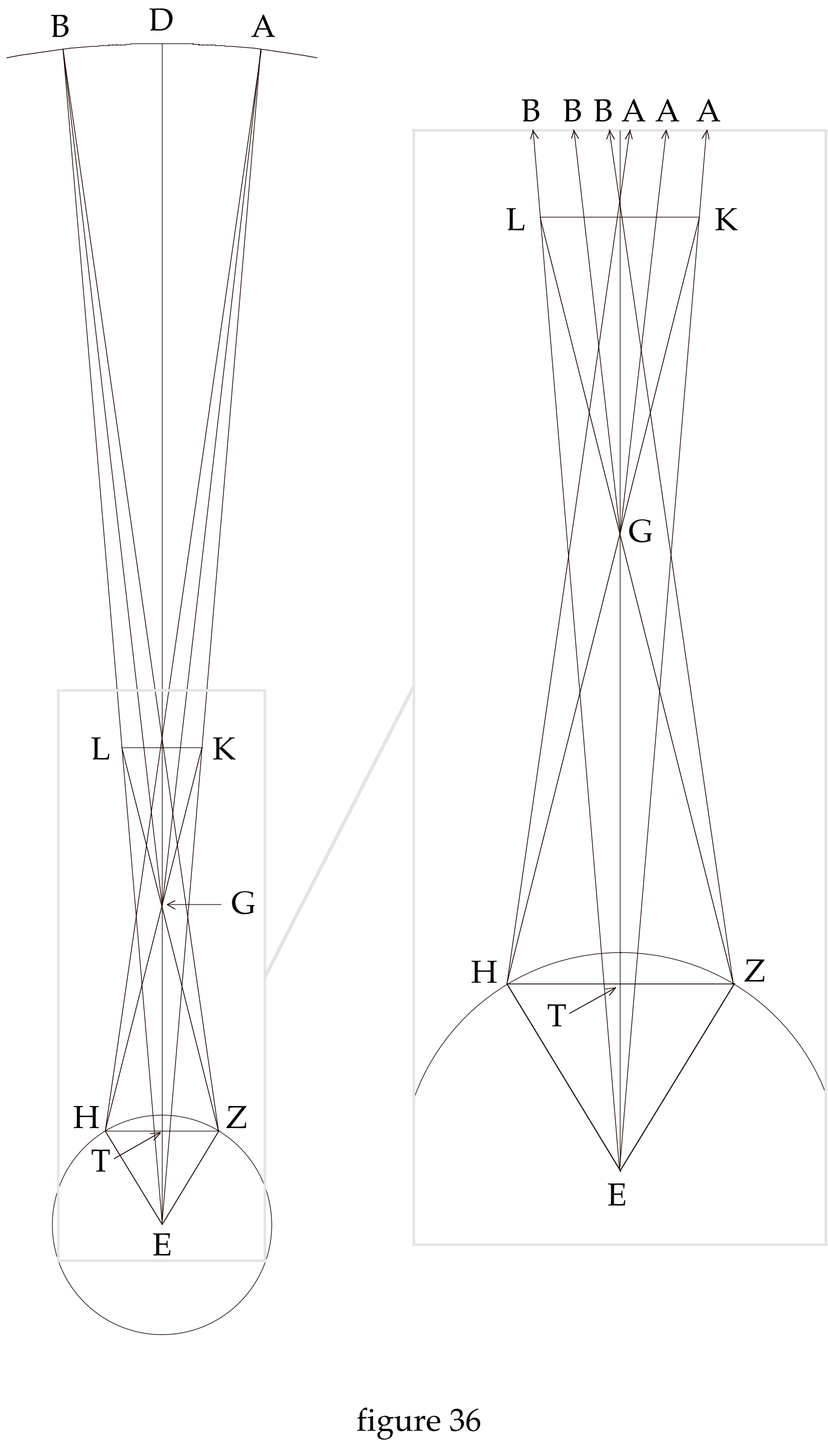

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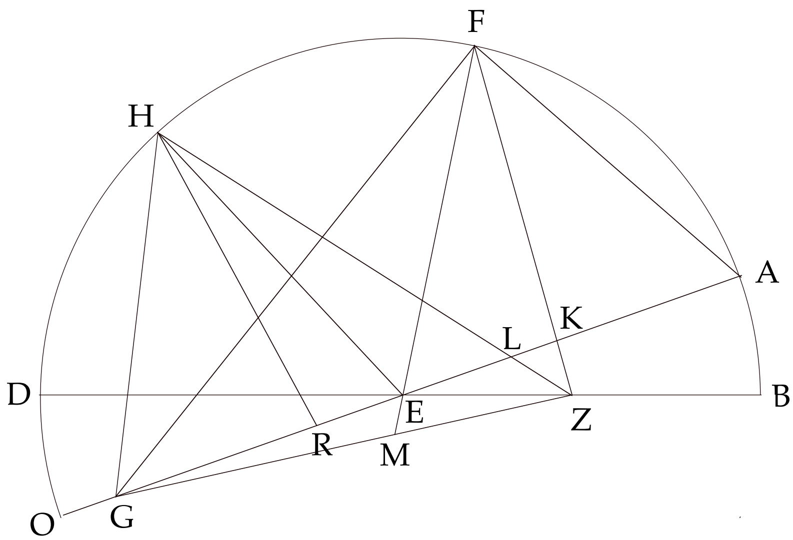

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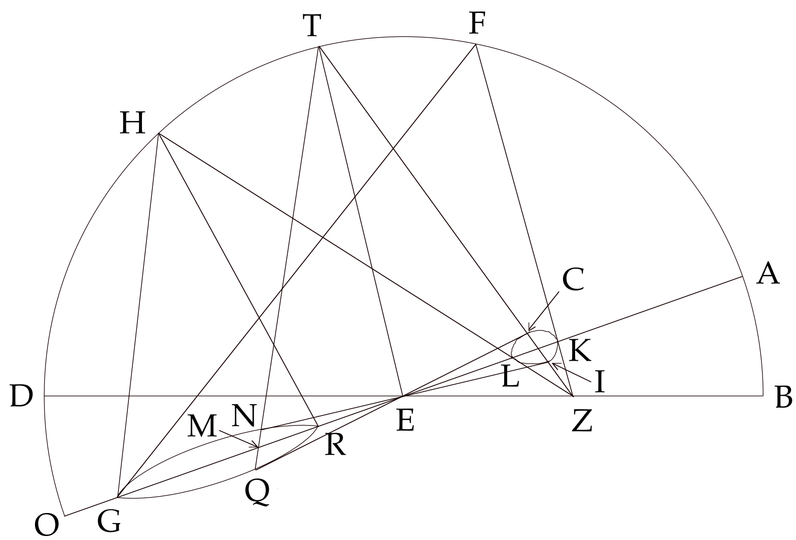

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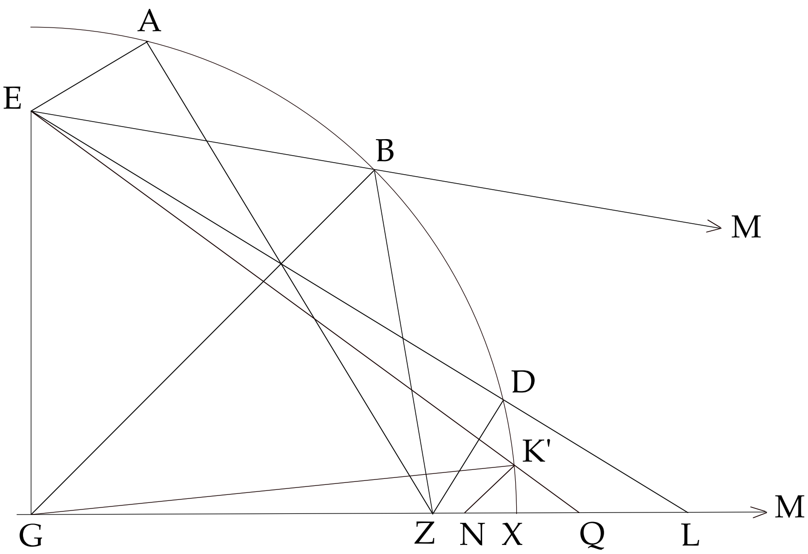

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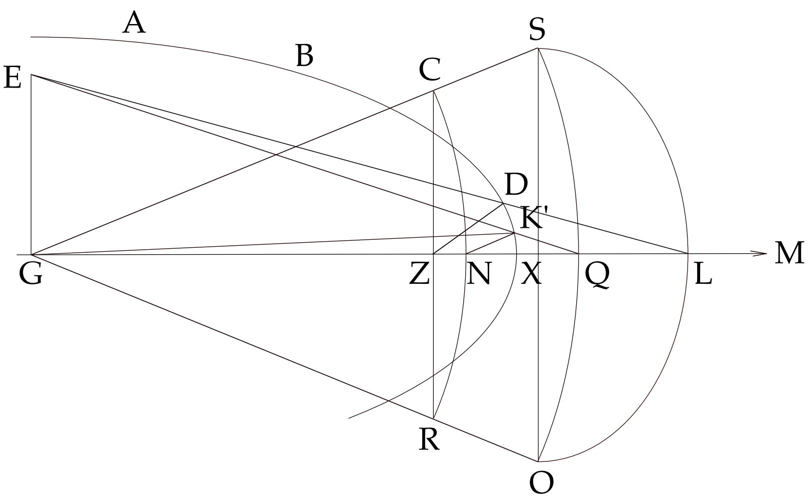


figure 39a

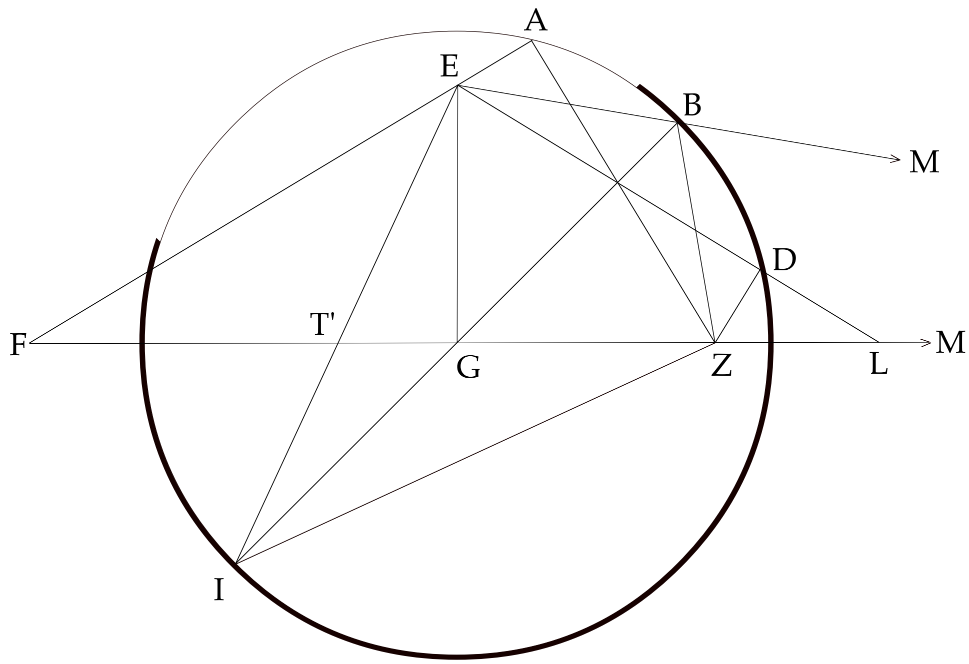


figure 39b

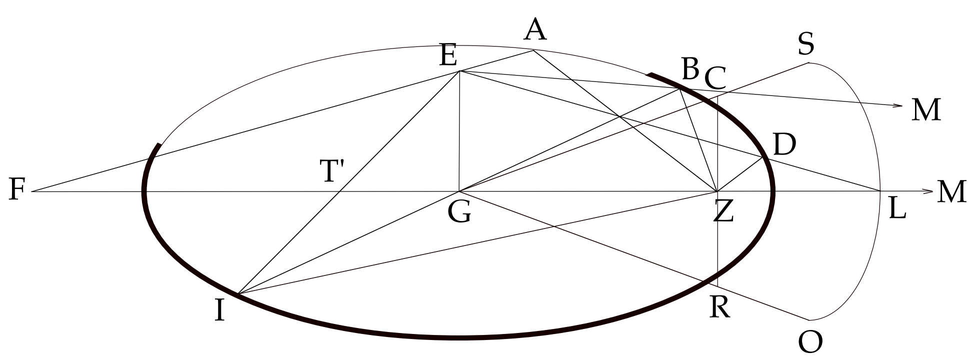
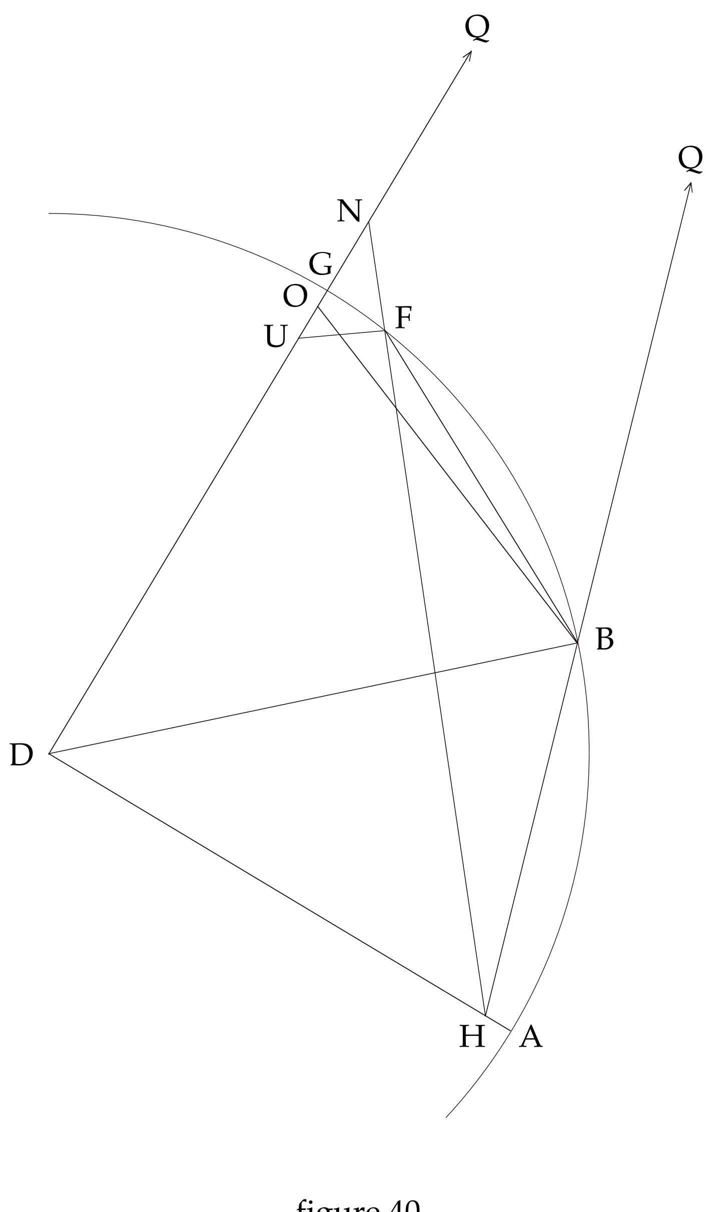


figure 39c



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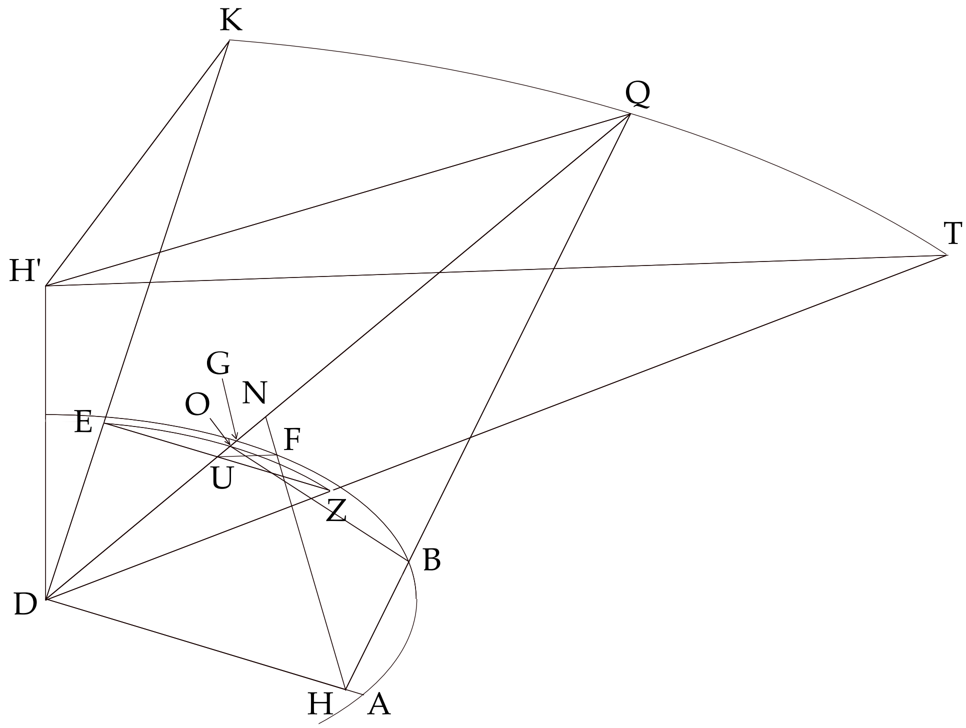

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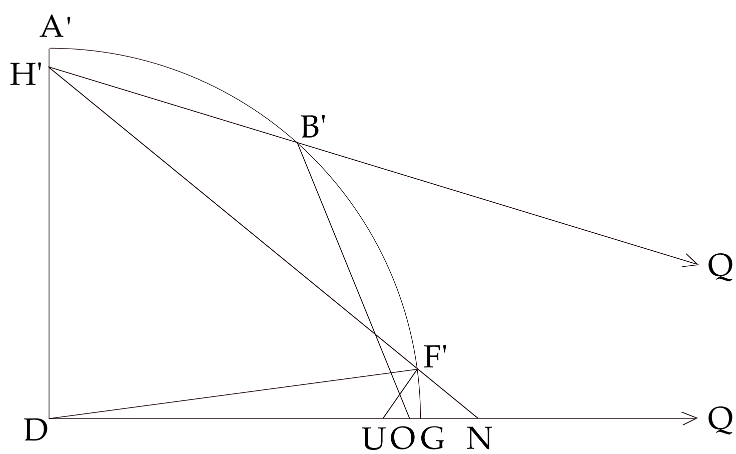
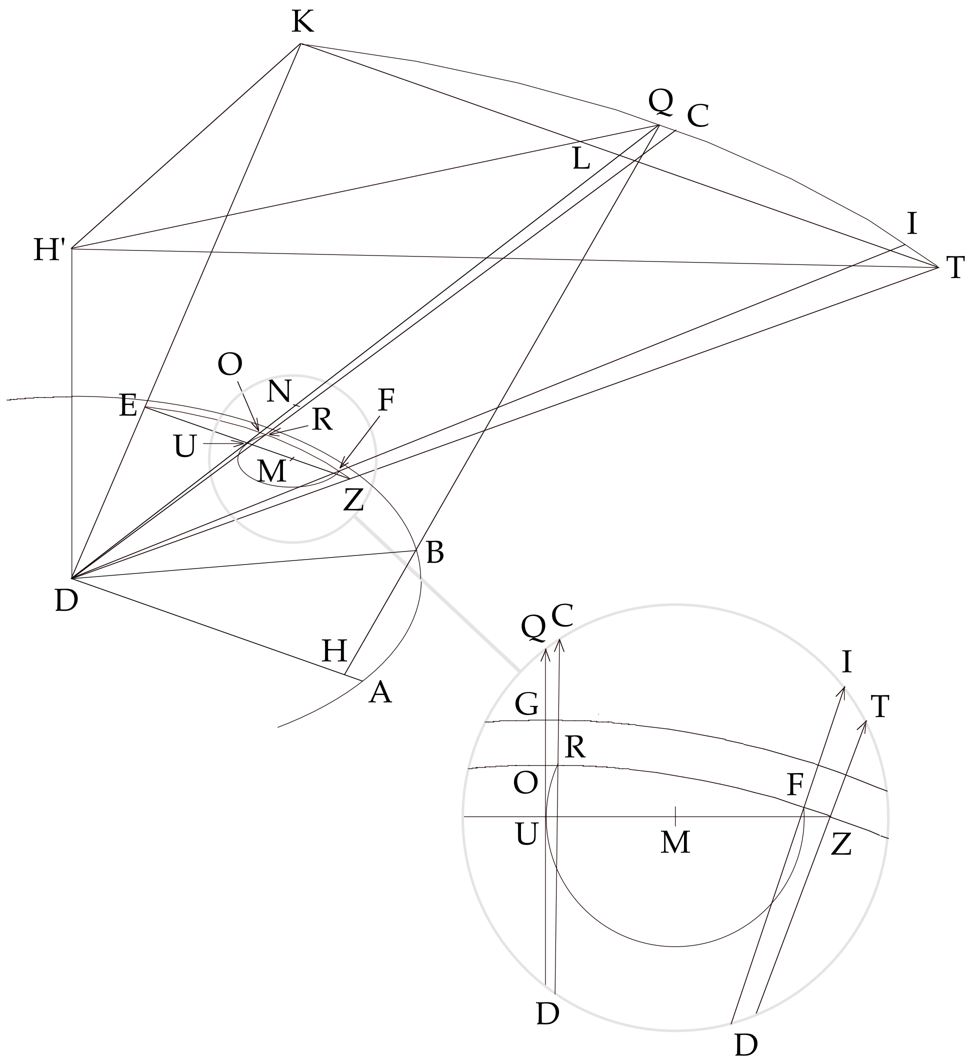


figure 40b



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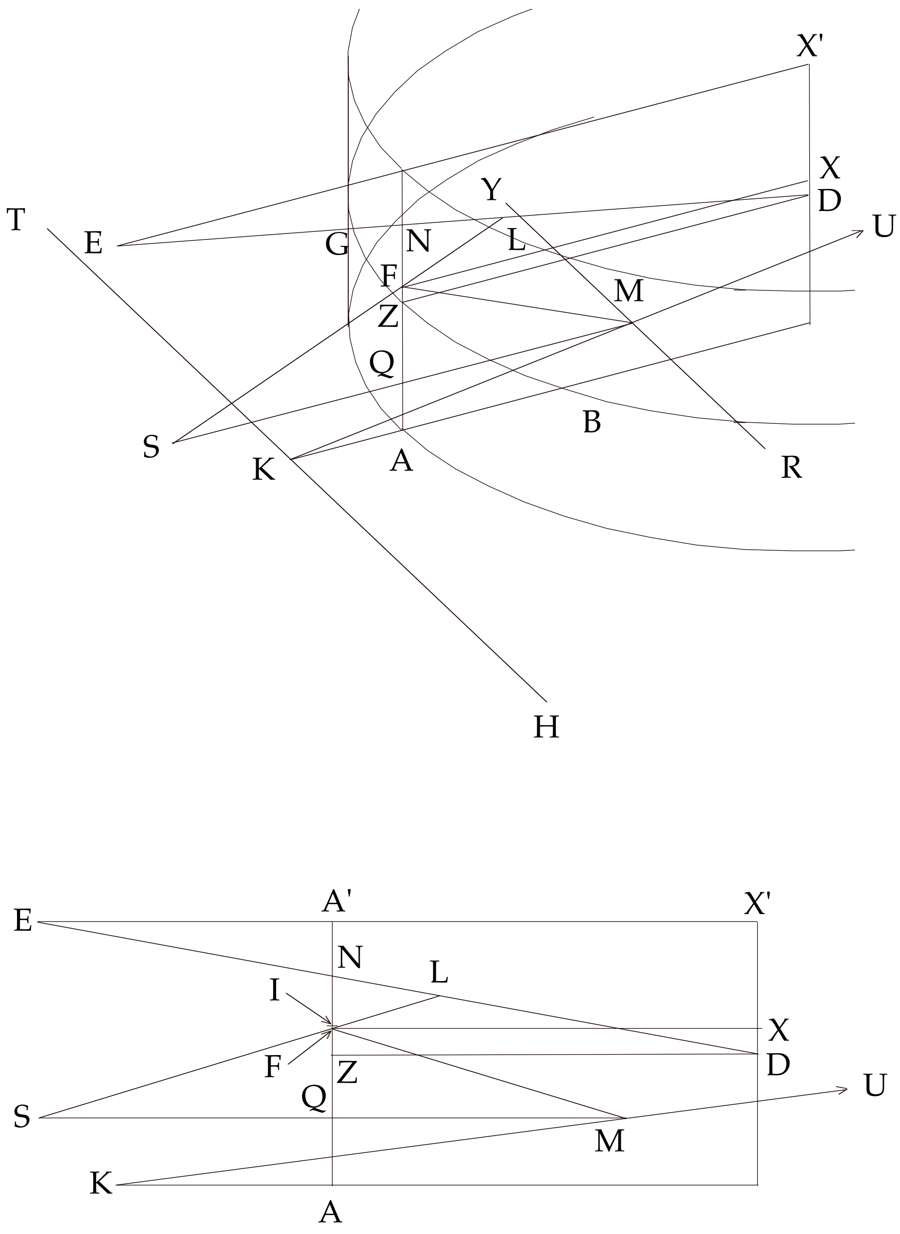

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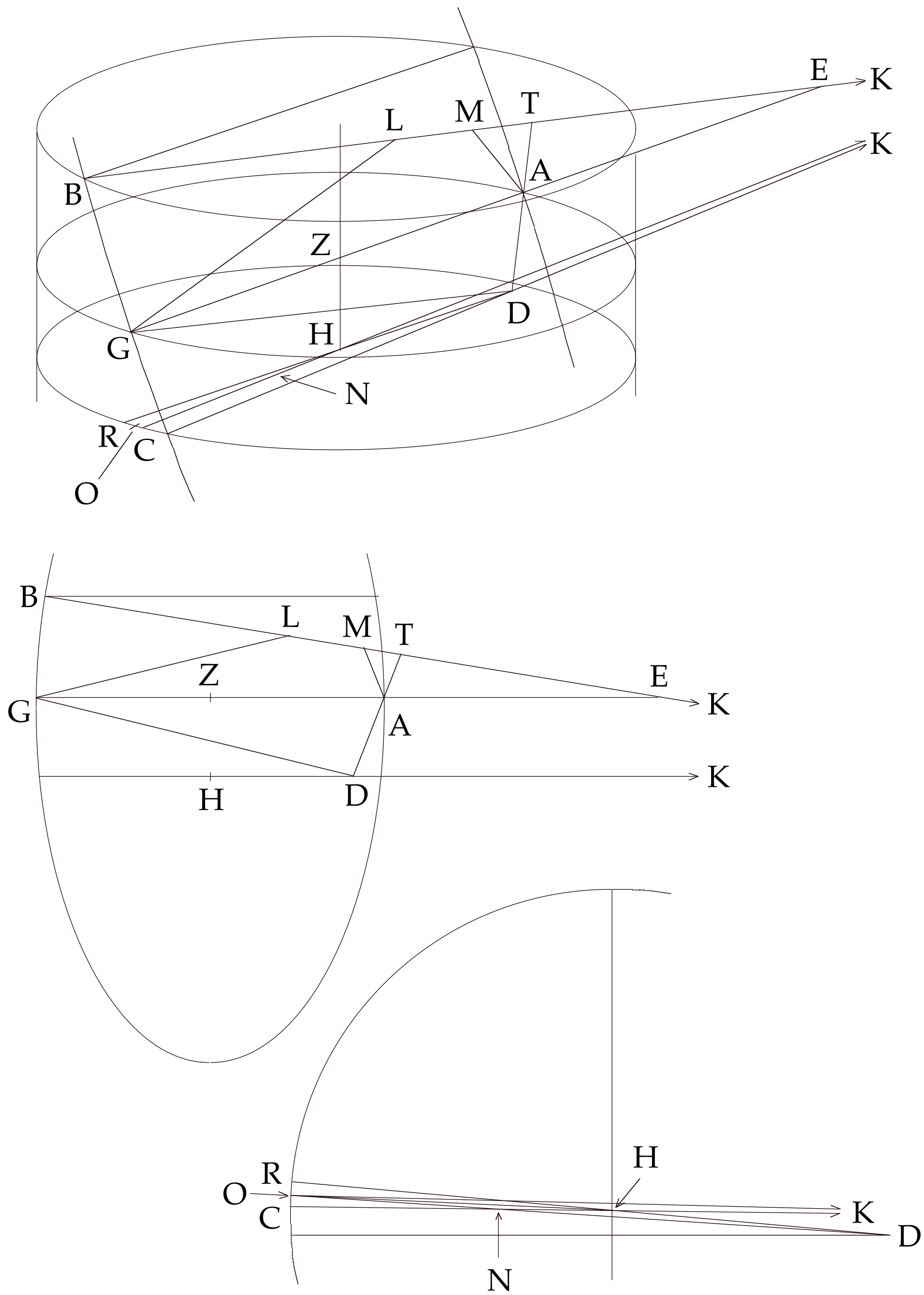

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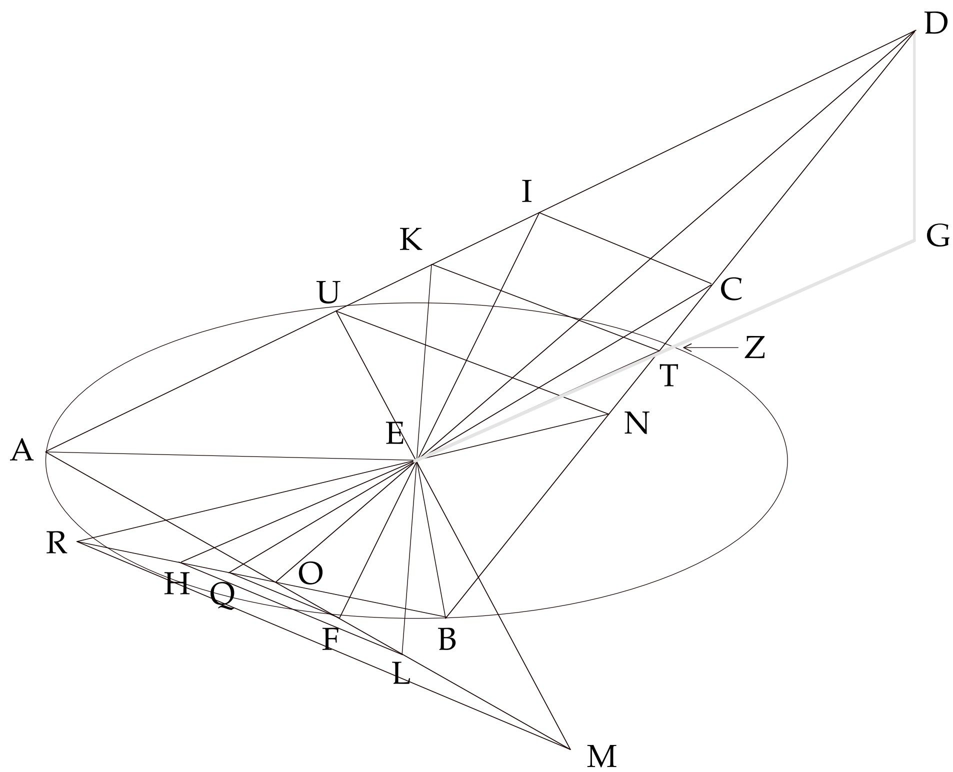

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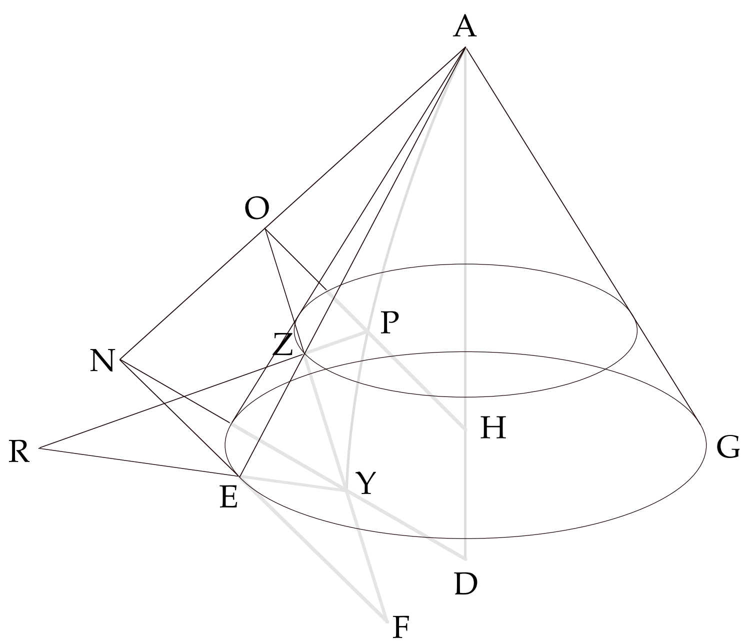

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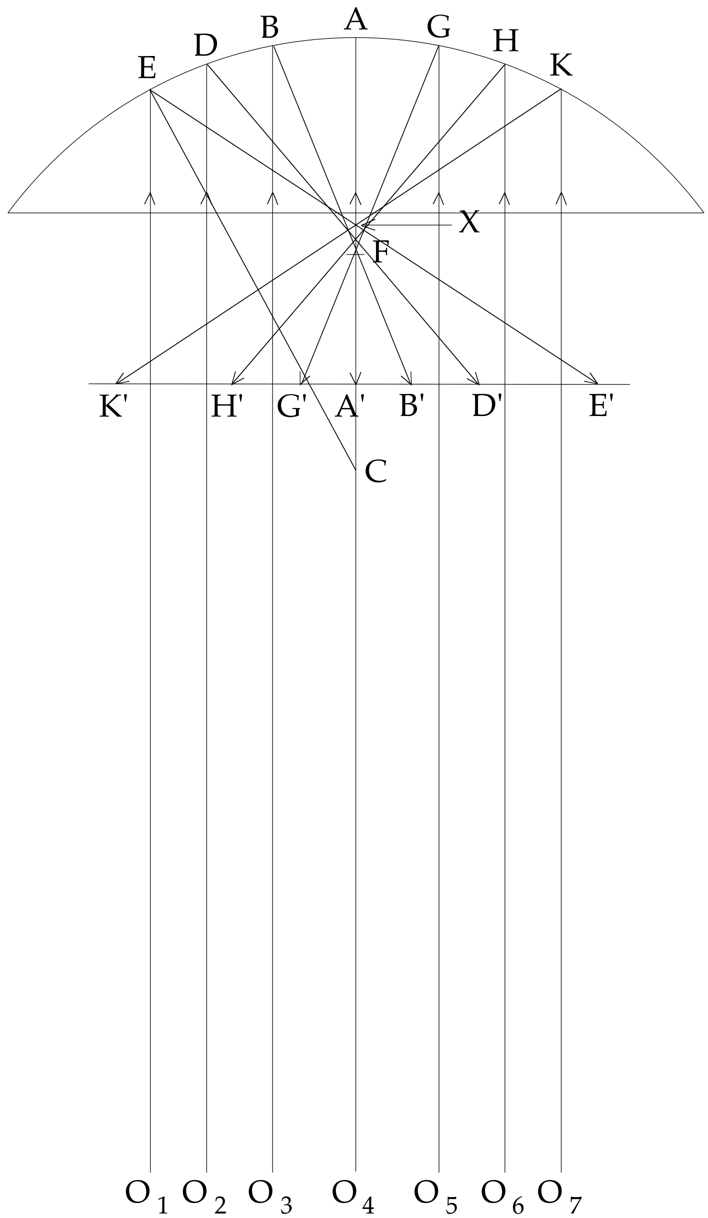

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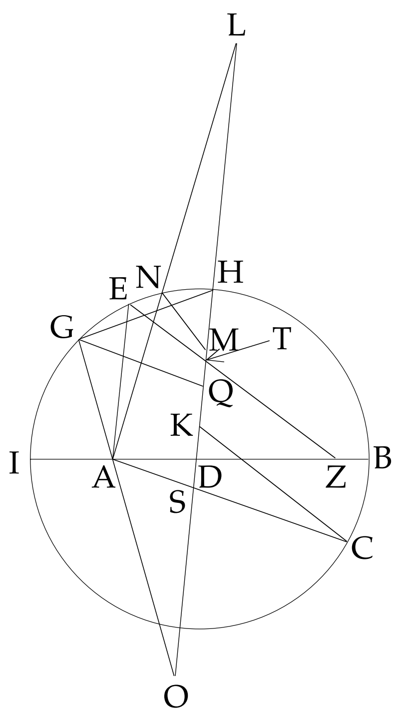

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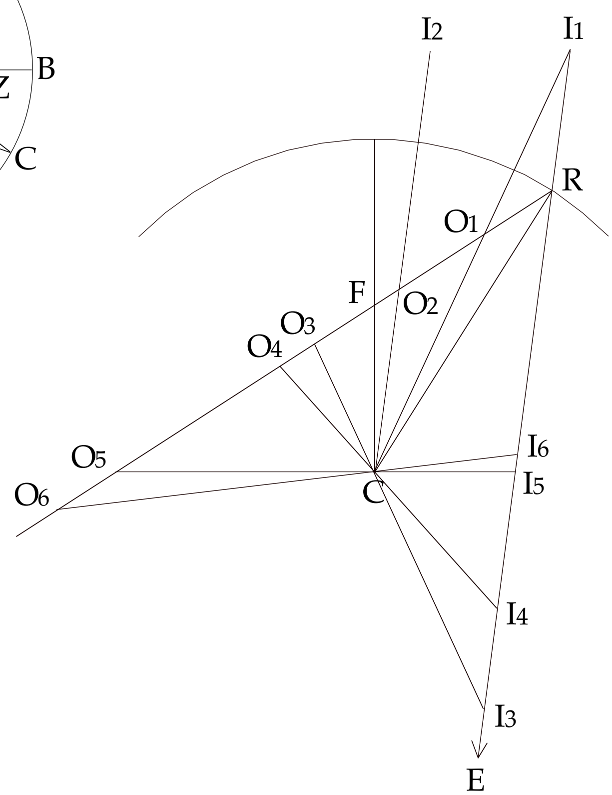


figure 47a

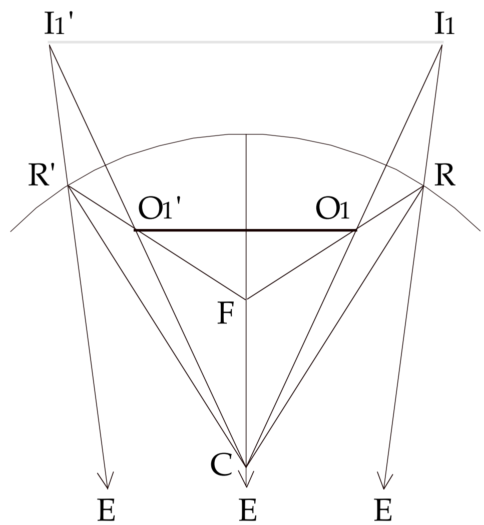
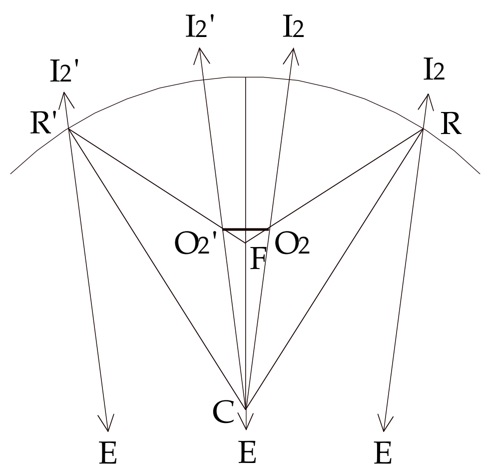


figure 47b



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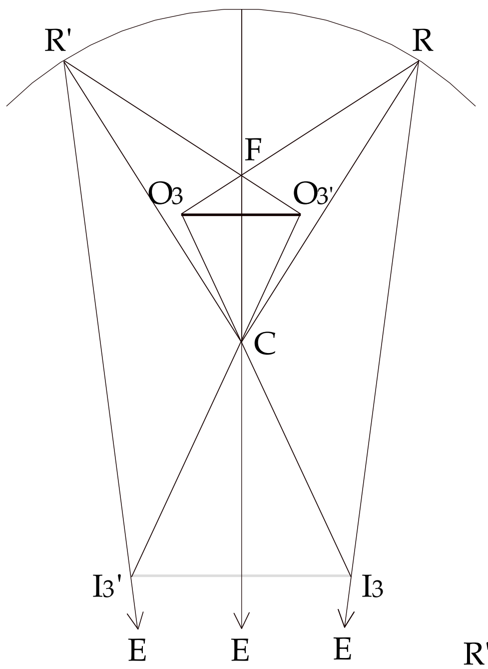

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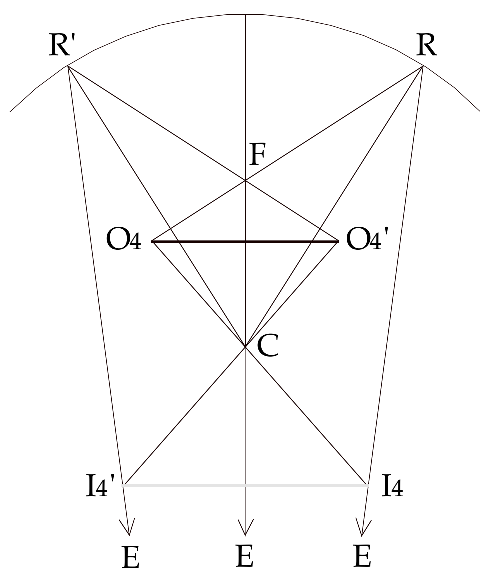

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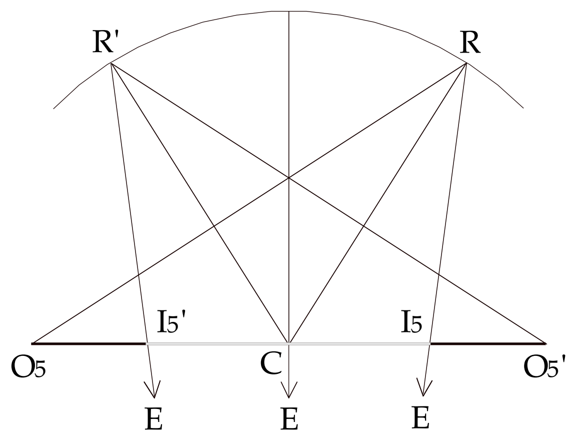

figure 47f

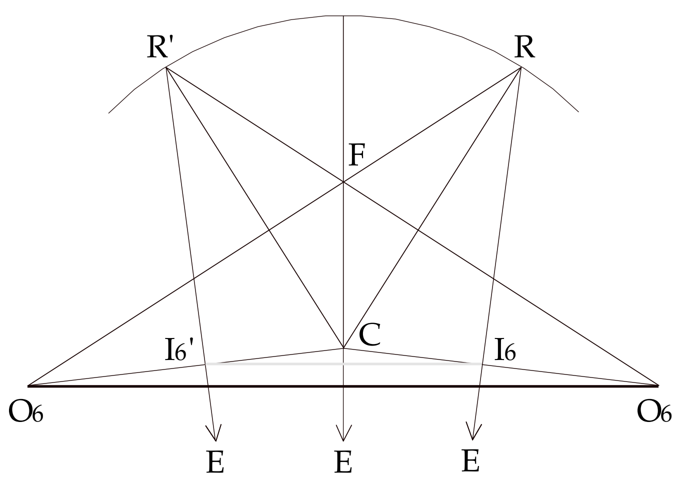

figure 47g

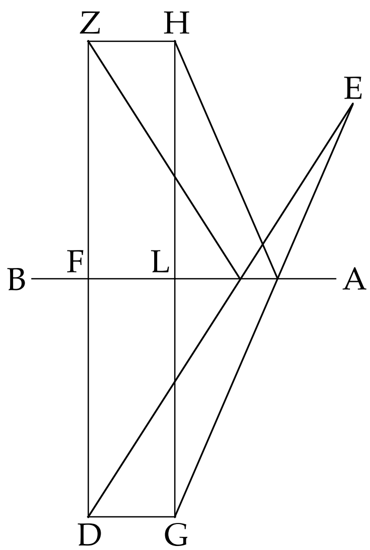

FIGURE 6.3.1

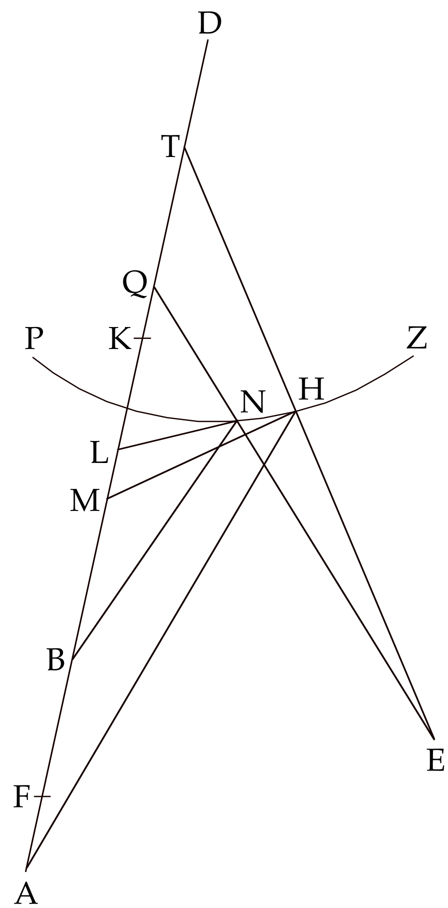

FIGURE 6.4.2

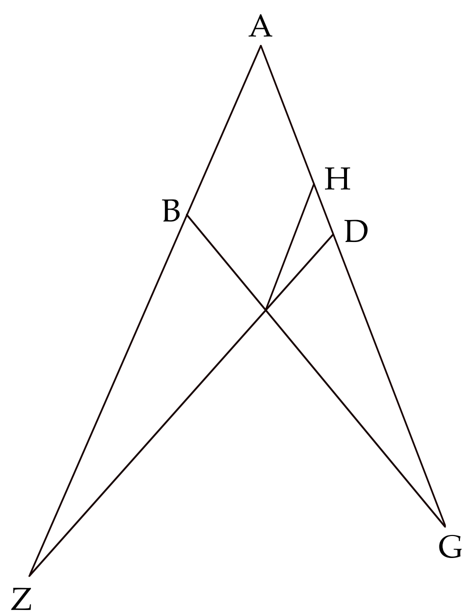

FIGURE 6.4.2a

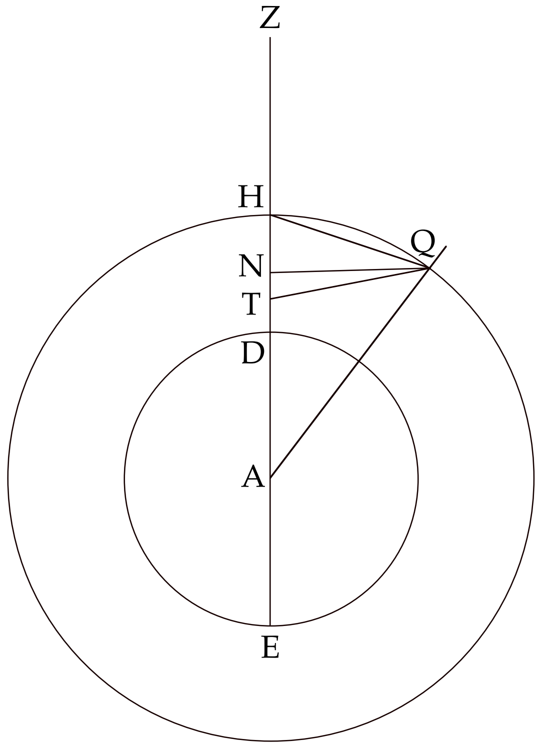

FIGURE 6.4.3

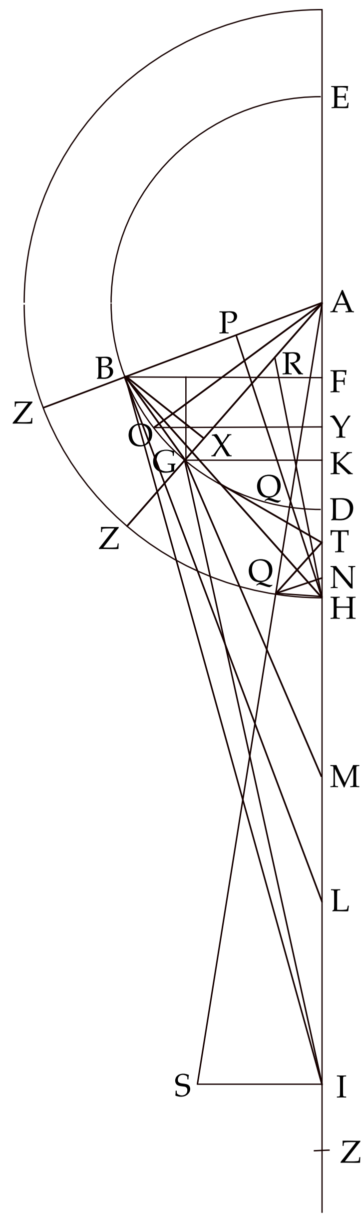

FIGURE 6.4.3a

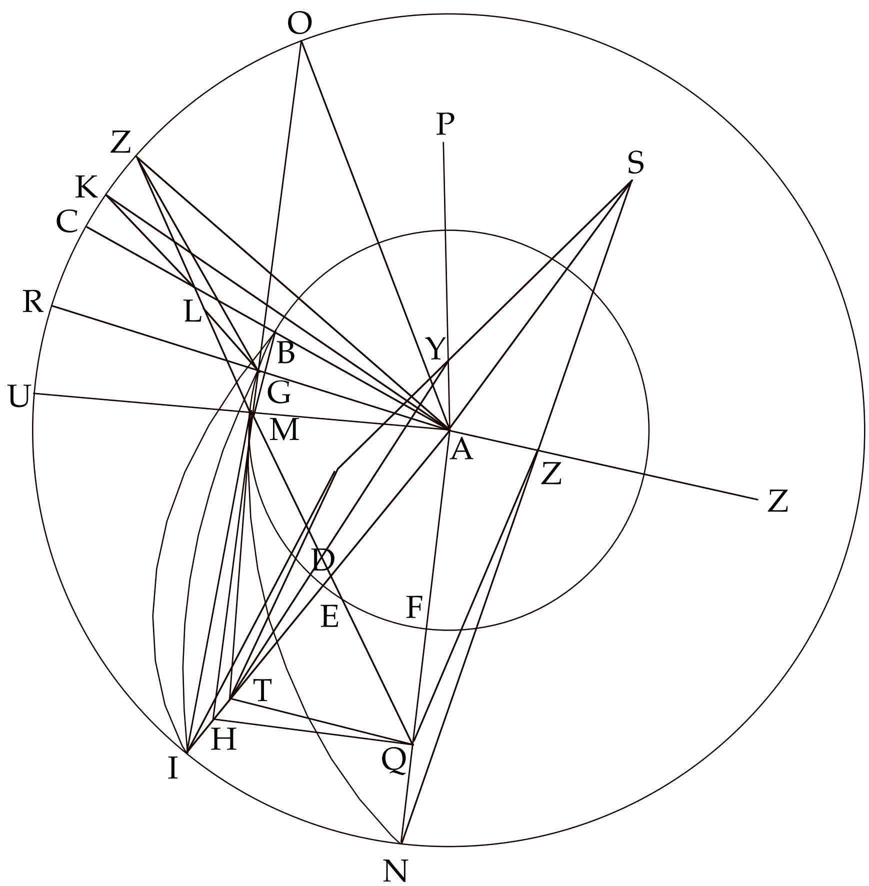

FIGURE 6.4.3b

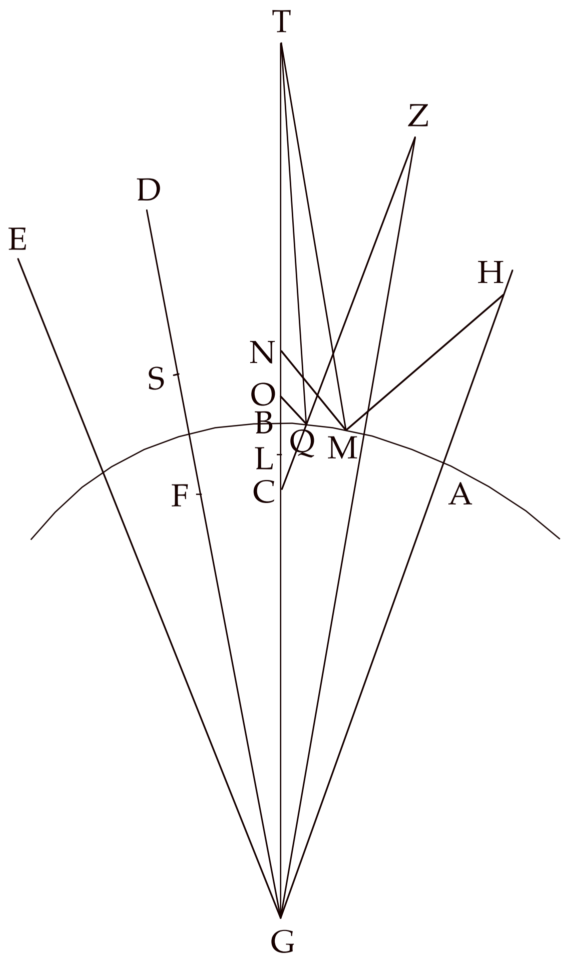

FIGURE 6.4.4

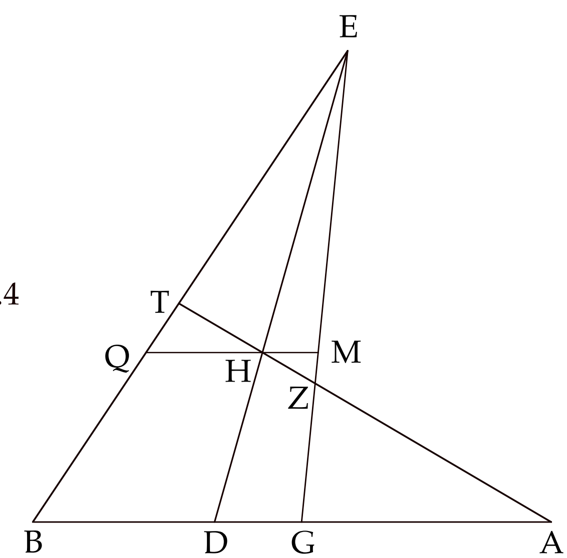

FIGURE 6.4.5

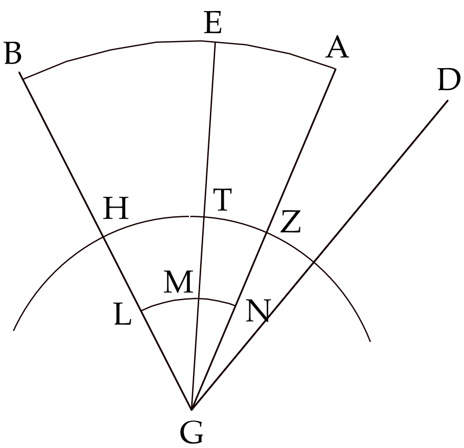

FIGURE 6.4.8

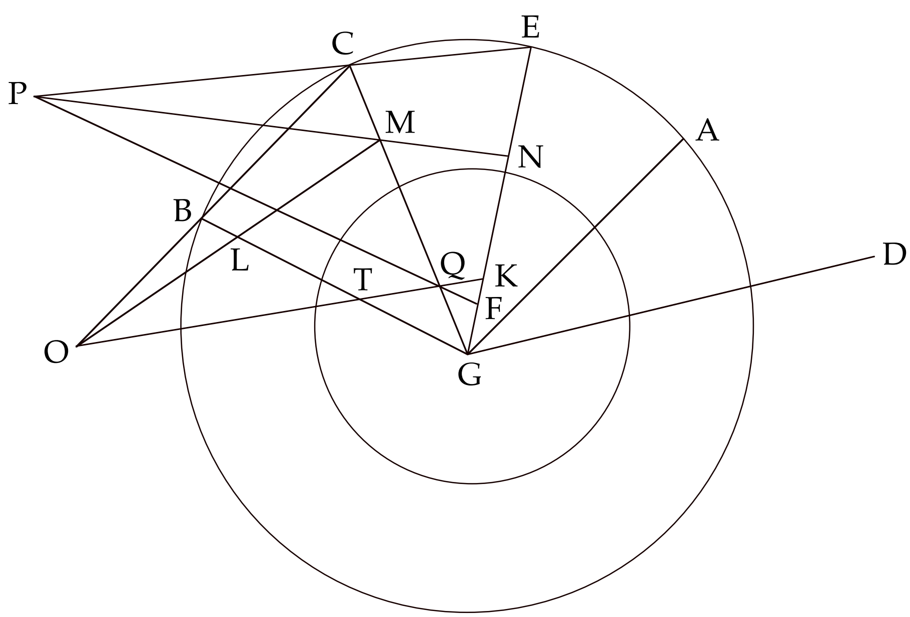

FIGURE 6.4.8a

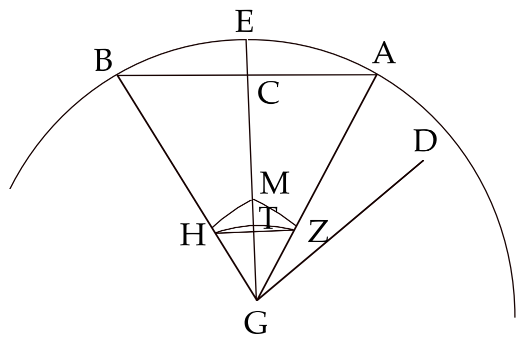

FIGURE 6.4.10

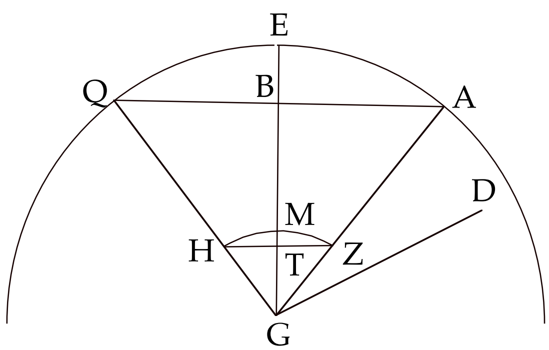

FIGURE 6.4.11

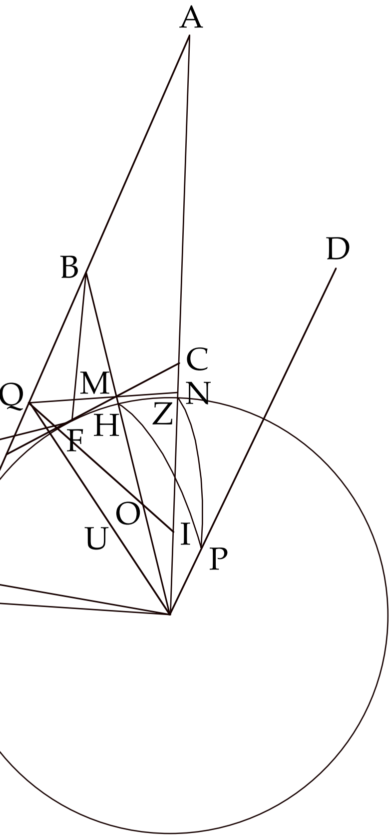

FIGURE 6.4.12

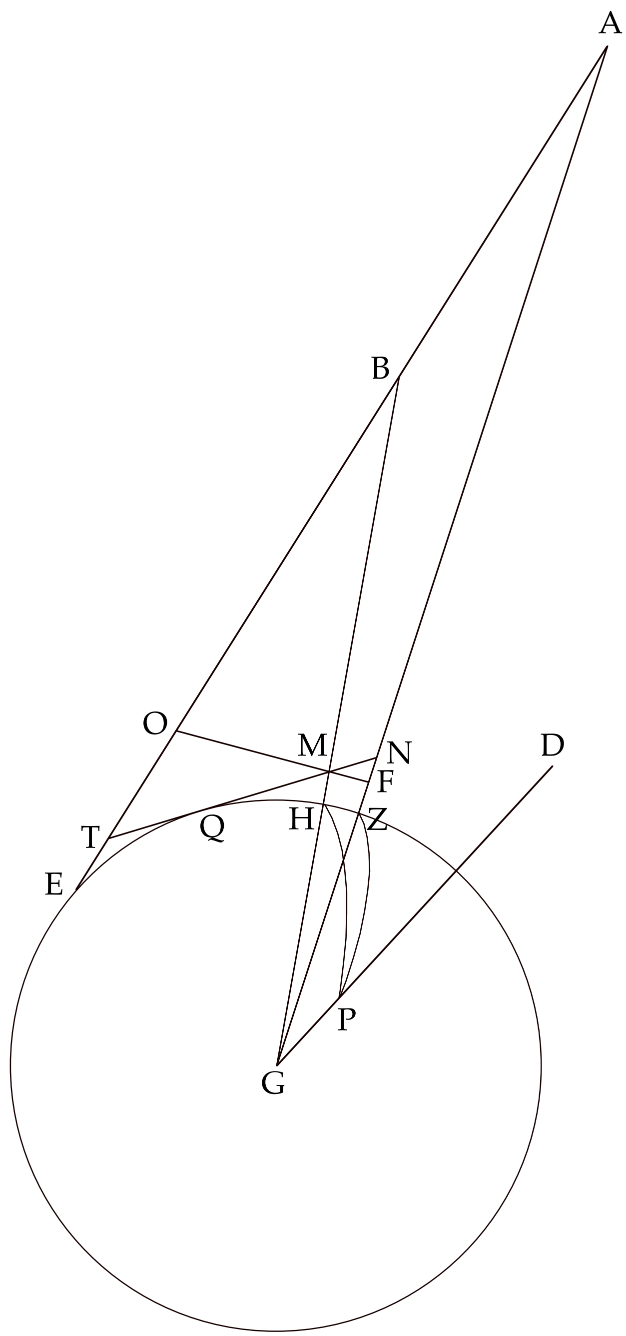

FIGURE 6.4.13

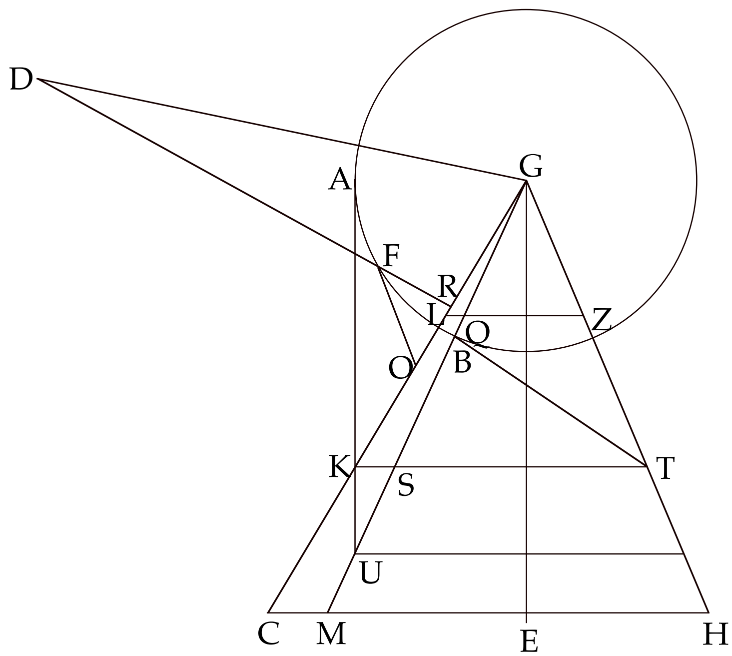

FIGURE 6.4.15

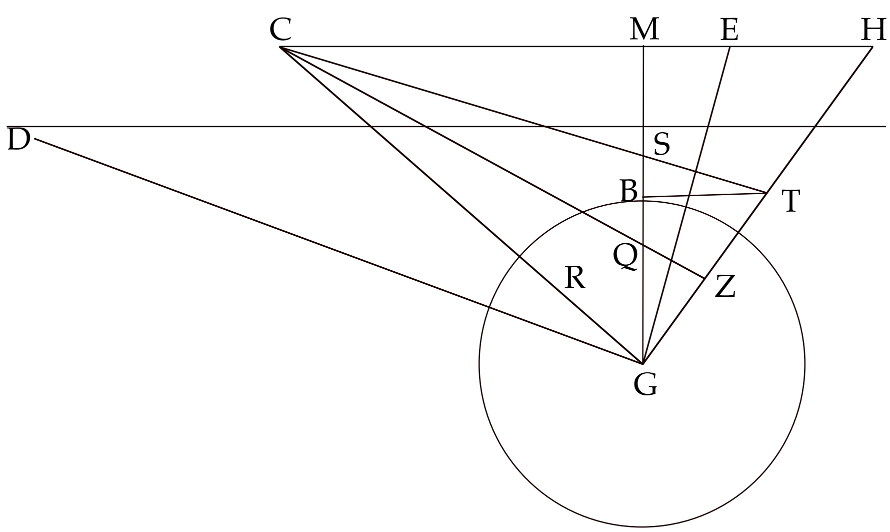

FIGURE 6.4.15a

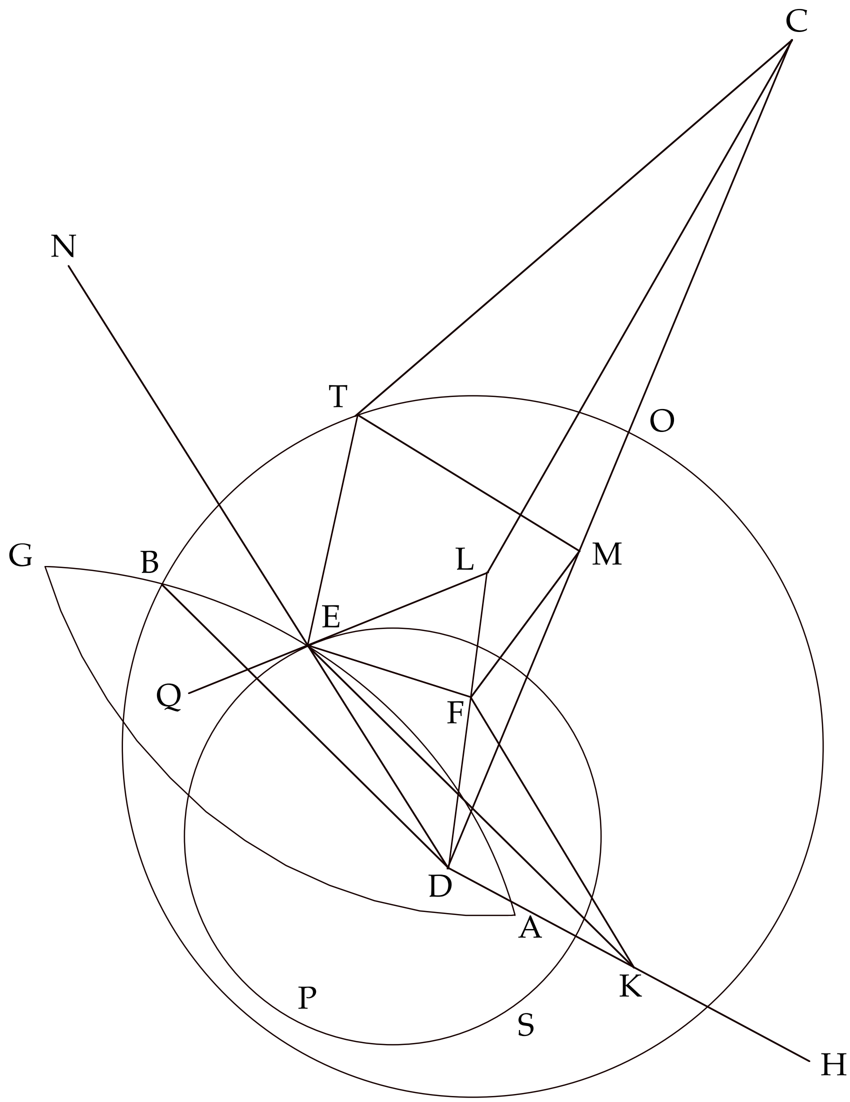

FIGURE 6.5.16

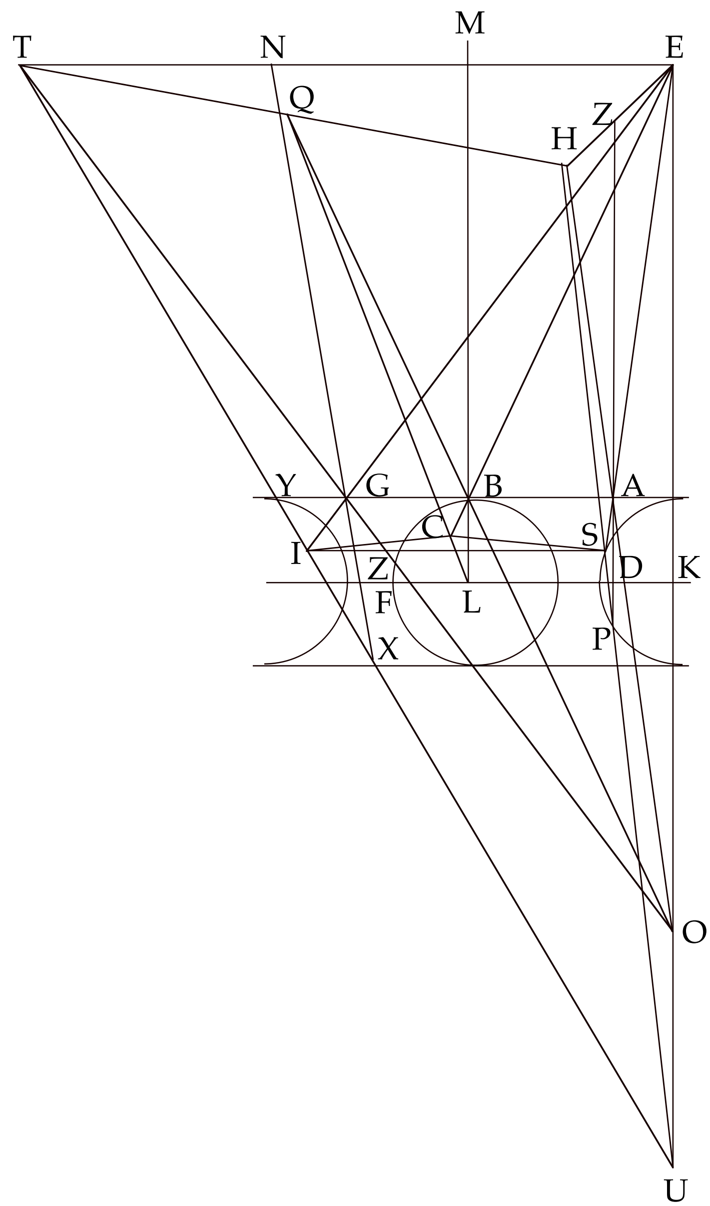

FIGURE 6.5.17

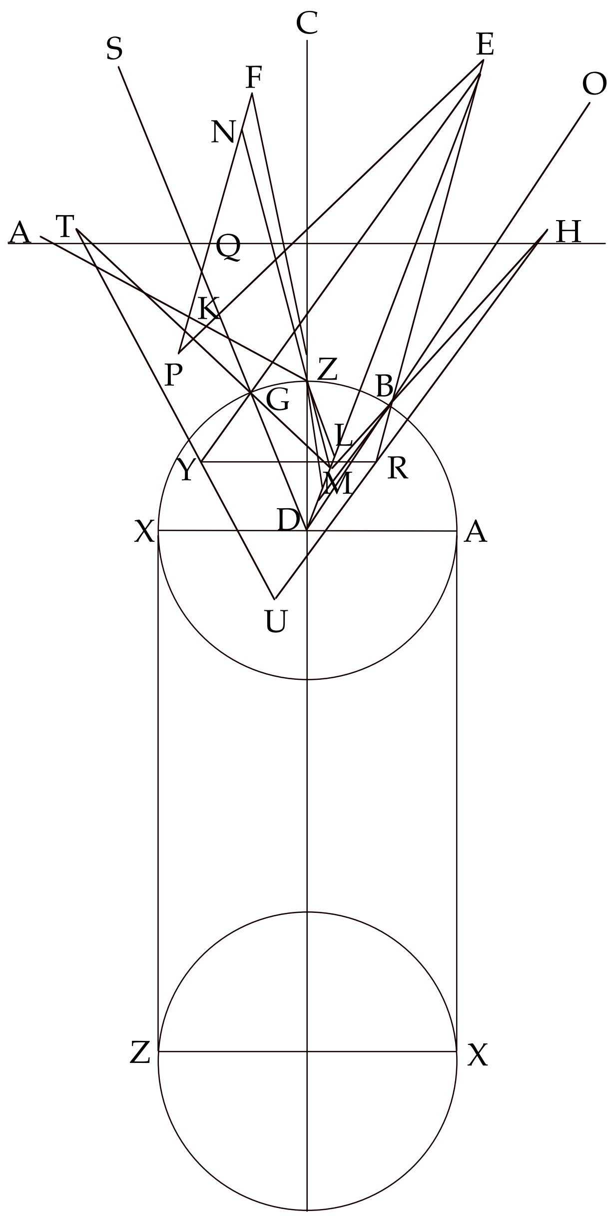

FIGURE 6.5.19

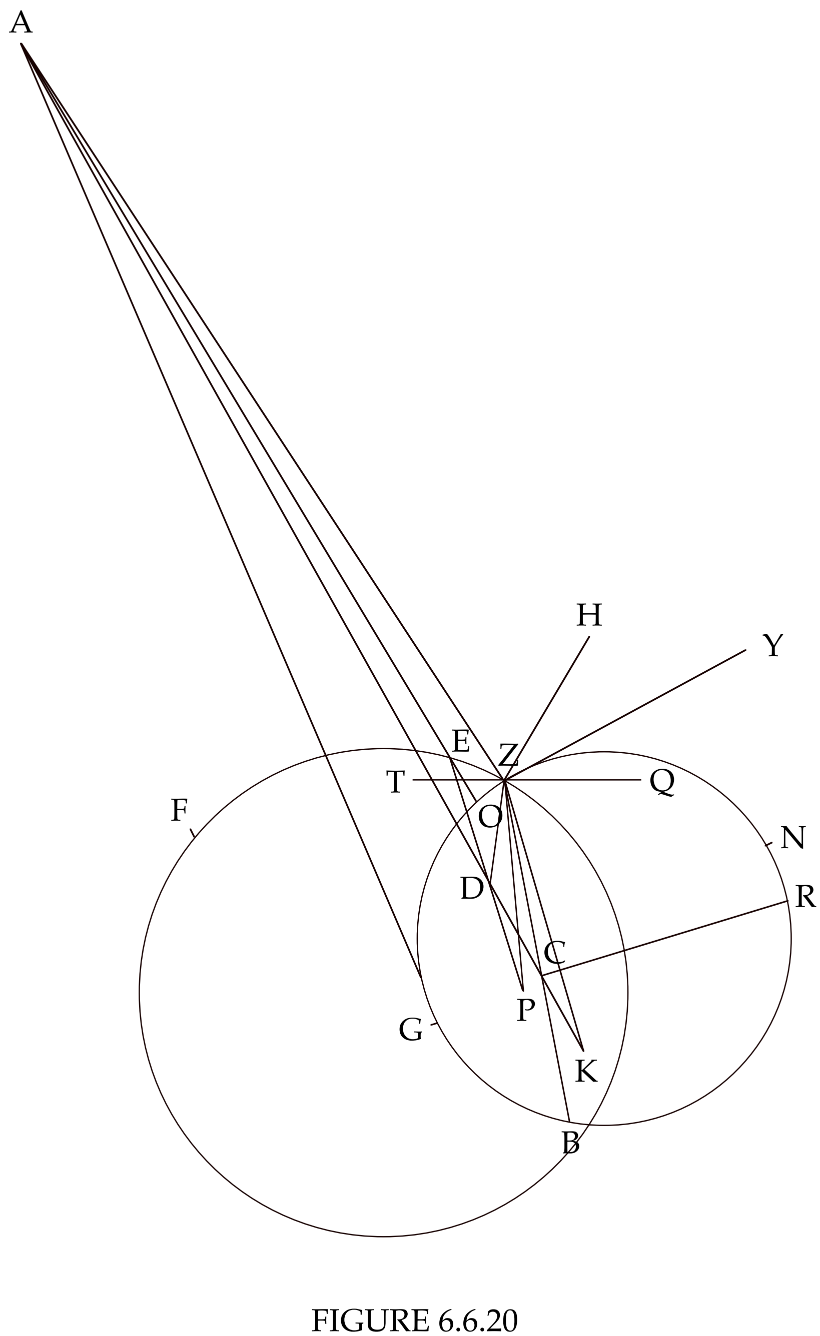

FIGURE 6.6.20

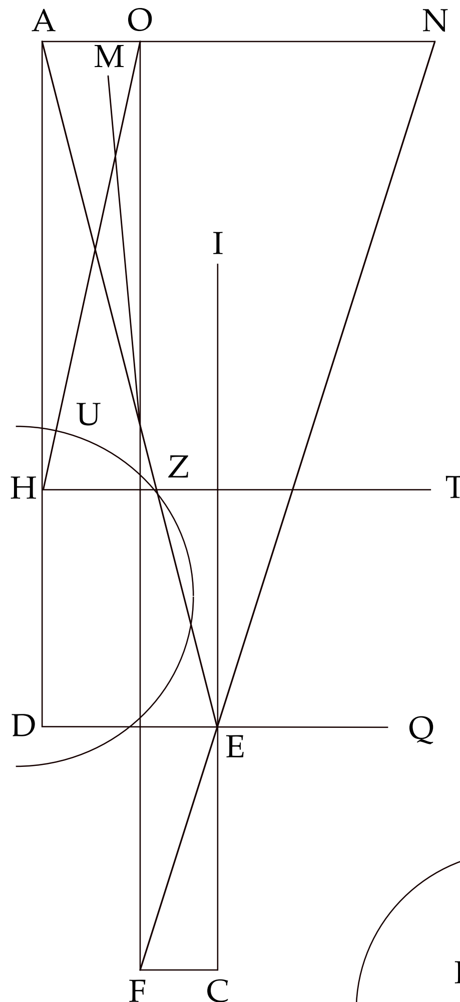

FIGURE 6.6.21

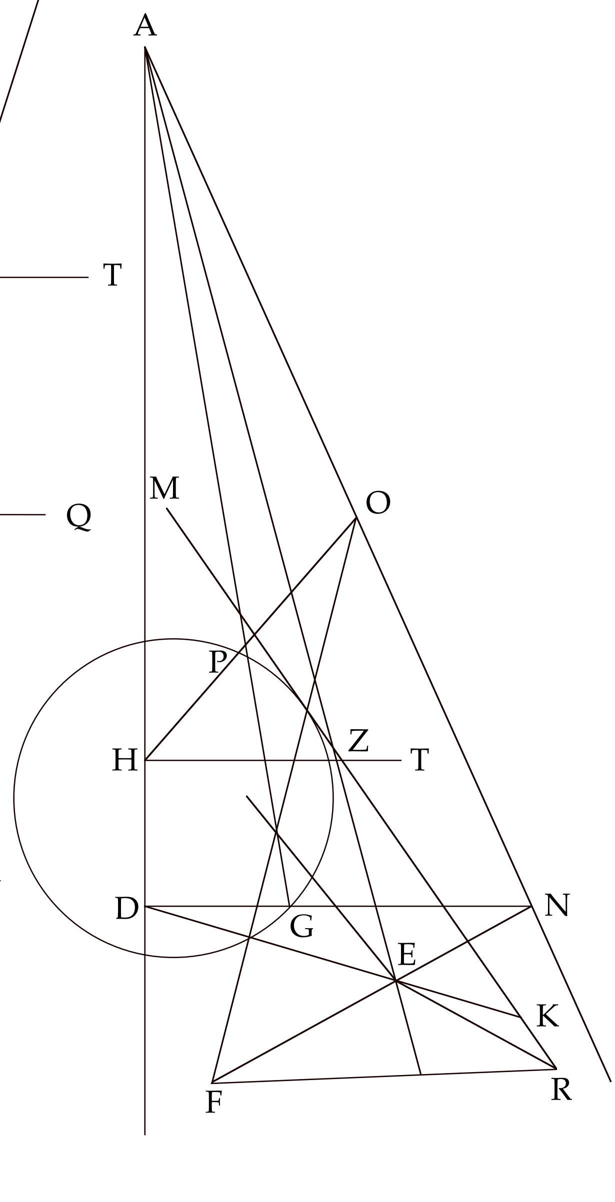

FIGURE 6.6.22

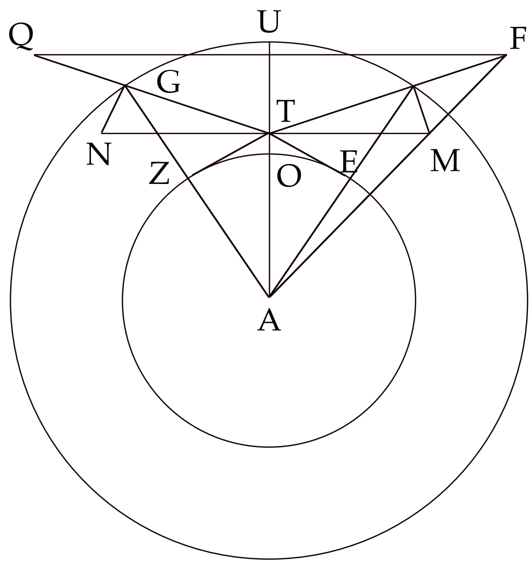

FIGURE 6.7.23

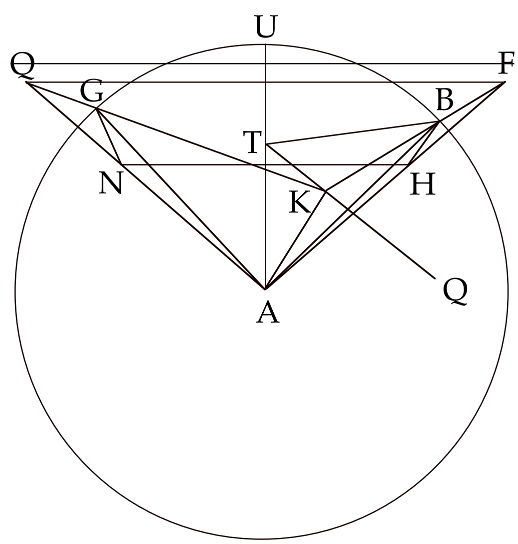

FIGURE 6.7.24

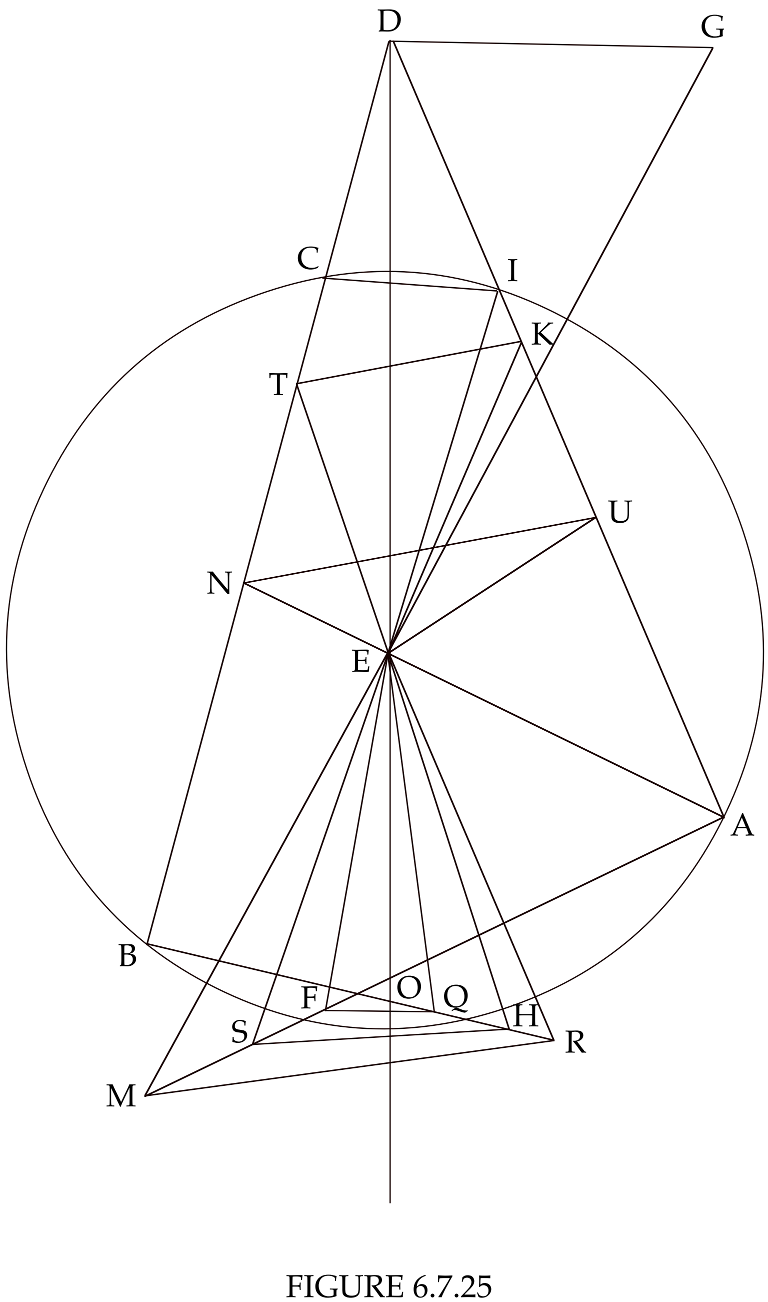

FIGURE 6.7.25

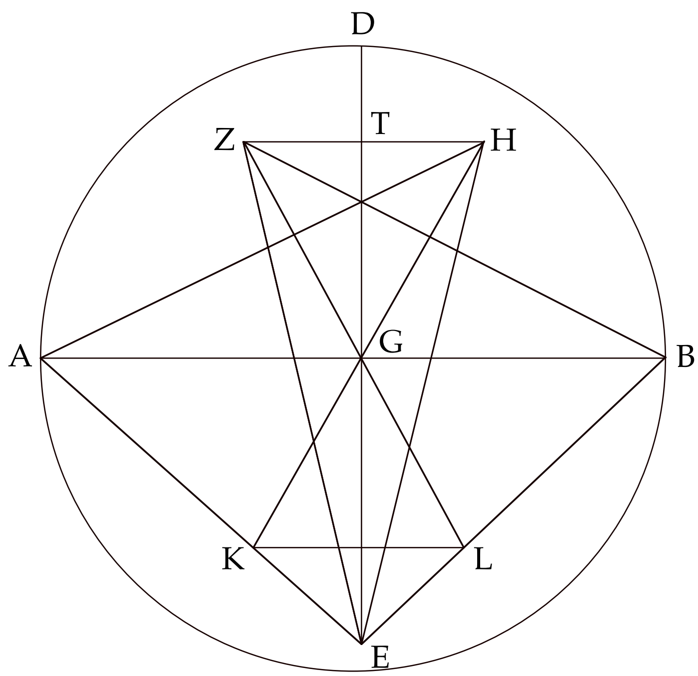

FIGURE 6.7.28

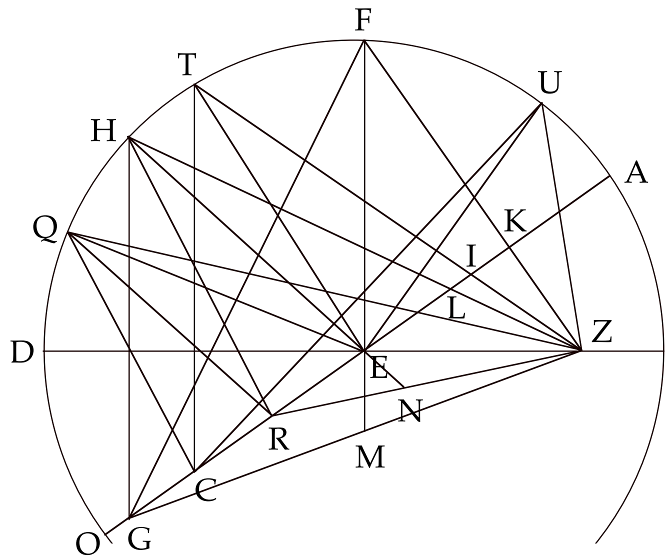

FIGURE 6.7.29

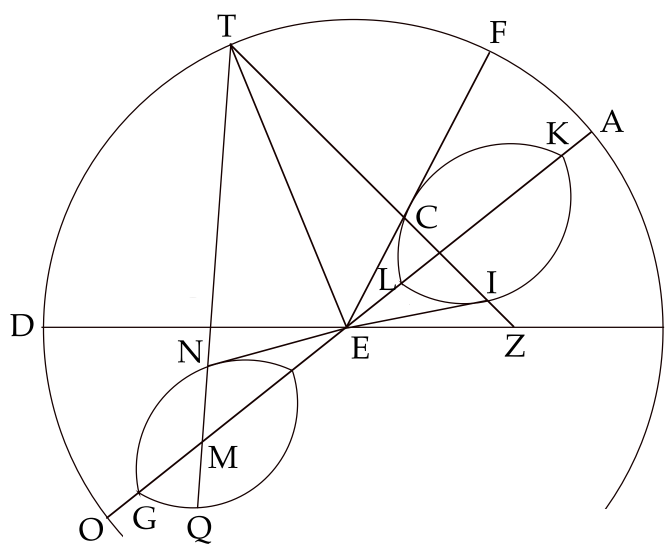

FIGURE 6.7.30

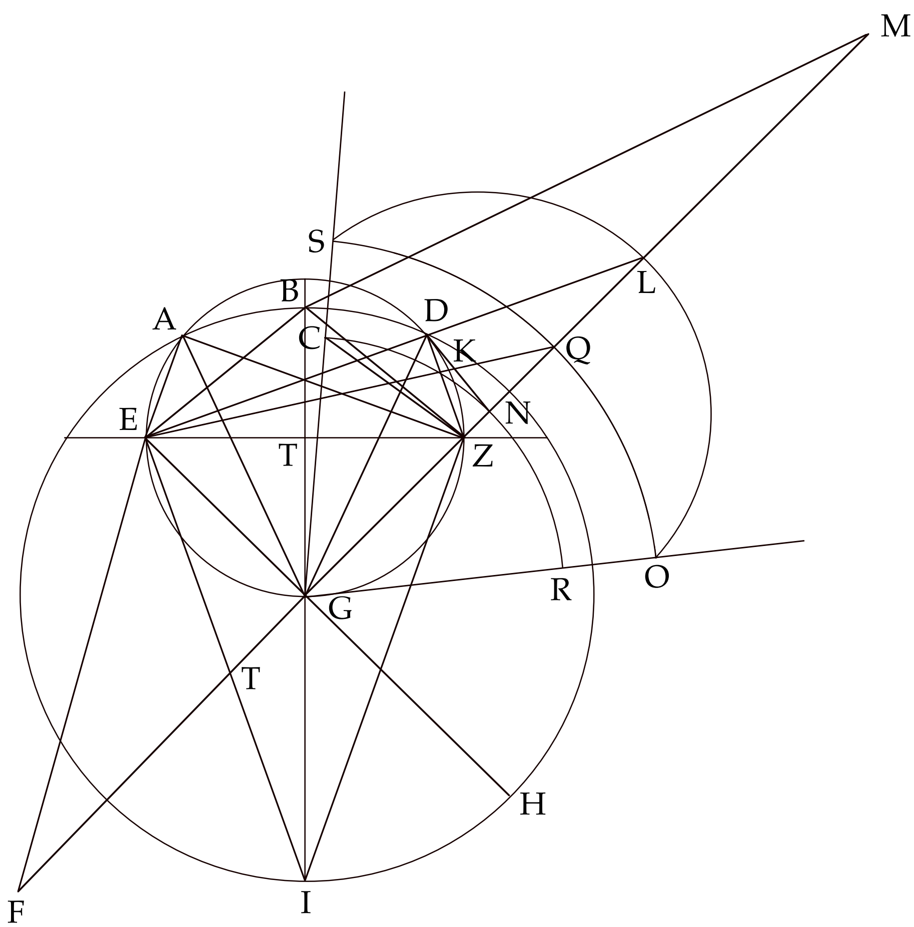

FIGURE 6.7.31

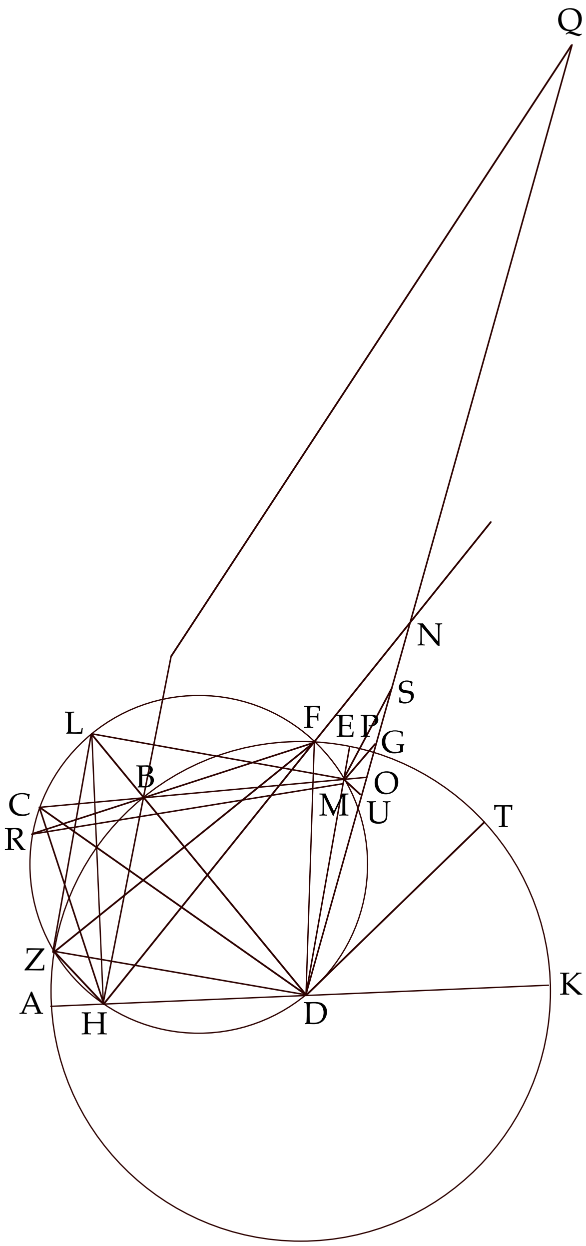

FIGURE 6.7.32

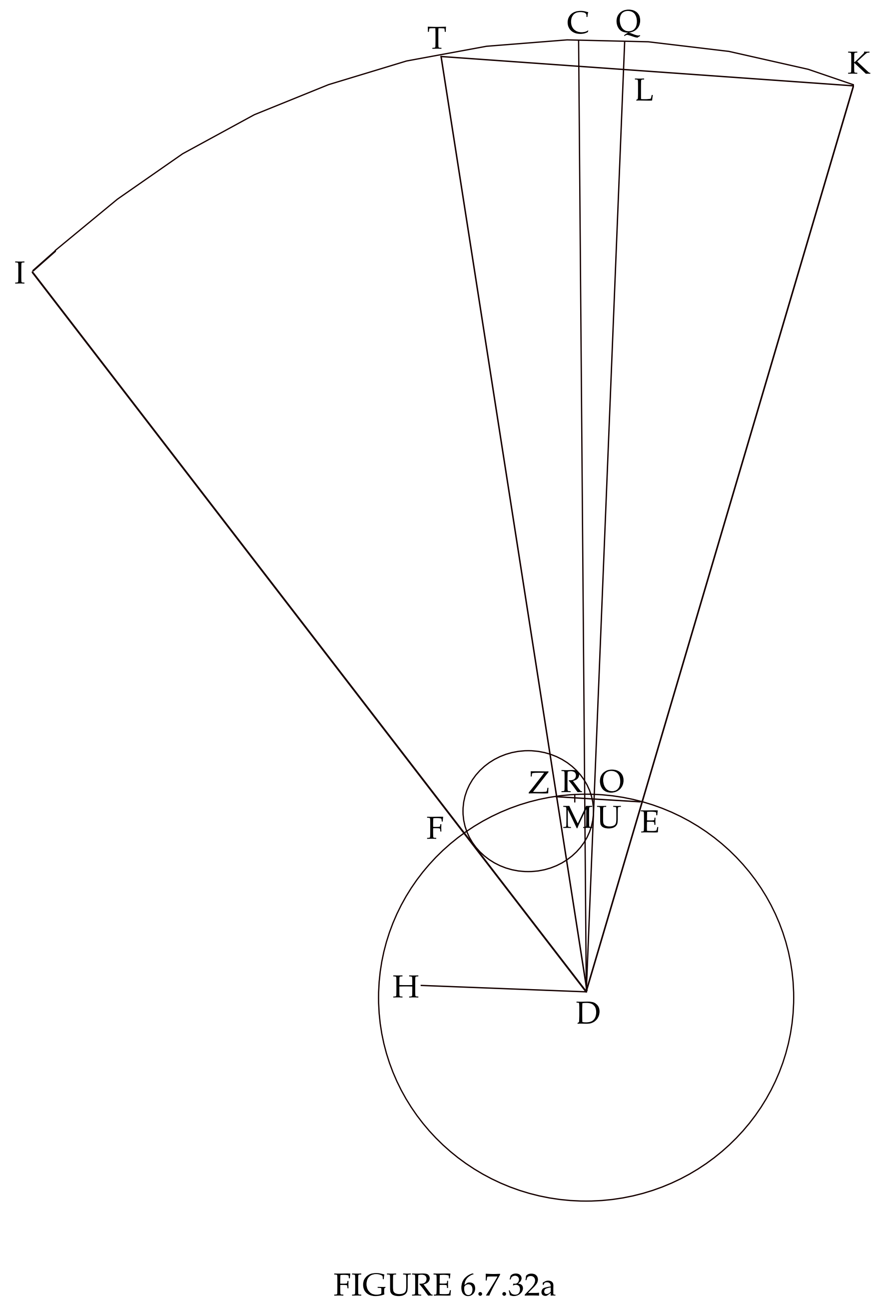

FIGURE 6.7.32a

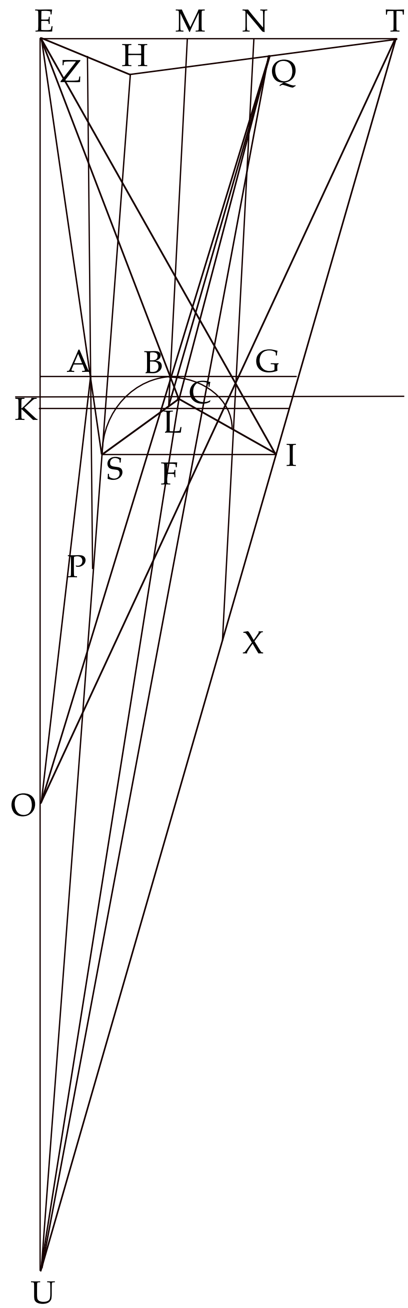

FIGURE 6.8.33

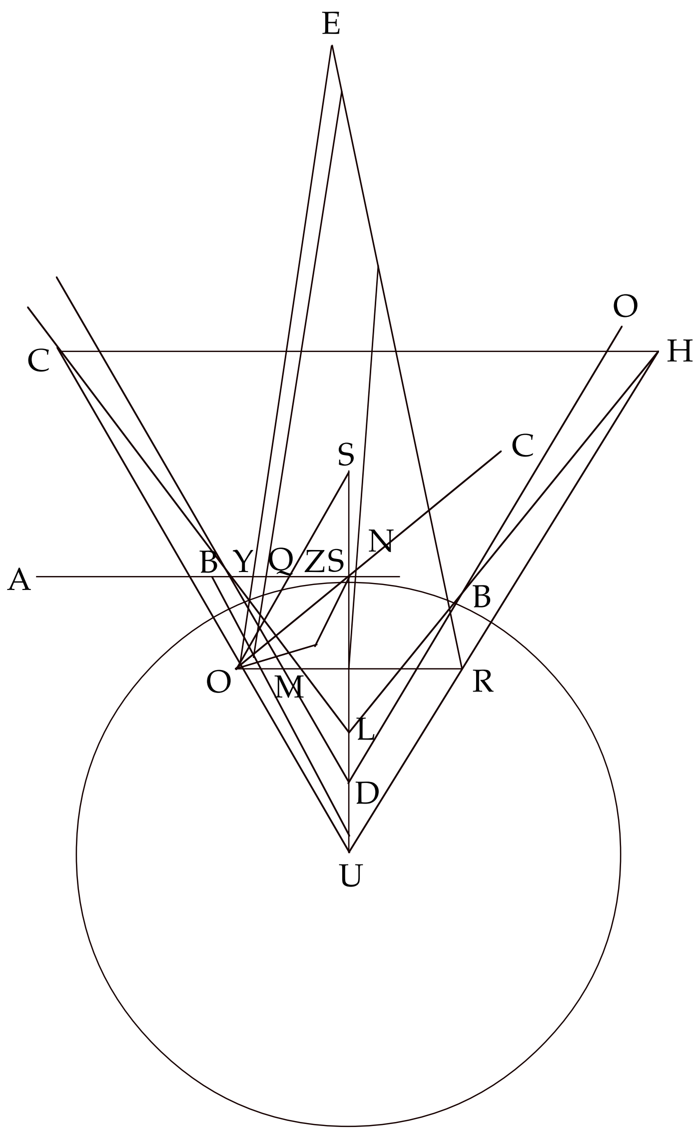

FIGURE 6.8.34

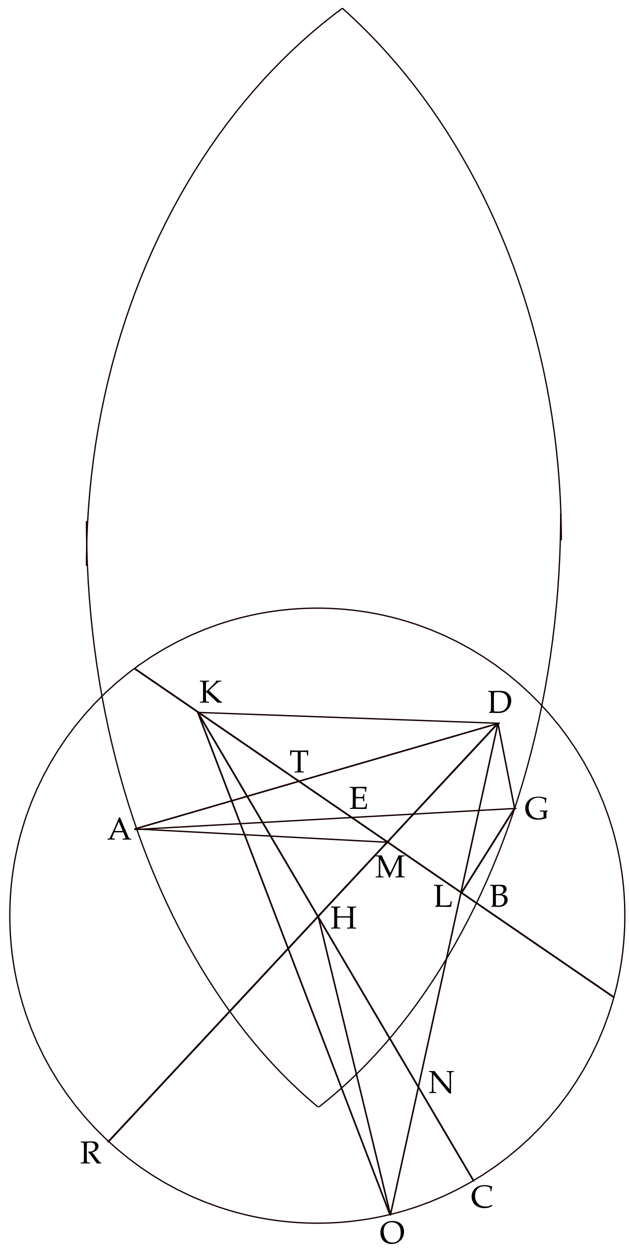

FIGURE 6.8.35

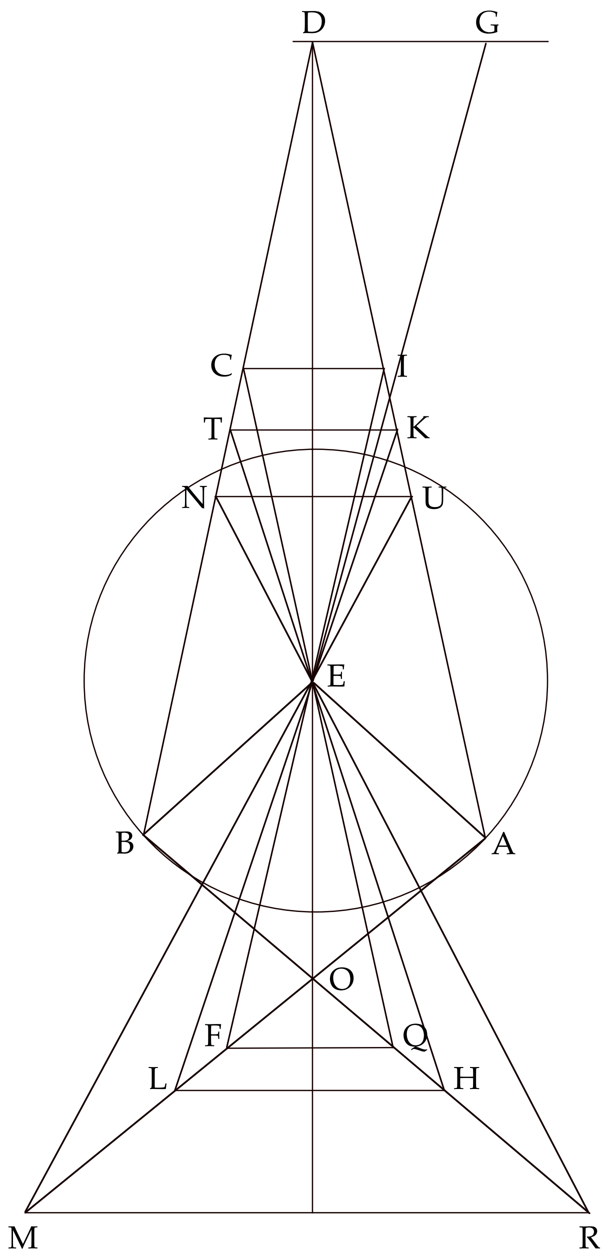

FIGURE 6.8.36

APPENDIX

CAPITULUM SEXTUM

De fallaciis que accidunt in speculis piramidalibus convexis erectis

[1] In istis autem accidunt deceptiones quarum causa est conversio, ut accidunt in speculis columpnaribus convexis, in nullo enim differunt preter quam in hoc quod forme que comprehenduntur in hiis sunt magis declinantes ad piramiditatem, et quod est in capite speculi ex forma est strictius.

[2] In ceteris vero rebus adequantur linearum rectarum, enim forme convertuntur a superficiebus istorum speculorum a lineis rectis, et ymagines earum sunt sicut ymagines linearum rectarum que equidistant vel appropinquant longitudini speculi columpnalis, scilicet quare erunt convexe parum, et convexitas earum erit ex parte visus.

[3] Et ymagines etiam linearum rectarum equidistantium latitudini speculi piramidalis erunt etiam sicut ymagines linearum rectarum que equidistant latitudini speculis columpnalis, scilicet quod linee quarum forme convertuntur erunt convexe convexitate manifesta, et erit centrum visus extra superficies in quibus est convexitas, et erunt diametri earum multum minores ipsis lineis. Ad huius autem demonstrationem premittamus hanc propositionem.

[4] **[PROPOSITIO 20]** Cum ceciderit aliquis sector in speculo piramidali convexo erecto, et in superficie eius fuerit extracta perpendicularis super superficiem contingentem piramidem in puncto conversionis, et fuerit etiam extracta alia linea perpendicularis super lineam contingentem circumferentiam sectoris in puncto remotiori a capite piramidis puncto conversionis, tunc, si ista perpendicularis extracta fuerit recte, concurret cum perpendiculari extracta a puncto conversionis super punctum existens sub puncto quod in axe.

[5] Sit ergo speculum ABG [figure 6.6.20alt, p. 153], et caput eius A, et cadat in ipsum sector ex sectoribus a quorum circumferentia

4 istis: quibus *O* / conversio: convexio *L3* 7 ex forma est: est ex forma *L3* 9 rectarum enim *transp.* (rectarum *inter.*) *O* 10 speculorum: speculum *L3* 11 earum: eorum *L3* 12 vel appropinquant *inter. O* / columpnalis: columpnaris *L3* 18 erunt: erit *L3* 19 *post* visus *add.* et *L3* 20 multum: multo *L3* 21 propositionem: proportionem *OL3* 22 sector: sectior *L3* 23 erecto: erecta *L3* / in *inter. O* 26 sectoris: sectionis *L3* 27 piramidis: piramidali *L3* 28 extracta[1]: exstracta *O* 29 *post* quod *add.* est *L3* 31 ipsum: speculum *L3* / sector: sectoris *O*

CHAPTER SIX

Concerning the misperceptions that occur in right convex conical mirrors

[1] In these [sorts of mirrors] there occur the misperceptions whose cause is reflection [itself], as happens in convex cylindrical mirrors, for [those mirrors] differ only insofar as the forms that are perceived in them are more inclined toward a conical shape, and the part of the form that lies at the vertex of the mirror is most acutely narrowed.

[2] As for the remaining phenomena, however, they are appropriately explained by rectilinear radiation, for forms are reflected from the surfaces of these mirrors according to straight lines, and their images are like the images of straight lines that are parallel or nearly parallel to the [length along the] longitude of a cylindrical mirror, i.e., because they will be somewhat convex, and their convexity will lie toward the center of sight.

[3] In addition, the images of straight lines posed along the width of conical mirrors will also be like the images of straight lines posed along the width of cylindrical mirrors, that is, the lines whose forms are reflected will be convex with a pronounced curvature, and the center of sight will lie outside the surface containing the convexity, and their cross-sections will be much shorter than the lines themselves. To demonstrate this, let us set forth this proposition.

[4] **[PROPOSITION 20]** When a given [conic] section falls on [the surface of] a right convex conical mirror, and when a normal is dropped to the plane tangent to the cone's surface at the point of reflection, and when another line is also dropped perpendicular to a line tangent to the periphery of the [conic] section at a point farther from the cone's vertex than the reflection-point, then, if this latter perpendicular is extended in a straight line, it will intersect the normal dropped from the point of reflection at a point lying outside [any] point on the [cone's] axis.

[5] Accordingly, let ABG [in figure 6.6.20alt] be the mirror, A its vertex, and let one of the conic sections from whose periphery forms are reflected

convertuntur forme. Sit ergo BFZ, et punctum conversionis sit E, et sit perpendicularis exiens a puncto conversionis in superficie sectoris linea ED. Et tangat linea TZQ sectorem BFZ in Z, et sit Z remotius a puncto A quam E. Et transeat per Z superficies equidistans basi piramidis, et faciet circulum LZB. Iste ergo circulus secat sectorem BFZ, quia circulus est perpendicularis super axem AD, et sector est obliquus super ipsum.

[6] Et continuemus ZA, AE, et extrahamus AE donec concurrat cum circumferentia LZB in O. O ergo est remotius a puncto A quam E, quia AO est sicut AZ. Cum ergo exiverit ex O perpendicularis super superficiem contingentem superficiem piramidis transeuntis lineam AO, concurret cum axe ultra punctum D. Sit ergo concursus K, et continuemus KZ. Erit ergo perpendicularis super superficiem contingentem piramidem transeuntem per AZ, quia angulus AZK est rectus, et linea QZ est in hac superficie contingente. Angulus ergo QZK est rectus.

[7] Et extrahamus ex Z differentiam communem superficiei circuli et superficiei contingenti piramidem, et sit ZY. Et sit centrum circuli C, et continuemus CZ. Angulus ergo CZY est rectus. Et extrahamus ex C lineam in superficie circuli continentem cum linea CZ angulum rectum, et sit CL. Linea ergo CL est equidistans linee ZY, et linea CL est perpendicularis super superficiem AZK, quia angulus ZCL est rectus positione. Et angulus ZCA est rectus, quia AC est perpendicularis super superficiem circuli. Linea ergo ZC est perpendicularis super superficiem ACL. Et ZY est equidistans CL; linea ergo ZY est perpendicularis super superficiem AZC. Ergo linea ZQ est obliqua super superficiem AZC.

[8] Et extrahamus a puncto Z in superficie sectoris lineam continentem cum linea ZQ angulum rectum, et sit ZH. Et quia D in superficie sectoris ex axe est aliud a puncto K, ut predictum est, erit K extra superficiem sectoris. Sed DZ est in superficie sectoris; ergo KZ est

32 ergo *om. L3* / BFZ: BEG *O*; FZ *L3* 34 TZQ: TZK *O*; CZQ *L3* / BFZ: BEF *L3*; *alter. ex* BG *in* BEG *O* 35 per: super *L3* 36 *post* LZB *inter.* F *O*; *add.* FG *L3* 37 BFZ: BEG *O*; BEF *L3* 38 obliquus: oblitus *L3* 39 *post* continuemus *add.* DZ *O* / ZA: AZ *O*; DZA *L3* / AE[1]: ZAE *L3* / AE[2]: AZ *L3* 40 LZB: LZF *O*; LZG *L3* / O[2] *om. O* / O ergo *transp. L3* 41 exiverit: exierit *L3* 43 concurret: concurrat *L3* / sit ergo *transp. O* 44 K: Q *O* / et *om. L3* / KZ: QZ *deinde inter.* F in arabico *O* / *post* KZ *add.* DZ *L3* / ergo *inter. O* 45 AZK: AZQ *OL3* 46 QZ: KZ *OL3* / contingente: contingentem *O* 49 ZY: ZK *O*; *alter. ex* ZRQY *in* ZRY *L3* / et[3] *om. L3* / et sit[2] *inter. O* 51 ex: in *L3* / circuli *om. L3* / continentem: contingentem *L3* 52 CL[1]: OZ *O*; CZ *L3* / CL[2]: CZ *OL3* / ZY: ZK *OL3* 53 CL: CZ *OL3* / AZK: AZC *OL3* / ZCL: AZ *L3* 56 ACL: AZQ *OL3* / ZY[1,2]: ZF *L3* / est equidistans *mg. O* / CL: CZ *OL3* (*mg. O*) 57 AZC: AZQ *OL3* 58 AZC: AZQ *OL3* 59 Z: DZ *OL3* / continentem: contingentem *L3* 60 ZQ: ZK *OL3* 61 est[1] *om. L3* / aliud: illud *L3* / K[1]: E *OL3* / K[2]: Q *O*; *om. L3* 62 DZ *corr. ex* DEZ *L3* / KZ: QZ *O*; quia *L3*

fall on it. Let it be BFZ, let E be the point of reflection, and let the normal dropped from the point of reflection within the plane of the [conic] section be line ED. Let line TZQ be tangent to [conic] section BFZ at Z, and let Z lie farther from point A than E [does]. Then let a plane pass through Z parallel to the base of the cone, and it will form circle LZB. This circle therefore intersects [conic] section BFZ, because the circle is perpendicular to [the cone's] axis AD, and the [conic] section is oblique to it.

[6] Let us then draw ZA and AE, and let us extend AE until it intersects the circumference [of circle] LZB at O. O thus lies farther from point A than E [does], because AO = AZ. Hence, if a normal is dropped from O to the plane tangent to the surface of the cone and passing along line AO, it will intersect the axis below point D. Let K be the [point] of intersection, then, and let us draw KZ. It will therefore be perpendicular to the plane tangent to the cone and passing through AZ, because angle AZK is right, and line QZ lies in this tangent plane. Angle QZK is therefore right.

[7] Now from Z let us draw the common section of the circle's plane and the plane tangent to the cone [at Z] and let it be ZY. Let C be the center of the circle, and let us draw CZ. Angle CZY is therefore right. Let us then draw a line from C in the plane of the circle that forms a right angle with line CZ, and let it be CL. Line CL is thus parallel to line ZY, and line CL is perpendicular to plane AZK, because angle ZCL is right, by construction. Angle ZCA is also right, because AC is perpendicular to the plane of the circle. So line ZC is perpendicular to plane ACL. In addition, ZY is parallel to CL, so line ZY is perpendicular to plane AZC. Consequently, line ZQ is oblique to plane AZC.

[8] From point Z let us now draw a line within the plane of the [conic] section that forms a right angle with line ZQ, and let it be ZH. Since D on the axis within the plane of the [conic] section is distinct from point K, as was claimed earlier, K will lie outside the plane of the [conic] section. But DZ lies in the plane of the [conic] section, so KZ lies outside the plane of the [conic] section. ZH, on the other hand, lies in the plane of

extra superficiem sectoris. Sed ZH est in superficie sectoris; ergo KZ secat ZH, nec continuatur cum ipsa. Sunt ergo in eadem superficie preter superficiem sectoris secante superficiem sectoris super lineam ZH. Ergo ZD, que est in superficie sectoris, est extra superficiem in qua sunt linee KZ, ZH.

[9] Sed ZK continet cum ZQ angulum rectum, quia ZK est perpendicularis super superficiem contingentem piramidem, que transit per lineas AZ, ZQ. Sed linea QZ continet cum ZH angulum rectum. Ergo ZQ est perpendicularis super superficiem in qua sunt linee HZ, ZK. Et superficies QZH secat superficiem AZQ super lineam ZQ, et secat lineam KZ in Z.

[10] Et puncta T, Z, K sunt a lateribus superficiei HZQ. Ergo superficies HZQ secat superficiem in qua sunt linee DZ, ZK. Differentia ergo communis superficiebus DZK, HZQ est in superficie HZQ. Continet ergo cum ZQ angulum rectum, nam ZQ est perpendicularis super superficiem HZK, et differentia communis hiis superficiebus est media inter duas lineas ZK, ZD. Ergo angulus HZD est obtusus, et linea HZ est in superficie in qua sunt linee DZ, ZQ, que est superficies sectoris, et continet cum ZQ angulum rectum.

[11] Ergo ZH extracta recte in parte Z secabit angulum DZK, et linea HZ concurret cum ED, nam declaratum est in quinto tractatu, in undevicesima figura, in capitulo de ymagine, quod omnes due perpendiculares super duas lineas contingentes sectorem debent concurrere ultra arcum sectoris in qua est punctum contactus. Cum ergo linea HZ concurrit cum ED, et HZ secat angulum DZK, ergo linea HZ concurret cum ED sub puncto D, et hoc est quod voluimus.

63 ZH: HZ *L3* / KZ: QZ *OL3* 65 preter *alter. in* post *O* / secante: secantem *L3* 66 *post* ZH *inter*. G in arabico *O*; *add*. verbi gratia HZG secantem *L3* 67 KZ: QZ *O*; quia *L3* 68 ZK: ZQ *OL3* 70 ZQ: ZK *OL3* / QZ: KZ *OL3* 71 ZQ: ZK *OL3* 72 ZK: ZQ *OL3* 73 *post* KZ *add*. CQ *O* 74 et *om*. *L3* / HZQ: HZIQ *OL3* 77 ZQ[1,2]: ZK *OL3* 78 HZK: HZQ *OL3* / *post* hiis *add*. duabus *L3* 79 est[1]: cum *L3* / ZK: KZ *L3* / HZD: KZD *OL3* 81 ZQ: ZK *OL3* 82 ZH: ZKH *OL3* / recte: recto *O* / et *inter*. *O* 83 ED *corr. ex* D *O* 85 debent: dabent *O* 87 concurrit: concurret *O* 88 concurret: concurrit *L3* / est *om*. *L3*

the [conic] section, so KZ intersects ZH and is not continuous with it. They therefore lie in the same plane outside the plane of the [conic] section and intersecting the plane of the [conic] section along line ZH. ZD, which lies in the plane of the [conic] section, therefore lies outside the plane containing lines KZ and ZH.

[9] But ZK forms a right angle with ZQ, because ZK is perpendicular to the plane tangent to the cone and passing through lines AZ and ZQ. Line QZ, however, forms a right angle with ZH. Therefore, ZQ is perpendicular to the plane containing lines HZ and ZK. Moreover, plane QZH intersects plane AZQ along line ZQ, and it intersects line KZ at Z.

[10] Moreover, points T, Z, and K lie to the side of plane HZQ. Therefore, plane HZQ intersects the plane in which DZ and ZK lie. Hence, the common section of planes DZK and HZQ lies in the plane of HZQ. It therefore forms a right angle with ZQ, because ZQ is perpendicular to the plane of HZK, and the common section of these planes lies in between the two lines ZK and ZD. Thus, angle HZD is obtuse, and line HZ lies in the plane that contains lines DZ and ZQ, which is the plane of the [conic] section, and it forms a right angle with ZQ.

[11] Accordingly, if ZH is extended in a straight line from Z, it will cut angle DZK, and line HZ will intersect ED, because it has been demonstrated in the nineteenth proposition of chapter [2] on image[-formation] in book 5 that any two lines perpendicular to two lines tangent to a [conic] section must intersect beyond the arc of the section where the point of contact is. Hence, since line HZ intersects ED, and since HZ cuts angle DZK, line HZ will intersect ED outside of point D, and this is what we wanted [to demonstrate].

LATIN-ENGLISH INDEX

LATIN-ENGLISH INDEX

ablatio 16.54 **subtraction** 170
ablatus *see* **aufere**
abscidere/abscindere 75.133, 136; 76.154, 168, 173 **to cut** 216, 217
accedere 22.208; 27.37; 35.266; 41.131; 47.7; 48.9 **to approach** 175, 185 **to get to something** 190 **to proceed** 179 **to verge towards** 196
accessus 3.10; 48.13 **closeness** 196 **incidence** 161
accidere 3.3; 4.27, 29; 5.51, 53; 6.82-84, 92, 97; 7.104, 108, 112, 116; 9.156, 157; 19.138; 23.230; 39.67, 68, 71; 48.16; 57.272, 276; 58.279, 280, 290-292, 294, 1; 59.10, 14, 22; 68.248; 81.293-295, 298-3; 92.294, 295, 2, 3, 5, 6, 8; 94.66, 67 **to arise** 163, 164, 221, 230 **to be** 204 **to happen** 162, 164, 166, 173, 203-205, 221, 230 **to occur** 161, 162, 164, 188, 196, 203, 204, 211, 221, 229, 230, 232
acumen 33.192; 48.24, 29; 50.73 **vertex** 183, 196, 198
acutus 8.153; 9.156; 10.187, 190; 13.280; 15.32; 16.47; 22.200, 202; 37.22; 39.87; 40.93; 41.127; 44.200; 50.78; 51.87, 91, 101; 52.132, 136; 53.142, 146, 149; 55.206, 207; 61.63; 63.133; 64.136, 137; 76.161; 78.206, 207; 83.62, 63; 84.67; 86.122; 87.152, 153; 89.214, 217, 220, 223; 93.40 **acute** 165, 166, 168, 170, 174, 175, 187, 189, 190, 192, 198-201, 206, 208, 217, 218, 223, 225, 227, 231
addere 9.169, 170; 18.105; 77.185 **to add** 166, 171, 218
additamentum 7.101 **increase** 164
adiacere 35.248 **to lie upon** 185
adinvicem 6.87 **corresponding** 163
adiungere 10.194 **to adjoin** 167
adquisitio 3.9; 4.23 **apprehension** 161
altitudo 25.272 **height** 177
angulus *frequently recurring* (8-10, 13-22, 24-29, 32-34, 37, 39-44, 46, 47, 49-55, 60-79, 82-84, 86, 87, 89, 90, 93) **angle**
angulus communis 17.76 **common angle** 171
angulus corporalis 20.164 **solid angle** 173
angulus rectus 17.77; 22.202; 53.145; 64.142, 145; 75.133, 134, 138 **right angle** 171, 175, 200, 208, 216
angulus reflexionis 13.270, 280 **angle of reflection** 168
anima 4.27 **soul** 162
antecedentia 23.239 **preliminary points** 176
apparentia 36.294 **appearance** 186
apparere 3.12; 4.34; 6.72; 7.109, 120; 23.236, 237; 34.232; 35.273; 36.289, 290, 295, 296; 38.57-59; 39.65, 72, 73, 75, 81; 45.234; 59.27; 81.7; 88.198; 89.204; 93.31 **to appear** 161-164, 176, 184, 186, 188, 189, 193, 205, 221, 227, 230 **to be apparent/visible** 164, 185

apponere 6.95 **to apply** 164
apprehendere 62.107 **to apprehend** 207
appropinquare 57.254, 255, 258, 259, 277 **to approach** 203
appropriare 92.298 **to apply to** 229
aptare 20.154 **to apply** 173
arcualis 88.192, 193; 90.244, 250 **curved** 226, 228
arcualitas 88.198 **curvature** 227
arcus *frequently recurring* (6, 15, 18, 19, 22, 27-38, 61, 66-80, 90) **arc**
aspiciens 67.225, 234, 235; 74.117 **viewer** 211, 216
assignare 4.37; 34.243 **to designate** 184 **to point out** 162
assimulare 58.282 **to take the shape of** 203
attingere 35.262 **to reach** 185
aufere 16.38 **to subtract** 170
augmentare 48.13, 18 **to augment** 196
axis *frequently recurring* (6, 39-50, 52-54, 56, 82, 83, 85-91, 93, 94) **axis, visual axis**
axis visualis 6.96 **visual axis** 164

basis 18.110, 111; 21.170, 171, 186; 27.49; 40.94, 106; 42.145, 165; 48.29, 31; 50.82; 52.127; 57.260, 271, 274; 67.217; 75.150; 82.25; 87.149, 150, 161; 89.228; 90.246, 249, 253 **base** 172, 174, 179, 189, 191, 196, 198, 199, 203, 210, 222, 225-228
brevis 29.105; 30.107, 108 **short** 180, 181

cadere 8.129, 130, 140; 10.185, 195, 196; 12.255; 13.258; 14.291; 15.26; 17.70; 18.103; 19.118, 120; 22.200, 201, 205, 206, 216-218; 23.221, 222, 224, 225; 24.262; 25.294; 28.63; 29.80, 81, 89, 90; 30.125; 31.136, 137, 162; 32.163, 189; 33.191, 204, 214; 34.230, 237; 37.23, 33-35; 41.128; 42.151; 46.260-262; 49.34; 51.89; 52.114; 57.262, 265; 84.76; 87.151 **to drop** 167, 180 **to fall** 165, 167-172, 174-177, 179-184, 187, 190, 194, 197-199, 203, 223, 225 **to intersect** 191 **to lie** 166, 171, 175, 176, 203 **to touch** 181
capitulum 58.289, 300; 59.17, 24, 31; 65.162; 68.246; 69.280, 287; 78.216; 81.297; 84.72; 85.92, 98; 90.255; 91.283; 92.299, 1, 8; 94.59, 63, 69 **chapter** 204, 205, 209, 211, 219, 221, 223, 224, 228-232
caput 7.107, 109, 111, 112; 10.195; 30.109; 36.289; 45.234, 236, 239; 52.120, 124; 56.231, 241; 57.258, 266, 273; 92.12 **end** 164, 186 **endpoint** 167, 181 **head** 193 **terminal segment** 193 **vertex** 199, 202, 203, 230
casualis 60.45 **random** 205
casus *see* **cadere**
causa 3.15; 7.104; 58.284, 294; 59.30; 92.6 **cause** 204, 230 **reason** 161, 164, 203, 205
centrum *frequently recurring* (7-9, 15, 22-25, 27-31, 34-36, 38, 43, 45, 49, 57, 59-61, 63, 66-68, 72, 73, 75, 78, 79, 85, 90, 91, 93, 94) **center, centerpoint, center of sight** *see also* **punctum centrum**
centrum visus 5.59; 23.241, 242, 245; 24.247; 27.44, 50; 31.153; 34.225, 240; 35.255,

274; 36.286, 6; 45.229; 57.250; 59.27; 85.96, 97; 90.254; 93.22, 33, 42; 94.56 **center of sight** 163, 176, 179, 182, 184-186, 193, 203, 205, 224, 228, 230, 231

certificare 59.27 **to be clear** 205

circuitus 63.110; 67.226 **(area) around** 207, 211

circulus *frequently recurring* (7-9, 13, 15, 17-22, 24, 29-31, 33-37, 39-45, 49-53, 57, 59-64, 66-68, 71-73, 75, 76, 78-80, 82, 83, 85, 87, 89-91) **circle, great circle**

circulus communis 37.9 **common section** 186

circulus magnus 61.83; 73.95 **great circle** 206, 215

circumferentia 53.138; 54.191, 193; 55.214, 219; 56.225; 59.25; 61.84; 63.115; 64.152; 76.165, 167, 178; 80.267, 269; 82.31; 84.75, 88; 86.130; 89.212; 90.234 **circumference** 199, 205-208, 217, 218, 220 **periphery** 201, 202, 222, 223, 225, 227, 228

coalternus 42.156 **alternate** 191

cognoscere 4.25 **to know** 162

collateralis 13.270 **vertical (angle)** 168

collatio 4.26 **correlation** 162

color 4.23, 35, 36, 38, 39, 42, 43; 6.74, 76-78, 82, 91; 58.293; 59.13; 81.1; 92.4 **color** 161-164, 204, 221, 230

columpna 3.7; 40.94, 95, 98, 106; 41.132, 134, 137, 138; 42.145, 165; 43.196; 45.235, 236, 242, 243, 245; 46.257; 47.289, 6, 8; 48.10; 83.43, 44, 59; 84.69, 74, 78; 86.123, 124, 127, 136, 138, 142; 87.145, 146, 149, 150, 161, 162; 88.182, 183, 185, 186, 188, 194, 200; 89.205, 208-210, 218, 228, 229; 90.245-247, 249, 254; 91.264, 270, 271 **cylinder** 189-196, 222, 223, 225-229 **cylindrical mirror** 161, 222, 227

columpnalis 56.247; 58.280; 81.10; 84.83; 85.114, 118; 90.258; 91.262; 92.294, 4, 7, 9; 93.15, 27, 30; 94.66 **cylindrical** 203, 221, 223, 224, 228-230, 232

columpnaris 3.5; 39.70, 76-78, 84; 48.16; 81.298; 84.73 **cylindric/cylindrical** 161, 188, 189, 196, 221, 223

communis 5.57; 9.179; 16.38, 44, 54; 17.77; 18.105; 21.178, 191; 30.113; 31.151, 154, 156; 34.227; 37.9; 41.118, 137; 42.169; 44.219; 45.235; 49.32, 50, 54; 50.60, 62, 65, 66, 69; 53.144; 55.199; 56.226; 57.269; 61.83; 82.27; 83.42; 85.111; 86.137; 88.176, 178; 90.238 **common** 166, 170, 171, 174, 181, 182, 190, 191, 193, 196, 197, 200-203, 206, 222, 224-226, 228 **common arc** 171 **common section** 163, 184, 186, 193 **common term** 170 *see also* **angulus communis, circulus communis, differentia communis, linea communis, superficies communis**

compar 63.113, 116; 64.153; 67.228, 229; 73.95; 76.156; 78.222, 229 **counterpart** 215 **equivalent** 207, 208, 211, 215, 217, 219

componere 58.287; 81.295, 296; 94.67 **to compound** 204, 221, 232

comprehendere 3.12; 4.32, 46; 5.53; 6.78, 85, 89; 7.119; 9.158, 162; 36.1; 52.120; 56.241; 57.272; 58.282, 287, 296, 297, 3; 59.5, 11, 19-21, 32, 33; 60.34, 35, 37; 61.60, 76; 62.108; 63.117, 122; 64.148, 153, 161; 65.163, 165; 66.193, 201, 210; 67.234, 235, 238, 239, 241; 68.244, 254; 71.21, 39, 43, 45; 72.73; 74.117; 75.126; 80.274, 275, 281-283, 285, 288; 81.289, 290, 293; 83.60; 84.70, 80; 86.129; 90.257; 92.295; 94.56-58, 65 **to perceive** 161-164, 166, 186, 199, 202-216, 220, 221, 223, 225, 228, 229, 231, 232

comprehensibilis 5.47 **perceptible** 162

comprehensio 3.14; 4.19, 21; 5.51; 36.3; 59.7, 8 **perception** 161, 162, 186, 204
concavitas 83.58, 65; 84.66, 76, 77; 85.94; 86.123; 87.145, 157; 88.195-197; 89.217; 93.18, 32 **concave** 223 **concavity** 223-227, 230 **curvature** 231
concavus 3.6, 7, 12; 58.290, 297, 4; 59.6, 10, 18, 21; 60.36, 37, 40; 63.122, 123; 65.165; 66.193, 195; 67.234; 68.243, 246; 71.42, 44-46; 72.47; 74.100, 104, 110, 111, 121; 75.125-128; 79.260; 80.272, 275, 281-284, 288; 81.289, 290, 293, 298, 299, 3, 5, 8; 83.59, 60; 85.93, 106, 115, 116; 86.127; 89.204, 208; 90.258; 91.260, 262; 92.293, 294, 296, 2, 4, 7, 9, 10, 13; 93.15, 27, 29, 31, 36; 94.60, 67 **concave** 161, 204, 205, 207-211, 214-216, 220, 221, 223-225, 227-232
concurrere 8.151; 25.281; 26.7, 8, 10, 19, 21, 22; 27.27-30; 28.71, 72, 75; 29.79, 84; 33.201, 202; 34.227-229; 35.272, 275; 36.285, 7; 37.14-16, 18, 22, 23, 28, 29; 38.36, 42, 50; 40.89, 102; 41.129; 42.153; 43.189, 191, 192; 44.198, 201, 202, 204-206, 208-211, 215, 216; 45.252; 46.256, 280; 47.288, 291, 292, 296-299; 48.26; 49.37, 39; 50.70, 71, 76, 81; 51.90, 91; 52.128, 129, 136; 53.138, 139, 148, 149, 165, 166; 55.210, 214; 56.227; 57.267; 61.64-66; 62.87, 89; 63.135; 64.137, 139, 143, 146; 68.265, 266; 72.66, 68, 69, 71, 72; 75.141, 142, 146; 76.161-163; 80.265; 85.94; 86.143; 89.219, 222, 223 **to fall** 198 **to intersect** 177-180, 183, 184, 186-203, 206-208, 212, 215, 217, 220, 224, 225, 227 **to meet** 165, 180, 185, 186, 192
concursus 8.151; 28.76; 29.79; 37.16, 28; 38.50, 52; 40.89; 42.153; 43.190, 192; 44.199; 47.298; 58.285 **intersection** 165, 180, 187-189, 191, 192, 195, 204
confundere 6.80 **to confuse** 163
confusio 6.76 **blend** 163
coniecturatio 4.26 **conclusion** 162
coniungere 82.13; 85.109 **to converge** 224 **to intersect** 221
constare 25.288, 292, 295; 26.300, 2, 3 **to be compounded** 177, 178
constituere 58.284 **to determine** 204
conterminabilis 35.277; 36.278, 280 **bordering** 185, 186
continere 52.131, 134; 53.145, 164; 63.129, 130, 134; 64.139, 142, 144; 66.204; 75.137, 140, 144; 86.122; 89.213; 93.39 **to form** 199, 200, 208, 210, 216, 217, 225, 227, 231
contingentia 22.218; 23.219, 222, 224, 226, 243, 244; 24.260, 271; 25.272, 276, 277; 28.67, 77; 29.86, 94; 31.160; 33.210; 37.13, 19, 26, 27, 33; 38.42 **tangency** 175-177, 179, 180, 182, 184, 186, 187
contingere 8.127, 128; 13.259, 269; 17.73; 19.118, 120; 22.216, 217; 24.259, 261; 31.149, 152, 159, 161, 162; 32.163, 164, 170, 188; 33.190, 191, 214; 35.274; 37.12, 20, 31; 39.88; 40.93, 98, 100, 103, 104; 41.128; 43.197; 45.250; 47.290; 48.25; 49.36, 41, 47, 53-55; 50.75, 78, 85; 51.88, 95; 52.130, 132; 53.142, 144, 151; 55.205, 208, 212, 215, 218; 57.268; 82.15, 31; 84.69, 73-75, 78, 84; 86.123, 130, 141; 87.162, 168; 89.210, 212; 91.264; 93.19, 38 **to be tangent to** 165, 168, 171, 172, 175, 176, 182-187, 189, 190, 192, 194-203, 221-223, 225-227, 229-231
continuare 53.141; 54.171, 188, 191; 55.204; 60.47-49; 61.68, 80; 62.86, 88, 90; 63.128, 131; 64.147; 65.168, 172, 174, 184, 185; 66.208, 211; 68.261, 262; 69.268; 70.294, 3,

14; 71.28, 32, 33; 72.51, 56; 73.76, 77, 87; 74.113; 75.148; 76.155, 157, 160, 165, 177; 77.183; 79.234, 237; 80.265; 82.23, 34; 84.88; 86.133, 140; 87.158, 164; 90.232, 236, 238 **to be continuous** 201 **to connect** 206-210, 212-218, 222, 223, 225, 226, 228 **to draw** 200, 201, 206, 210, 212, 217, 219, 220, 222, 225, 226, 228 **to extend** 201, 205 **to produce** 199

continuus 7.108, 112 **on the surface/plane of** 164

contrapositio 46.264 **vertical angle** 194

contrarius 6.87; 13.264; 23.237 **contrary** 168 **opposite** 163, 176

conversio 58.282, 294; 81.300; 83.42; 84.86, 90; 85.95; 92.297, 298, 6 **reflection** 203, 204, 221-224, 229, 230 **reversal** 229 *see also* **punctus conversionis**

conversus 11.222; 60.34; 64.157; 65.180; 66.192; 67.237, 242; 68.245, 253; 72.59; 81.4, 7; 90.251, 252, 256, 258; 91.278; 93.23; 94.45, 50, 62 **converse** 167 **inverted** 205, 208-211 **reflected** 214, 230, 231 **reversed** 221, 228, 229, 231

convertere 54.176, 183, 187, 188; 56.234, 235, 242, 246; 58.283; 59.25; 60.59; 61.60, 82, 84; 62.101, 102; 69.277, 278, 283, 286, 287, 289, 291; 70.293, 300, 2, 3, 10, 12-14; 71.27; 72.60, 61; 73.80, 96; 74.114; 77.200; 78.227, 229; 79.244; 80.267, 269; 82.12; 84.70, 80, 81, 88; 85.94, 119; 86.120, 128, 130, 132, 136, 139; 87.167, 169; 89.210, 225, 227; 90.234, 235; 91.271, 273, 275; 93.20; 94.46 **to reflect** 200-203, 205-207, 212-216, 218-221, 223-231

convincere 27.29 **to make indisputable** 178

copulare 54.185 **to connect** 201

corda 9.176; 15.19 **chord** 166, 169

corporalis 20.164; 45.233 **having bodily dimensions** 193 **solid** 173

corpus 4.35, 46; 6.85, 96; 7.99, 120; 9.159; 38.59-61, 63; 39.65 **body** 188 **object** 162-164, 166, 188

dare 26.15; 29.102, 104; 30.107, 110, 113-115, 117-119; 40.92 **to give** 178, 180, 181, 189

debere 70.12 **to be obligated (must)** 213

debilis 4.30, 42 **diminished** 162 **weak** 162

debilitare 4.30, 38, 43 **to weaken** 162

debilitas 58.293; 59.13; 81.1; 92.4 **weakening** 204, 221, 230

debilitatio 6.82 **weakening** 163

deceptio 59.9, 10, 15 **deception** 204, 205

declarare 9.164; 27.39; 52.119; 53.157; 54.169, 172; 56.244; 57.261; 58.299; 59.16, 17, 30, 32; 60.38; 61.65; 62.105; 65.162; 67.232; 68.253; 70.18; 77.191; 78.215; 80.277; 81.296, 8; 84.73, 82; 86.131; 88.195; 89.208; 92.299; 93.25; 94.63, 69 **to analyze** 212 **to demonstrate** 166, 199, 203-207, 213, 218, 221, 223, 227, 229, 230 **to discuss** 205 **to establish** 200 **to explain** 232 **to prove** 202, 211, 225 **to show** 179, 200, 205, 209, 219, 220, 223, 227, 232

declaratio 94.68 **proof** 232

declinare 27.47; 28.64; 34.230, 234, 237; 36.295, 1; 90.250 **to incline** 179, 184 **to lie outside of** 186 **to slant** 179, 184, 186, 228

declinatio 34.234, 238, 242-244; 35.250, 263, 269; 36.281, 295; 50.68 **slant** 184-186, 197 *see also* **linea declinationis**
declinis 89.206; 90.245; 94.43 **inclined** 231 **slanted** 227, 228
defectus 18.96 **lack** 171
demonstratio 69.280; 78.218; 83.62; 87.147; 92.11; 93.14, 26 **demonstration** 230 **proof** 212, 219, 223, 225, 230
describere 72.53 **to circumscribe** 214
determinare 22.214 **to determine** 175
dexter 6.86-89; 64.157, 158; 67.239 **right(-hand side)** 163, 208, 209, 211
differentia 53.145; 55.200; 82.28; 94.44 **common section** 200, 201, 222, 231
differentia communis 53.144; 55.199; 56.226; 57.269; 61.82; 82.27; 86.137; 88.176, 178; 90.238 **common section** 200-203, 206, 222, 225, 226, 228
dignitas 58.286 **role** 204
dimidium 12.248, 249, 251, 253; 76.179 **one-half** 168, 218
diminuere 4.45; 36.294; 39.72; 48.18 **to diminish** 186, 188, 196 **to shorten** 162
dimissus 20.141; 28.78; 32.189; 37.26, 27, 32; 38.46 **below** 173, 180, 183, 187
directio 4.23, 25, 32, 39, 45; 5.47, 48, 54; 6.79, 81 **direct vision** 161-163
directus 4.19, 22, 28, 41; 6.78, 83, 93, 94, 96; 7.100, 105, 108, 112 **direct** 161-163 **direct vision** 161, 163, 164
discernere 6.75, 80; 9.163; 38.60, 61; 39.64 **to detect** 188 **to discern** 163, 166 **to perceive** 163
dispositio 3.13; 58.280 **disposition** 161 **way** 203
disquirere 3.16 **to discuss** 161
distantia 4.24; 7.110; 59.13; 60.44; 83.61 **distance** 161, 204, 205, 223 **separation** 164
distare 5.64, 69, 70; 29.92, 93 **to lie at a distance/far** 163, 180
distinctio 18.96; 75.136 **designation** 171 **sub-angle** 216
distinctus 85.110 **distinct** 224
distortus 3.13 **distorted** 161
diversare 23.245; 29.91; 30.107; 80.279; 81.5, 6; 83.60; 93.25 **to be opposite** 180, 181 **to differ** 176, 220 **to vary** 221, 223, 230
diversitas 3.16; 58.293, 295, 298; 59.12, 13; 60.38; 81.1, 2; 83.61; 92.4 **change** 204 **different types** 161 **variation** 204, 205, 221, 223, 230
dividere 9.170; 12.243; 15.14, 18, 23; 21.175; 22.194; 25.280, 283; 26.9, 13, 16; 27.31, 33; 29.95; 33.193, 199; 60.43; 66.203; 67.230; 69.273; 70.297, 7; 72.49; 75.135; 82.23, 24; 87.156 **to bisect** 166, 169, 174, 205, 212, 213, 216, 222 **to cut** 177, 178, 180, 183, 210, 214, 225 **to divide** 168, 177, 178
divisio 12.250; 26.17, 18; 29.83; 33.200 **division** 168, 178, 180, 183 *see also* **punctus divisionis**
dubitare 78.219 **to confuse** 219
ducere *frequently recurring* (5, 8-10, 13-15, 17-19, 22-33, 35-37, 39-52) **to combine, to draw, to drop, to extend, to produce, to project**
ductus. . .in. . . 9.170, 172, 173, 181; 10.186, 191, 194, 197-201, 206; 11.208, 209, 211,

214, 225, 228; 14.295, 296; 15.33, 35; 16.36, 39, 44, 49, 51-55, 59; 17.71-73; 41.124, 126; 46.268, 271 **rectangle** 166, 167, 169-171, 190 *see* **forming a rectangle**
duplex 36.299 **both** 186
duplicare 16.57, 60 **to duplicate** 170
dyameter 9.167, 169; 29.105; 40.105; 51.95; 56.239; 57.251; 61.68, 75; 62.105; 64.149; 65.176; 66.211; 67.223, 233; 68.258; 69.286; 71.20, 21; 83.64; 85.102, 110, 111; 87.160; 91.274, 276, 280, 281; 93.17, 23 **cross-section** 202, 203, 206-211, 224, 229, 230 **diameter** 166, 180, 189, 198, 212, 213, 223, 226

econversus 57.271; 67.240; 68.253 **opposite** 203 **vice-versa** 211, 212
efficere 4.28; 11.207, 208, 220, 228; 12.243; 33.218 **to cause** 162 **to equal** 167, 168 **to form** 184
egredi 4.20, 28; 6.78 **to exceed** 161-163
elevatus 24.270; 86.141; 87.146; 88.194 **lying above/beyond** 177, 225, 226
elongare 6.98 **to lie outside of** 164
elongatio 48.13 **lying far** 196
equalitas 32.167, 175, 176; 46.280; 87.156 **equal (measure)** 182, 183, 195, 226
equidistans *frequently recurring* (8, 13, 15, 17, 18, 25-28, 35-54, 56, 57, 60-62, 67, 77-79, 82, 83, 85, 87-90) **parallel**
equidistantia 15.28; 17.65; 87.159; 89.216 **parallelism** 170, 226, 227
equidistare 36.286; 42.164; 53.163; 61.74 **to be parallel** 186, 191, 200, 206
erectus 20.165; 48.28; 52.123; 60.35 **erect/erected** 174, 205 **right** 199 **upright** 196
erroneus 5.51 **misperception** 162
error 3.3, 4, 14, 15, 17; 4.21, 28, 29, 41, 44, 46; 5.49, 52, 53; 6.76, 83, 84, 91, 92; 7.104, 116; 39.67, 70, 77; 48.15 **error** 161-164, 188, 196
Euclides 10.192; 15.22, 34; 17.74 **Euclid** 166, 169-171
evenire 3.4; 6.97; 7.116; 48.16 **to arise** 161 **to happen** 164, 196 **to occur** 164
evitare 18.95 **to avoid** 171
excedere 9.157; 14.288, 292, 294, 296; 20.159; 30.110, 111, 113; 68.259; 74.116; 77.188, 192-196 **to be larger/longer** 166, 181 **to exceed** 169, 173, 218 **to extend past** 212, 216
excessus 10.201, 205; 11.216, 217, 219; 14.299-3, 6, 8; 15.11 **remainder** 167, 169
exemplum 6.95; 94.68 **example** 164, 232
exire 57.268; 66.203; 73.82; 82.30, 33; 83.55; 87.148-151, 161, 172; 88.185, 188 **to be dropped** 203 **to extend** 210, 222, 223 **to pass along/through** 215, 222, 225, 226
existere 20.142; 34.226; 36.279, 293, 297, 299, 2, 3, 5; 38.55; 57.253; 80.287; 91.260 **to be** 203 **to exist** 221 **to leave be** 228 **to lie** 173, 184-186, 188
explanare 3.16; 23.231 **to explain** 161, 175
explanatio 23.232, 238 **explanation** 175, 176
exponere 4.21 **to analyze** 161
extendere 56.243; 57.254; 60.59; 61.60; 62.100, 102; 82.14; 84.70, 79, 81, 86, 90; 86.127,

132; 87.169; 92.11 **to extend** 202, 203, 206, 207, 221, 223, 225, 226, 230 **to pass along** 223

exterior 3.5, 6; 7.117; 13.261; 18.103; 29.99, 103; 30.116, 118; 39.70, 71; 45.237, 240; 48.15, 16 **convex** 161, 164, 188, 193, 194, 196 **outer** 168, 171, 180, 181

extrahere 52.124, 127, 130, 131, 133; 53.140, 141, 147-149, 152, 153, 162, 163; 54.171, 177, 178, 181, 189; 55.202, 209, 212, 215, 218, 221; 56.241; 60.42, 46-48, 50; 62.85-87, 89; 63.118, 119, 124-126, 129, 130, 134; 64.138, 142-145; 65.167, 169, 172; 66.197, 200, 205, 209; 68.256, 258; 69.268, 269; 70.4; 71.30, 32, 33; 72.49, 50; 73.75, 80, 83, 88; 74.105, 113; 75.130, 131, 137, 139, 144; 76.164, 167, 173, 175, 176; 77.202, 203; 78.221, 224, 231; 79.235; 86.142; 87.154, 155, 158; 89.216 **to continue** 200, 209 **to draw** 200, 205, 206, 210, 212, 214-216, 219 **to drop** 199, 202, 208 **to erect** 201 **to extend** 199-202, 205-210, 212-219, 225-227 **to extrapolate** 207 **to produce** 199, 200, 208, 212, 219

extremitas 38.62; 85.105, 109; 89.210; 91.284, 287; 92.289; 93.24 **boundary** 188 **endpoint** 224, 227, 229, 230

extrinsecus 46.265 **external** 194

facere 4.21-23, 26; 5.52; 9.175, 177, 181; 10.194; 13.270; 15.26, 33; 17.71-73; 18.97; 20.164; 21.167; 22.202; 30.108, 124; 31.135; 32.166; 39.78, 80, 87; 40.94, 106; 41.144; 46.257, 266; 48.23; 50.78, 80, 82; 51.97, 100, 102; 52.127; 53.154; 60.42, 44; 63.111, 125; 64.152; 66.197; 67.227; 68.257; 73.85, 89, 94, 96; 75.129, 148; 78.226, 233; 79.236, 240, 263; 80.267, 268; 89.207, 229; 91.271 **to construct** 171, 181 **to cut** 215 **to depend upon** 162 **to draw** 181 **to form** 166-169, 173-175, 182, 189, 194, 198-200, 205, 208, 210, 212, 215, 217, 227, 229 **to happen** 162 **to occur** 161, 189 **to produce** 166, 194, 196, 198, 207, 208, 211, 215, 216, 219, 220 **to project** 191

facies 3.13; 67.225, 233 **face** 161, 211

fallacia 58.279, 287, 288, 290; 59.11; 81.293, 295, 298, 300, 10; 84.82; 85.118; 91.260; 92.294, 298, 2, 3; 93.16, 30, 35; 94.66, 67 **deception** 204 **misperception** 203, 204, 221, 223, 224, 228-232

fieri *see* **facere**

figura 4.24; 5.54; 6.73; 7.104; 18.95; 31.143, 146; 50.72; 55.208; 63.110, 121; 64.151; 68.243; 69.281, 285; 75.125; 78.215; 79.250; 80.280; 81.9-11; 85.117, 118; 87.147; 89.208; 90.248; 91.259; 93.16, 30, 35 **diagram** 171, 221, 224, 225, 230 **figure** 182, 207, 208, 212, 231 **proposition** 198, 201, 207, 211, 212, 216, 219-221, 224, 227, 228, 230 **shape** 161-164 **theorem** 220

finire 94.69 **to end** 232

finis 22.218; 23.219, 222, 226, 243; 24.260, 271; 25.272, 275, 277, 278; 28.67, 77; 29.86, 94; 31.160; 33.210; 37.13, 19, 26, 33; 38.42 **endpoint** 175-177, 179, 180, 182, 184, 186, 187 **limit** 175

foramen 7.109, 110 **window** 164

forma *frequently recurring* (3-5, 9, 52, 54, 56-64, 66-71, 74, 75, 78, 80, 84-87, 89-94) **diagram, form, image, shape**

fortis 4.42; 37.16; 39.73 **pronounced** 188 **strong** 162

fortitudo 4.41 **intensity** 162
funis 7.107, 109, 110 **rope** 164

generalis/generaliter 4.31; 6.91; 7.118 **general** 162 **overall** 164

habere 12.238; 13.262; 17.76, 77; 20.162; 26.22; 31.145; 38.59; 46.255; 47.300; 57.255, 256, 259, 260; 58.286; 59.9, 15, 18, 28, 29; 63.113; 66.196; 67.231; 68.249-251; 70.17; 74.111, 112. 120; 75.127; 80.276, 278; 85.112; 91.284-287; 92.288-291; 94.64 **to be** 194, 204, 207, 210 **to be subject to** 205 **to have** 168, 171, 173, 178, 182, 188, 195, 203, 205, 211, 213, 216, 220, 224, 229, 232 **to play** 204 **to yield** 211, 229
hora 59.12 **time** 204

immobilis 63.111; 64.151; 67.226 **stationary** 207, 208, 211
immotus 21.167 **in place** 174
impedimentum 4.33 **constraint** 162
impossibilis 13.265 *reductio ad absurdum* 168
impossibilitas 13.266 **impossibility** 168
improbare 13.265 **to disprove** 168
incidere 3.14 **to occur** 161
inducere 4.21, 29, 41, 44, 46; 6.92, 93 **to cause** 162 **to lead to** 161, 162, 164
inequalis/inequaliter 23.241; 30.133 **different** 176 **not equal** 181
infinitum 82.33 **infinity** 222
infinitus 23.227 **infinitude** 175
initium 52.116 **start** 199
intelligere 6.95; 15.25; 23.233, 237; 28.59; 35.272; 36.299 **to imagine** 169, 179 **to think about** 175 **to understand** 164, 176, 185, 186
intendere 94.69 **to intend** 232
interiacere 35.254, 273 **to lie between** 185
intermedia 48.18 **intermediate position** 196
intricatio 18.95 **tangle** 171
invenire 20.155; 93.17, 19 **to find** 173, 230
iterare 34.221; 61.77; 71.23; 78.218; 81.9; 85.117; 91.259; 93.15, 35 **to copy** 213 **to duplicate** 206 **to recapitulate** 221, 224, 228, 230, 231 **to redraw** 219 **to repeat** 184
iudicium 4.27 **judgment** 162

latere 4.38 **to be less clear** 162
latitudo 4.44, 45; 38.59, 62; 39.66; 47.8; 48.12, 18; 56.245; 57.256, 274; 93.31 **breadth** 188 **range** 162 **width** 196, 203, 231
latus 9.154; 10.188, 192-195; 13.279; 18.107, 110; 21.170, 172, 173, 178, 181; 25.285; 27.49; 43.181; 46.270; 51.107; 66.202; 71.24 **side** 165-168, 172, 174, 177, 179, 192, 194, 199, 210, 213
legere 23.232 **to read** 175

liber 3.1, 2, 9, 15; 4.19, 21; 13.262; 24.257; 41.138; 50.72 **book** 161, 168, 176, 191, 198
linea *frequently recurring* (3, 5-15, 17-57, 60-94) **axis, line, section**
linea communis 21.191; 31.155; 41.118, 137; 44.219; 50.60, 62, 65, 66, 69 **common section** 174, 182, 190, 193, 197
linea declinationis 34.238, 242-244; 35.250, 263 **slanted line** 184, 185
linea longitudinis 39.81; 40.95, 99; 41.134, 136; 42.150, 159; 43.184, 194; 45.226; 46.257; 47.283, 6; 49.52, 53; 50.74, 79 **line of longitude** 189-198
linea radialis 56.234; 65.182; 66.204; 84.85, 89; 85.95; 94.56 **radial line** 202, 209, 210, 223, 224, 231
linea recta 18.114; 19.133; 23.234; 30.120; 33.204; 34.226, 231; 38.47, 52, 57; 39.68, 71, 75, 78; 41.139, 141; 47.5, 7; 52.120; 54.185, 189; 55.200, 201; 56.240, 243; 70.19; 71.20; 74.103, 109; 75.125; 79.260; 80.274, 280, 286, 287; 81.290; 82.14, 35, 36; 85.104, 114; 90.248; 92.11; 93.17, 20, 24, 32; 94.59 **straight line** 172, 175, 181, 183, 184, 187-189, 191, 196, 199, 201, 202, 213, 215, 216, 220-222, 224, 228, 230, 231
linea reflexionis 3.10; 8.128; 34.236, 237, 243, 244, 247; 35.248, 251, 254, 256, 264, 271, 276; 36.283; 38.41 **line of reflection** 161, 165, 184-187
linea visualis 36.289, 291 **visual axis** 186
littera 18.96, 100; 78.219; 81.11; 91.260; 93.36 **letter** 171, 221, 228, 231 **letter-designation** 219
locum/locus 3.11; 10.195; 14.290; 18.99, 100; 28.54; 33.206, 207; 38.48, 49; 56.230; 58.285; 72.74; 74.119 **location** 161, 202 **place** 171, 183, 188, 204, 215, 216 **point** 179 **where** 167
locus ymaginis 5.64, 65, 68; 6.86; 8.131, 146, 147; 13.264, 265, 267, 269; 19.115, 134, 136; 20.143, 144, 151; 21.192; 22.197, 207, 215; 24.263, 266; 29.86; 58.284; 59.20; 93.24 **image-location** 163, 165, 168, 172-177, 180, 204, 205, 230
longitudo 6.83; 7.99, 100, 102, 106; 9.158, 159, 163; 23.240, 241; 27.50; 28.51; 32.171; 38.62; 39.65, 79-81; 40.95, 99; 41.134, 136; 42.150, 160; 43.177, 184, 194; 44.212; 45.226, 244, 245, 247; 46.253, 257; 47.283, 6; 48.10, 17; 49.52, 54; 50.74, 79; 56.243; 57.254; 73.85, 89; 78.233; 79.235, 263; 82.14; 92.12 **distance** 163, 164, 166, 176, 179, 183, 192-194 **length** 188, 203, 230 **longitude** 189-198, 202, 221 **radius** 215, 219, 220 *see also* **linea longitudinis**
longus 38.59 **long** 188
loqui 84.72 **to discuss** 223
lux 4.23, 30, 35, 37-39, 41-43; 6.76, 77, 82, 91; 58.293; 59.13; 81.1; 92.4 **light** 161-164, 204, 221, 230

magnitudo 4.24 **size** 161
magnus 7.110; 38.62; 39.64, 73; 61.83; 73.95 **considerable** 188 **great** 206, 215 **large** 188 **significant** 164
maioritas 5.49 **degree** 162
manere 21.167 **to remain** 174
manifestus/manifeste 6.84; 36.294; 38.60; 57.249, 256, 275; 88.199 **clear** 163, 186, 188, 203, 227 **evident** 203 **manifest** 203

medietas 9.171, 176; 10.184, 197; 12.236-239, 241-243, 247, 248; 15.21, 22; 17.67, 80-84; 21.177; 37.21; 72.50 **one-half** 166-169, 171, 174, 187, 214
medius 11.221; 16.58; 17.68; 29.90; 85.104, 110; 87.148; 91.285, 286; 92.288; 93.25 **halfway** 225 **intermediate** 229, 230 **mean** 167, 170 **midpoint** 180, 224 **one-half** 171 *see also* **punctus medius**
mensurare 78.227 **to measure off** 219
minor/minus *frequently recurring* (4-9, 11, 12, 17, 19, 20, 22, 28, 32, 33, 37, 39, 45, 56-59, 62, 65-70, 72, 75, 91, 94) **less (than), small**
minoritas 7.101 **diminution** 164
minuere 15.33; 35.266; 48.12 **to be less than** 170 **to decrease/diminish** 185, 196
minutia 4.46; 6.79 **tiny feature** 162, 163
mirabilis 93.32 **remarkable** 231
miscere 4.35 **to mingle** 162
mixtura 4.36 **mingle** 162
modicus 35.263; 36.1; 38.59, 63; 39.76, 81; 44.224; 47.4, 8; 56.240; 85.113 **inconsiderable** 195, 196 **slight** 186, 188, 189, 193, 202 **small** 185 **some** 224
modus 3.9, 15; 6.91; 13.272; 16.48, 50; 22.195, 211; 26.10, 15; 27.28, 45; 29.94, 96; 33.207; 34.222; 35.270, 271; 36.296, 1; 45.240; 52.115, 125, 126; 57.253; 59.7; 60.39, 43, 46; 63.120, 126, 128; 80.277; 81.295, 8; 85.111 **case (of/for)** 164, 207, 221 **extreme (means)** 203 **how (something occurs)** 161, 178, 224 **reasoning** 170 **token** 168, 175, 186 **way** 174, 178, 180, 183-186, 194, 199, 204, 205, 221
multiplicare 39.77; 78.219 **to add** 219 **to compound** 188
multiplicatio 9.168; 10.184, 185; 14.289, 293, 294, 5-7, 9; 15.10-12; 16.36, 38, 41-43, 45; 41.120, 123, 124 **rectangle** 166, 169, 170, 190
multiplex 18.96 **excessive** 171
multiplus 75.135; 76.179, 180; 77.181, 182, 186-188, 192, 193, 195 **several/several times larger** 216, 218
multitudo 92.6 **multitude** 230
mutare 18.95 **to revise** 171

neuter 91.268, 269 **neither** 229
nomen 18.97, 99 **key point** 171 **letter** 171
notare 31.143 **to note** 182
nullus 23.225; 35.252; 70.17; 82.33 **no/not any** 175, 185, 213, 222
numerus 5.49; 6.77; 59.15, 17; 81.7; 92.291, 297 **number** 162, 163, 205, 221, 229

obliquus 52.121; 53.154; 54.190; 56.241; 57.253 **oblique** 200 **slanted** 199, 201-203
obtusus 10.190, 193; 13.280; 14.288; 51.101; 53.146; 55.206, 209; 63.130; 65.168; 67.221; 72.70; 75.145; 78.207 **obtuse** 166-169, 198, 200, 201, 208, 209, 211, 215, 217, 218
occultare 3.13; 6.80; 34.247; 35.261, 276; 36.278, 288; 38.62; 45.233, 238 **to block** 193 **to hide/make invisible** 163, 185, 186, 188, 193 **to obscure** 161 **to occlude** 185
oculus 4.34 **eye** 162

opponere 6.86; 7.106, 108 **to face** 163, 164
oppositus 4.34; 10.192; 27.42 **facing** 179 **lying in front of** 162 **opposite** 167
ordinatio 7.119; 59.5, 8, 24, 28; 60.38 **arrangement** 164, 204, 205
oriri 3.4; 4.26 **to draw** 162 **to originate** 161
ortogonalis/ortogonaliter 5.58, 59; 27.46, 48; 39.88; 40.97; 41.116-118, 127, 128; 44.203; 45.229, 231, 241 **orthogonal** 189, 192-194 **perpendicular** 163, 179, 189, 190
ostendere 41.138; 81.6; 83.48; 91.282 **to show** 191, 221, 222, 229

pars 3.2; 7.119; 9.170-172; 10.183-185, 194; 11.225-229; 12.233, 234, 236, 238, 239, 242, 244, 249, 250; 21.191; 26.12, 13; 28.63, 64; 29.84, 88, 89, 91, 95, 100, 104; 30.107; 31.137, 143, 144, 146; 32.165, 181, 186; 33.215; 34.235; 35.251, 270, 275; 36.281, 282, 287, 290; 37.8; 38.51; 45.238, 239; 50.69; 52.116; 56.238; 57.266, 271, 273, 274; 58.296; 59.5, 8, 24, 28; 60.38; 64.143, 145; 66.198; 71.20, 21, 31, 38, 42, 43; 72.66, 72; 73.84; 74.100, 101, 104, 117; 75.139; 76.167, 173, 176; 80.272; 83.55, 58; 85.106, 107; 86.143; 87.152, 155 **chapter** 161 **direction/orientation/respect to** 180, 184-186, 197, 199, 202, 203, 208, 214-216, 220, 223, 224 **part** 164, 166, 168, 193, 204, 205 **segment** 167, 174, 180, 182, 213 **side** 178-186, 188, 210, 214-218, 223, 225
particula 75.133 **sub-angle** 216
partire 3.2 **to divide** 161
parvus 39.74; 44.225; 56.239; 75.133 **slight** 202 **small** 188, 193, 216
patere 3.9; 5.55; 8.133; 19.116, 117, 125; 24.257; 31.138; 54.178, 183, 186, 187; 55.208; 56.233, 247; 57.248; 59.10; 63.121; 65.165; 66.193; 67.233; 68.243; 69.287; 70.12; 75.125; 79.260; 80.280; 81.292; 82.12; 84.87; 85.93, 98, 114; 87.147; 90.235, 248, 257; 92.292; 93.28, 30; 94.59 **to be clear/evident/obvious** 165, 182, 200, 201, 203, 204, 207, 209-211, 216, 220, 221, 224, 228, 230, 231 **to demonstrate** 172, 212 **to show** 161, 162, 172, 176, 202, 213, 221, 223-225, 228, 229
percipere 4.36; 7.107 **to perceive** 162, 164
perficere 78.218 **to finish** 219
perpendicularis/perpendiculariter *frequently recurring* (5, 8, 10, 13-15, 22, 23, 28, 29, 34, 39-45, 47-58, 60, 61, 63, 66, 72, 73, 75, 78, 79, 82, 84, 86-89, 91, 93) **axis, normal, orthogonal/orthogonally, perpendicular**
pertransire 74.122 **to pass through** 216
pervenire 8.144; 74.106; 83.55, 56; 84.70, 80; 86.121, 128 **to converge** 225 **to reach** 165, 216, 223, 225
piramidalis 3.6, 7; 48.15; 51.98; 52.120, 123; 56.241, 245; 57.265, 272; 92.2; 93.16, 18 **conic/conical** 161, 196, 198, 199, 202, 203, 230
piramidata 57.273 **conical form** 203
piramidatio 58.281 **conical shape** 203
piramis 48.17, 22, 24, 28, 29, 31; 49.36, 53; 50.73, 75, 77, 78, 85; 51.97, 99; 52.127, 129, 132; 53.143, 151, 154; 54.189, 190; 55.198, 199, 205; 56.231, 243; 57.254, 256, 258, 260, 266, 271; 92.12; 93.19, 37, 38; 94.44 **cone** 196-203, 230, 231
planum 3.14; 5.63; 8.150; 9.169; 39.64 **clear/evident/plain/obvious** 161, 163, 165, 166, 188

planus 3.4; 5.53, 56; 7.116; 39.67, 69; 41.142; 80.288; 81.5; 94.69 **flat** 221 **plane** 161-164, 188, 191, 221, 232
pluralitas 4.45 **increase in number** 162
polus 49.49 **pole** 197
ponere 16.44; 18.93, 99, 100, 107, 114; 27.25, 27, 29; 33.206, 207; 38.48; 46.274; 53.147; 54.184; 60.44, 45; 62.88; 63.118; 67.232; 68.259, 260; 69.275, 291; 71.26; 73.85; 79.262; 87.172; 89.218, 221; 93.17, 31 **to assume** 172, 206 **to choose/select** 205, 212, 213 **to designate** 170 **to lie** 201 **to place/replace** 171, 183, 188, 230, 231 **to posit** 171, 195, 226 **to stipulate** 172 **to suppose** 178, 211 **to take** 200, 205, 207, 212, 213, 215, 220, 227
portio 67.230; 72.70; 82.31 **section** 222 **segment** 211, 215
positio 58.299; 59.12-14; 60.34; 81.294; 83.40; 88.193; 92.5 **construction** 222, 226 **location** 230 **position** 204 **situation** 204, 205, 221
precedere 35.248 **to lie in front of** 185
predictus 16.47, 50; 20.154, 158; 22.195; 25.284; 26.10, 13, 15; 29.85, 94, 96; 30.112, 125; 34.221; 35.263; 43.187; 46.263; 53.157; 55.209; 58.288; 68.248; 81.292; 91.282; 92.299 **aforesaid/earlier/foregoing/preceding/prescribed/previous** 173, 174, 177, 178, 180, 181, 184, 185, 192, 194, 200, 201, 221, 229 **previously discussed/reasoned** 170, 181, 204, 211
prefigere 4.25 **to ensconce** 162
premittere 23.239; 27.37; 39.83; 41.131; 48.20; 81.10; 90.248 **to establish/make** 179, 189 **to provide** 221 **to set out** 176, 190, 196
preostendere 78.218; 79.250; 83.44, 57 **to establish** 219, 223 **to show** 220, 222
pretendere 6.75 **to present** 163
probare 7.122; 16.50; 19.137; 22.194; 27.28; 29.94; 30.121; 31.142; 33.207; 34.222; 36.5; 38.49, 54, 55; 39.82; 41.141; 43.180, 183, 186; 48.20; 50.71; 52.113 **to demonstrate** 165, 182-184, 188, 189, 192, 196, 198 **to prove** 170, 173, 174, 178, 180, 181, 186, 188, 191, 192, 199
probatio 5.55; 19.116; 20.145, 154; 24.246; 25.285; 26.6, 20; 27.25, 34; 29.90; 31.145, 147; 34.221; 39.82; 52.115 **demonstration** 162, 189 **proof** 172, 173, 176-178, 180, 182, 184, 199
procedere 10.194; 29.80; 37.27; 93.26 **to extend** 167, 180, 187 **to follow** 230
producere 5.61; 8.125, 129, 144, 151; 9.167, 176; 13.260, 270, 275; 15.18; 18.91, 99, 102; 19.133; 20.148; 21.169, 182, 183; 24.262; 25.294; 26.11; 28.71; 29.84, 103; 31.136; 32.164, 166, 176; 33.192, 204; 36.283; 37.11, 14, 19, 29, 34; 43.192; 44.201, 205; 45.251; 49.33; 50.84; 51.90; 52.113 **to draw** 166, 168, 174, 186, 187 **to extend** 163, 165, 166, 168, 169, 171-174, 176-183, 186, 187, 192, 194, 196, 198, 199
propinquitas 48.19 **near(ness)** 196
propinquus 23.243, 244; 25.277, 278; 28.68; 29.100; 30.115, 116, 130; 33.196; 47.284; 48.11; 56.237; 57.263, 267; 82.19; 83.50, 52, 54; 88.189 **close/near** 176, 177, 179-181, 183, 195, 196, 202, 203, 222, 223, 226
proponere 3.15; 5.56; 6.73; 8.143; 9.155; 19.122; 20.145, 151; 22.198; 25.278, 279; 26.5; 27.23, 37; 28.54; 29.96; 30.119, 131; 31.142; 33.208; 34.223, 234; 38.53, 54, 56, 58,

63; 41.130-132; 44.223; 47.4; 48.14, 21; 50.72 **to assume** 190 **to imagine** 163 **to give** 177, 184 **to place/pose** 188 **to propose** 173, 179, 190 **to purpose** 161 **to set forth/out** 163, 165, 172-174, 177-184, 188, 190, 193, 195, 196, 198

proportio *frequently recurring* (8-18, 24-29, 33, 38, 41, 61, 65, 69, 70, 78, 79) **difference, proportion, ratio**

protrahere 30.134; 31.153; 34.240 **to extend** 181, 182, 184

provenire 7.102; 14.298; 81.300 **to arise** 221 **to be due to** 164 **to form** 169

punctum/punctus *frequently recurring* (5-10, 13-15, 17-35, 37-57, 59-74, 76-80, 82-86, 88-94) **object-point, point**

punctum centrum 30.114 **centerpoint** 181

punctus conversionis 84.86, 90; 85.95 **point of reflection** 223, 224

punctus divisionis 26.17, 18; 29.83; 33.200 **point of division** 178, 180, 183

punctus medius 10.197; 14.291; 30.113; 31.141; 91.286, 287; 92.290 **intermediate point** 229 **midpoint** 167, 169, 181, 182

punctus reflexionis 32.183; 33.211; 35.264, 267, 269; 39.85, 86; 40.92; 45.246; 47.285; 48.22, 24, 27, 29 **point of reflection** 183-185, 189, 194-196

punctus sectionis 17.86; 20.150; 21.189; 24.254, 255; 25.281; 31.159; 32.184; 35.257; 43.195; 44.204; 47.285, 289, 295; 48.30; 83.45; 84.91 **point of division** 177 **point of intersection** 171, 173, 174, 176, 182, 183, 185, 195, 222, 224 **point of/on a section** 192, 195, 196

punctus visus 40.92 **visible point** 189

punctus ymaginis 31.139 **image-point** 182

quadratum 9.168, 170, 172-174, 182; 10.183, 186, 191-193, 199-202, 204-206; 11.207-210, 212, 213, 215-220, 225, 228; 14.288, 289, 292-296, 300-3, 5, 6, 8; 15.11-13, 32-35; 16.37-42, 48, 50-53, 58; 17.71, 72, 74; 42.166-168; 46.268, 271, 272; 51.108, 109 **square (x^2)** 166, 167, 169-171, 191, 194, 199

quantitas 4.40; 5.54; 6.72; 9.163, 175; 18.98; 30.124; 31.136; 38.63; 39.72; 56.240; 58.295; 59.15; 81.2, 6; 85.113; 92.296 **amount** 224 **length** 166, 171, 181 **range** 162 **size** 162, 163, 166, 188, 204, 221, 229

radialis 56.234; 65.182; 66.204; 84.85, 89; 85.95; 94.56 **radial** 202, 209, 210, 223, 224, 231 *see also* **linea radialis**

rectitudo 47.7; 48.11; 82.28; 83.55; 85.105; 92.297 **proper orientation** 229 **straight line** 222-224 **straightness** 196

rectus *frequently recurring* (6, 8, 9, 15, 17-19, 22, 23, 29, 30, 33, 34, 37-43, 47-49, 51-56, 60, 62-66, 68, 70-83, 85, 86, 90-94) **correctly oriented, direct/directly facing, erect, right, straight, upright** *see also* **angulus rectus, linea recta**

redere/redire 20.151 **to arrive at** 173

referre 5.63, 65, 68; 19.115, 134; 20.143, 144, 148, 150, 155; 21.192; 22.197, 214; 23.222, 224; 24.250, 251, 254-256; 31.158; 32.168, 173, 179-181, 185, 187; 33.210; 34.241, 242, 245; 35.259; 37.12, 30; 42.146; 43.176, 182-184, 187, 188, 195; 44.204; 45.239, 243, 244; 47.282, 286, 287, 289, 294; 51.105, 111; 52.114, 117 **to reflect** 163, 172-

176, 182-187, 191-195, 199
reflectere 6.94; 7.124; 19.135; 22.205, 207; 23.220, 227; 24.258; 32.184, 185; 34.236, 246; 35.254, 257; 37.25; 45.234 **to reflect** 164, 165, 173, 175, 176, 183-185, 187, 193
reflexio *frequently recurring* (3-9, 13, 28, 32-36, 38-41, 45, 47, 48) **reflected vision, reflection** *see also* **linea reflexionis, punctus reflexionis**
reliquus 74.108 **other** 216
remanere 19.129; 46.267; 69.284; 75.152 **to be left/remain** 172, 212 **to follow (logically)** 194, 217
remotio 19.119; 24.268; 48.19; 58.293; 81.1; 92.5 **distance** 204, 221, 230 **far** 172, 177, 196
remotus 6.96; 22.207; 23.242, 243; 24.247; 25.276-278; 28.61, 64; 30.114, 129; 31.140; 33.197; 39.69; 48.24, 30; 55.213, 216; 57.277 **distant** 176 **far away/from/outside** 164, 175-177, 179, 181-183, 188, 196, 202, 203
res 4.25, 34, 36; 6.88, 97; 7.117, 119, 121; 9.157, 162; 11.221; 19.138; 23.231, 232, 238; 36.300-5; 39.72-74; 57.276; 58.282, 286, 296; 59.5, 19, 24, 28, 30; 60.38; 62.109; 63.116; 64.154-156, 160; 65.162, 165; 66.189, 194; 81.7 **case** 162 **matter** 175 **object** 162-166, 173, 175, 186, 188, 203-205, 207-210, 221 **phenomenon** 205 **sake** 176 **thing** 162
res visa 4.36; 6.88, 97; 7.117, 119; 9.162; 19.138; 23.231; 36.300-2, 4, 5; 39.72; 57.276; 58.282, 286; 59.5, 19, 24. 28; 60.38; 62.109; 63.116; 64.154-156, 160; 65.162, 165; 66.189, 194; 81.7 **visible object** 162-164, 166, 173, 175, 186, 188, 203-205, 207-210, 221
residuus 11.223; 31.144, 146 **remainder** 167, 182
respicere 4.42; 10.188; 15.20, 21; 22.203; 23.235, 237; 27.41; 28.58, 60, 62, 65; 29.93; 30.114, 116; 36.296, 298; 76.166 **to accord with** 179, 186 **to correspond** 175, 176, 181, 186 **to depend upon** 162 **to form (with respect to)** 179, 180 **to subtend** 166, 169, 175, 217
revertere 94.58 **to reflect** 231
revolvere 63.110; 64.151; 67.226; 71.24 **to circumscribe** 213 **to rotate** 207, 208, 211

scriptus 23.232 **(written) text** 175
secare *frequently recurring* (8, 9, 15, 17, 19-22, 24-38, 40, 42, 44-56, 60, 61, 63, 68-74, 76, 78-80, 82-84, 86, 87, 89-91) **to bisect, to cut, to extend, to intersect, to pass through, to touch**
sectio 17.86; 20.150; 21.189; 24.254, 255; 25.281; 31.160; 32.184; 35.257; 39.79, 84; 40.91; 41.117, 118; 43.195, 197; 44.204; 47.286, 289, 290, 294, 295; 48.23, 25, 29, 30; 49.41, 55, 57, 58; 50.60, 63, 69; 51.98; 83.45; 84.91 **division** 177 **intersection** 171, 173, 174, 176, 182, 183, 185, 195, 222, 224 **section** 189, 190, 192, 195-198 *see also* **punctus sectionis**
sectio columpnaris 39.84 **cylindric section (ellipse)** 189
sectio piramidalis 51.98 **conic section (hyperbola)** 198
sector 53.154, 156, 161; 54.169-171, 173, 180, 191-193; 55.208, 213, 214, 216, 219, 220; 56.222, 224, 225, 227; 57.265, 269, 271; 83.43; 84.75, 76, 84; 86.130; 89.207, 209,

212, 220; 90.239, 240, 242, 245; 91.271, 272; 94.44 **section** 200-203, 222, 223, 225, 227-229, 231
semicirculus 72.70 **semicircle** 215
sensus 57.249 **visual sense** 203
separare 85.110 **to separate** 224
signare 52.125; 63.127; 65.168, 183; 67.229, 230; 74.105; 89.219 **to draw** 216 **to mark/mark off** 199, 208, 209, 211, 227
similis/similiter *frequently recurring* (4, 6-9, 13-17, 21-29, 31, 32, 35-47, 52-54, 57, 58, 60, 62-64, 67, 70, 76-81, 84, 86, 90) **alike/likewise, corresponding, identical, same, similar/similarly**
singulum 4.20, 29, 44; 5.51; 6.94; 7.112 **each case** 162, 164 **particular factor/kind** 161, 162
sinister 6.85, 88-90; 64.157, 158; 67.239 **left(-hand side)** 163, 164, 209, 211
situs, -us/-us, -a, -um 3.10; 5.54; 6.84, 87, 91; 7.105, 110; 23.237; 44.211, 212, 214; 45.245, 247; 46.253-255; 47.295; 48.9, 12, 14; 57.254; 58.293; 59.7, 16; 63.113, 122; 67.228, 231; 71.45; 79.261; 81.1, 5 **circumstance** 221 **disposition/spatial disposition** 161-164 **location** 193 **orientation** 176, 196, 203, 204 **position** 211 **situation** 194, 195, 204, 205, 207, 214, 220, 221
specialis 4.33 **specific** 162
species 92.296 **form** 229
speculatus 83.59 **reflecting** 223
speculum *frequently recurring* (3-9, 19, 20, 23, 24, 27-32, 34-39, 41, 45, 47, 48, 52, 56-60, 63-69, 71-75, 78-85, 87, 88, 90-94) **mirror**
spera 21.180, 183, 189-191; 23.242; 34.225; 61.81, 83 **sphere** 174, 176, 184, 206
spericus 3.5, 6; 7.115, 117; 39.71, 73, 75, 77; 45.237, 240; 57.261; 58.290; 59.6, 9, 10, 18, 21; 60.40; 63.121, 123; 68.255; 71.22; 81.299, 3, 8; 85.93; 91.260; 92.292, 295, 7; 93.36 **spherical** 161, 164, 188, 193, 194, 203-205, 207, 208, 212, 213, 221, 224, 228-231
strictus 57.259, 266, 274 **narrow** 203
subtrahere 16.44; 19.128 **to subtract** 170, 172
sumere 4.26; 9.166; 11.207, 208; 12.250; 13.265; 24.249, 252, 255; 27.44; 28.66; 29.97, 99; 34.235, 236, 240, 247; 35.250, 253, 256, 259, 262, 263, 265, 268; 36.280; 37.8; 39.84, 88; 42.158; 43.177, 182; 47.287; 48.22, 24, 25; 52.116 **to assume** 185, 189 **to choose/select** 168, 176, 185, 186, 191, 192 **to find** 176, 184 **to give** 168, 185 **to make** 162 **to take** 166, 176, 179, 180, 184-186, 192, 195, 196, 199
superficialis 58.292, 2; 59.6, 22, 30 **plane** 204, 205
superficies *frequently recurring* (5, 9, 20, 21, 23, 24, 27-29, 31, 34, 36-38, 40-64, 66-68, 71, 73, 75, 78-84, 86-91, 93, 94) **plane, surface**
superficies communis 83.42 **common section** 222

tangere 11.222; 31.148; 60.47, 52; 61.82 **to be tangent to** 205, 206 **to touch** 167, 182
temperamentum 4.28, 40, 44; 6.93; 7.102 **threshold condition** 162, 164
temperantia 4.20, 45; 6.78 **threshold** 161, 163

terminus 34.243; 36.283; 39.65; 52.117; 74.107 **boundary** 188 **endpoint** 184, 186 **limit** 216 **terminal** 199
tersus 86.127 **polished** 225
tortuosus 57.258 **curved** 203
totalis 9.170; 20.154 **entire** 173 **whole** 166
transire 8.125, 127; 15.25; 30.109; 33.192, 215; 35.265, 267. 268, 272; 41.144; 43.178, 179; 47.283, 296; 48.23; 50.82, 83; 52.120, 126, 133; 53.143, 151; 55.204, 205; 56.231; 60.41, 48; 62.101, 103; 63.125; 65.173, 184, 185; 66.197, 198, 205; 68.257, 261; 71.37; 72.69; 73.84; 74.103, 105, 121, 123, 124; 75.129, 131, 149; 76.156; 79.259; 80.265, 271; 82.16; 83.47; 84.69, 84; 86.134, 136, 138; 87.160, 162, 171, 174; 88.175, 177, 179, 190, 192; 89.206, 228; 90.232, 233, 243, 244, 246, 254; 91.267, 269; 92.12; 93.20, 33, 42 **to bisect** 212 **to continue** 201, 205, 209, 212 **to extend** 210, 215, 216 **to pass along/through/to** 165, 169, 181, 183-185, 191, 192, 195, 196, 198-202, 205, 207, 208, 210, 212, 214-217, 220, 222, 223, 225-231
triangulus 9.178, 180; 10.188; 15.30; 17.76; 19.130; 21.171, 173, 179, 181, 186; 25.289-291; 26.297, 299; 32.173; 33.192, 218; 44.216, 217, 219, 221, 222; 47.300-2; 54.194, 195; 55.196-198, 203; 56.235; 63.132, 134; 64.138; 72.53; 75.148; 82.17, 20, 25, 26, 32, 34, 36; 83.38, 39, 41, 46; 87.151; 88.201, 202; 91.264, 267 **triangle** 166, 170-172, 174, 177, 183, 184, 193, 195, 201, 202, 208, 214, 217, 222, 225, 227, 229

universitas 7.116 **entire range** 164

valere 14.295; 42.167, 168; 51.108, 109 **to equal** 169, 191, 199
variare 6.75 **to differ** 163
variatio 6.75 **variation** 163
varietas 3.17; 5.52; 6.74, 77 **difference** 163 **different type** 161 **variety** 162, 163
venire 19.133; 49.33 **to reach** 172, 197
veritas 3.12; 4.32, 39; 5.54; 7.101, 119; 67.225 **actuality** 161, 162, 164, 211 **proper disposition** 162
via 57.248; 93.26 **reason** 203 **train (of logic)** 230
videre 4.35, 36, 38; 5.70; 6.85, 88, 89, 96, 97; 7.99, 100, 107, 109, 111, 118-121, 123; 9.158, 159, 162; 19.138; 23.231, 234; 30.122; 34.225, 226, 231, 238, 247; 35.249, 252, 263, 265, 269-271, 277; 36.279, 281, 282, 284, 291, 294, 300-2, 4, 5, 7; 37.21; 39.72, 75, 80, 81; 40.92; 41.139, 142; 45.226, 239; 47.5, 7; 48.16, 17; 56.230; 57.276-278; 58.282, 286, 1; 59.5, 19, 24, 29; 60.38; 62.109; 63.116; 64.154-156, 160; 65.162, 166, 181, 182, 188; 66.189, 194; 67.224; 68.252; 74.119, 120; 79.261; 81.3, 5, 7; 85.106; 89.225, 226; 90.250; 92.9, 10, 13; 93.29; 94.60 **to appear** 163-166, 175, 184, 185, 188, 191, 196, 202-204, 209, 211, 216, 220, 221, 228, 230, 231 **to be visible** 162-166, 173, 175, 184, 186, 189, 193, 196, 203-205, 207-210, 221, 224 **to look** 164 **to see** 162, 164, 181, 184-187, 191, 216, 227 **to view** 164 *see also* **punctus visus, res visa**
visibilia 59.26 **visible objects** 205
visibilis 56.229; 57.268; 58.1, 3; 60.58; 61.76; 62.100, 107; 64.148, 153; 65.175, 177, 179, 180, 187; 66.191; 72.73; 74.109; 80.274, 285; 81.4; 83.58; 85.115; 86.126; 89.224;

91.279; 92.296; 93.28; 94.47, 53, 64 **visible** 203, 221, 229, 231, 232 **visible object** 202. 204, 206-210, 215, 216, 220, 221, 223-225, 227, 229-231

visualis 6.97; 36.290, 291 **visual** 164, 186 *see also* **axis visualis, linea visualis**

visum 59.18, 20 **visible object** 205

visus, -us 3.3, 10, 13; 4.19, 20, 28, 34, 36, 41; 5.59; 6.76, 94, 96; 7.107, 108, 124; 19.135; 20.142, 150; 23.241, 242, 245; 24.247; 27.44, 50; 31.153; 34.225, 226, 235, 236, 240, 241, 245; 35.251, 255, 257, 270, 273, 274, 276; 36.281, 286-288, 290, 293, 295, 297, 299, 300, 2, 5, 6; 38.55, 58, 63; 41.135, 137, 140, 143, 144; 42.147; 45.229-231, 233, 238, 239, 241, 242; 46.258, 259; 52.120; 56.229, 238, 242; 57.250; 58.299; 59.27; 60.58; 61.60, 75, 76; 62.99, 106, 107; 63.115, 117; 64.147, 152, 154, 156, 160, 161; 65.163, 175, 176, 181, 187; 66.189, 191, 192, 198, 199, 210; 67.228, 231, 235, 238, 240; 68.267; 69.279; 71.38, 42; 72.72; 74.101, 104, 108, 116; 80.273; 81.289; 83.58; 84.71, 86, 90; 85.95-97, 106, 107; 86.126; 88.199; 89.203, 204, 224; 90.254; 91.262, 279; 93.20, 22, 28, 33, 41, 42; 94.47, 52, 56 **center of sight** 173, 184-186, 188, 190, 191, 193, 194, 202, 204-212, 215, 216, 220, 223-225, 227-231 **eye** 186, 188, 199, 206-209, 214, 221, 227, 231 **sight** 161, 163, 165, 176, 179, 182, 184-186, 193, 202, 203, 205, 224, 228, 230, 231 **viewer** 162 **vision** 161, 162, 164 **visual faculty** 161, 162, 164

volere 56.244; 67.242; 71.22; 81.296; 92.299 **to want** 202, 211, 213, 221, 229

ymago *frequently recurring* (3, 5, 6, 8, 13, 19-25, 27-31, 33-39, 41, 43-45, 47, 48, 56-59, 61, 62, 64-74, 77-80, 82-94) **image** *see also* **locus ymaginis, punctus ymaginis**

ENGLISH-LATIN GLOSSARY

ENGLISH-LATIN GLOSSARY

to accord with respicere
actuality veritas
acute acutus
to add addere, multiplicare
to adjoin adiungere
aforesaid predictus
alike/likewise similis / similiter
alternate coalternus
amount quantitas
to analyze declarare, exponere
angle angulus
angle of reflection angulus reflexionis
to appear apparere, videre
appearance apparentia
to apply apponere, aptare
to apply to appropriare
to apprehend apprehendere
apprehension adquisitio
to approach accedere, appropinquare
arc arcus
(area) around circuitus
to arise accidere, evenire, provenire
arrangement ordinatio
to arrive at redere / redire
to assume ponere, proponere, sumere
to augment augmentare
to avoid evitare
axis axis, linea, perpendicularis

base basis
to be accidere
to be apparent/visible apparere
to be clear/evident/obvious certificare, patere
to be compounded constare
to be continuous continuare
to be dropped exire
to be due to provenire
to be larger/longer excedere

to be left/remain remanere
to be less clear latere
to be less than minuere
to be obligated (must) debere
to be opposite diversare
to be parallel equidistare
to be subject to habere
to be tangent to contingere, tangere
to be visible videre
below dimissus
to bisect dividere, secare, transire
blend confusio
to block occultare
body corpus
book liber
bordering conterminabilis
both duplex
boundary extremitas, terminus
breadth latitudo

case (of/for) modus, res
cause causa
to cause efficere, inducere
center/centerpoint centrum, punctum centrum
center of sight centrum, centrum visus, visus
change diversitas
chapter capitulum, pars
to choose/select ponere, sumere
chord corda
circle circulus
circumference circumferentia
to circumscribe describere, revolvere
circumstance situs
clear/evident/plain/obvious manifestus / manifeste, planum
close/near propinquus
closeness accessus
color color
to combine ducere
common communis
common angle angulus communis
common arc communis
common section communis, circulus communis, differentia, differentia communis, linea communis, superficies communis
common term communis
to compound componere, multiplicare
concave concavus, concavitas

concavity concavitas
conclusion coniecturatio
cone piramis
to confuse confundere, dubitare
conic/conical piramidalis
conic section (hyperbola) sectio piramidalis
conical form piramidata
conical shape piramidatio
to connect continuare, copulare
considerable magnus
constraint impedimentum
to construct facere
construction positio
to continue extrahere, transire
contrary contrarius
to converge coniungere, pervenire
converse conversus
convex exterior
to copy iterare
correlation collatio
to correspond respicere
corresponding adinvicem, similis
counterpart compar
cross-section dyameter
curvature arcualitas, concavitas
curved arcualis, tortuosus
to cut abscidere/abscindere, dividere, facere, secare
cylinder columpna
cylindric/cylindrical columpnalis, columpnaris
cylindric section (ellipse) sectio columpnaris
cylindrical mirror columpna

deception deceptio, fallacia
to decrease minuere
degree maioritas
to demonstrate declarare, patere, probare
demonstration demonstratio, probatio
to depend upon facere, respicere
to designate assignare, ponere
designation distinctio
to detect discernere
to determine constituere, determinare
diagram figura, forma
diameter dyameter
to differ diversare, variare
difference proportio, varietas

different inequalis / inequaliter
different types diversitas, varietas
to diminish diminuere, minuere
diminished debilis
diminution minoritas
direct directus, rectus
direct vision directio, directus
direction pars
directly facing rectus
to discern discernere
to discuss declarare, disquirere, loqui
disposition dispositio, situs
to disprove improbare
distance distantia, longitudo, remotio
distant remotus
distinct distinctus
distorted distortus
to divide dividere, partire
division divisio, sectio
to draw continuare, ducere, extrahere, facere, oriri, producere, signare
to drop cadere, ducere, extrahere
to duplicate duplicare, iterare

each case singulum
earlier predictus
end/endpoint caput, extremitas, finis, terminus
to end finire
to ensconce prefigere
entire totalis
entire range universitas
to equal efficere, valere
equality equalitas
equivalent compar
erect/erected erectus, rectus
to erect extrahere
error error
to establish declarare, premittere, preostendere
Euclid Euclides
evident manifestus / manifeste
example exemplum
to exceed egredi, excedere
excessive multiplex
to exist existere
to explain declarare, explanare
explanation explanatio
to extend continuare, ducere, exire, extendere, extrahere, procedere, producere,

protrahere, secare, transire
to extend past excedere
external extrinsecus
to extrapolate extrahere
extreme (means) modus
eye oculus, visus

face facies
to face opponere
facing oppositus
to fall cadere, concurrere
far remotio
far away/from/outside remotus
figure figura
to find invenire, sumere
to finish perficere
flat planus
to follow (logically) procedere, remanere
foregoing predictus
form forma, species
to form continere, efficere, facere, provenire
to form (with respect to) respicere
forming a rectangle ductus…in… (*see also* **rectangle**)

general generalis / generaliter
to get to something accedere
to give dare, proponere, sumere
great magnus
great circle circulus. circulus magnus

half/one-half dimidium, medietas, medius
halfway medius
to happen accidere, evenire, facere
to have habere
having bodily dimensions corporalis
head caput
height altitudo
to hide/make invisible occultare
how (something occurs) modus

identical similis
image forma, ymago
image-location locus ymaginis
image-point punctus ymaginis
to imagine intelligere, proponere
impossibility impossibilitas

impossible impossibilis
incidence accessus
to incline declinare
inclined declinis
inconsiderable modicus
increase additamentum
increase in number pluralitas
infinitude infinitus
infinity infinitum
in place immotus
to intend intendere
intensity fortitudo
intermediate medius
intermediate point punctus medius
intermediate position intermedia
to intersect cadere, concurrere, coniungere, secare
intersection concursus, sectio
inverted conversus

judgment iudicium

key point nomen
to know cognoscere

lack defectus
large magnus
to lead to inducere
to leave be existere
left-hand side sinister
length longitudo, quantitas
less (than) minor/minus
letter littera, nomen
letter-designation littera
to lie cadere, existere, ponere
to lie at a distance/far distare
to lie between interiacere
to lie in front of precedere
to lie outside of declinare, elongare
to lie upon adiacere
light lux
limit finis, terminus
line linea
line of longitude linea longitudinis
line of reflection linea reflexionis
location locum/locus, positio, situs

long longus
longitude longitudo
to look videre
lying above/beyond elevatus
lying far elongatio
lying in front of oppositus

to make premittere, sumere
to make indisputable convincere
manifest manifestus/manifeste
to mark/mark off signare
matter res
mean medius
to measure off mensurare
to meet concurrere
midpoint medius, punctus medius
to mingle miscere
mingling mixtura
mirror speculum
misperception erroneus, fallacia
multitude multitudo

narrow strictus
near(ness) propinquitas
neither neuter
no/not any nullus
normal perpendicularis
not equal inequalis/inequaliter
to note notare
number numerus

object corpus, res
object-point punctum/punctus
oblique obliquus
to obscure occultare
obtuse obtusus
to occlude occultare
to occur accidere, evenire, facere, incidere
on the surface/plane of continuus
opposite contrarius, econversus, oppositus
orientation situs
orientation/respect to pars
to originate oriri
orthogonal/orthogonally ortogonalis/ortogonaliter, perpendicularis/perpendiculariter
other reliquus
outer exterior

overall generalis / generaliter

parallel equidistans
parallelism equidistantia
part pars
particular factor/kind singulum
to pass along/through/to exire, extendere, pertransire, secare, transire
to perceive comprehendere, discernere, percipere
perceptible comprehensibilis
perception comprehensio
periphery circumferentia
perpendicular ortogonalis, perpendicularis
phenomenon res
place locum / locus
to place/replace ponere, proponere
plane planus, superficialis, superficies
to play habere
point locum / locus, punctum / punctus
point of/on a section punctus sectionis
point of division punctus divisionis, punctus sectionis
point of intersection punctus sectionis
point of reflection punctus conversionis, punctus reflexionis
to point out assignare
pole polus
polished tersus
to pose proponere
to posit ponere
position positio, situs
preceding predictus
preliminary points antecedentia
prescribed predictus
to present pretendere
previous predictus
previously discussed/reasoned predictus
to proceed accedere
to produce continuare, ducere, extrahere, facere
to project ducere, facere
pronounced fortis
proof declaratio, demonstratio, probatio
proper disposition veritas
proper orientation rectitudo
proportion proportio
to propose proponere
proposition figura
to prove declarare, probare
to provide premittere

to purpose proponere

radial radialis
radial line linea radialis
radius longitudo
random casualis
range latitudo, quantitas
ratio proportio
to reach attingere, pervenire, venire
to read legere
reason causa, via
reasoning modus
to recapitulate iterare
rectangle ductus…in… (*see also* **forming a rectangle**), multiplicatio
to redraw iterare
to reflect convertere, referre, reflectere, revertere
reflected conversus
reflected vision reflexio
reflecting speculatus
reflection conversio, reflexio
to remain manere
remainder excessus, residuus
remarkable mirabilis
to repeat iterare
reversal conversio
reversed conversus
to revise mutare
right erectus, rectus
right angle angulus rectus
right-hand side dexter
role dignitas
rope funis
to rotate revolvere

sake res
same similis
section linea, portio, sectio, sector
to see videre
segment pars, portio
semicircle semicirculus
to separate separare
separation distantia
to set forth/out premittere, proponere
several/several times larger multiplus
shape figura, forma
short brevis

to shorten diminuere
to show declarare, ostendere, patere, preostendere
side latus, pars
sight visus
significant magnus
similar/similarly similis / similiter
situation positio, situs
size magnitudo, quantitas
slant declinatio
to slant declinare
slanted declinis, obliquus
slanted line linea declinationis
slight modicus, parvus
small minor / minus, modicus, parvus
solid corporalis
solid angle angulus corporalis
some modicus
soul anima
spatial disposition situs
specific specialis
sphere spera
spherical spericus
square (x^2) quadratum
start initium
stationary immobilis
to stipulate ponere
straight rectus
straight line linea recta, rectitudo
straightness rectitudo
strong fortis
sub-angle distinctio, particula
to subtend respicere
to subtract aufere, subtrahere
subtraction ablatio
to suppose ponere
surface superficies

to take ponere, sumere
to take the shape of assimulare
tangency contingentia
tangle intricatio
terminal terminus
terminal segment caput
theorem figura
thing res
to think about intelligere

threshold temperantia
threshold condition temperamentum
time hora
tiny feature minutia
token modus
to touch cadere, secare, tangere
train (of logic) via
triangle triangulus

to understand intelligere
upright erectus, rectus

variation diversitas, variatio
variety varietas
to vary diversare
to verge towards accedere
vertex acumen, caput
vertical (angle) collateralis, contrapositio
vice-versa econversus
to view videre
viewer aspiciens, visus
visible visibilis
visible object(s) res visa, visibilia, visibilis, visum
visible point punctus visus
vision visus
visual visualis
visual axis axis, axis visualis, linea visualis
visual faculty visus
visual sense sensus

to want volere
way dispositio, modus
weak debilis
to weaken debilitare
weakening debilitas, debilitatio
whole totalis
width latitudo
window foramen
written text scriptus

to yield habere

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GENERAL INDEX

actual object/object-line xvi, xli, 233 (n. 3)
ambient light 233 (n. 1)
angle *See* **angle of incidence, angle of reflection, equal-angles law/principle** *and* **visual angle** *below.*
angle of incidence xvii, xviii, xlii (n. 16), 246 (n. 72)
angle of reflection xvii, xviii, xlii (n. 16)
Apollonius of Perga xxxiv
Arabic manuscripts/sources xliii (n. 19), xlvii, xlviii, lii (nn. 7, 8), 242 (n. 48)
'Arafat, W. xliii (n. 21)
Archimedes xxxv
axis xxii-xxiv, xxx-xxxii, xli (n. 5), 244 (n. 61), 245 (nn. 63, 66, 67), 246 (nn. 70, 72), 248 (nn. 83, 85), 255 (nn. 123, 124), 256 (n. 126), 257 (nn. 131, 135) *See also* **major axis, minor axis,** *and* **visual axis** *below. See also* **axis** *in Latin-English Index.*

Bacon, Roger xxxiv, xlii (n. 15)
bird's eye view (relative to figures) xxiii, 247 (n. 76), 250 (n. 88), 255 (n. 124), 256 (n. 125)
brightness xvi, 233 (n. 1)
burning mirrors xxxv, xliii (n. 19), xliv (n. 24)
burning point xxxix

camera obscura xl, xlii (n. 16)
cathetus xxii-xxiv, xxvi, xxviii, xxix, xxxii, xxxvi, xxxvii, xli (nn. 8, 10), xliv (n. 23), 236 (n. 23), 242 (nn. 50, 51), 243 (n. 53), 245 (n. 64), 246 (nn. 70-72), 248 (n. 85), 250 (n. 90), 251 (n. 97), 254 (n. 117), 255-256 (n. 124), 257 (n. 130), 258 (nn. 136, 138) *See also* **cathetus of incidence, cathetus-rule** *and* **normal** *below.*
cathetus of incidence xxii, xxiv, xxxvi, xliv (n. 23)
cathetus-rule 247 (n. 74), 255 (n. 124)
catoptrics xxxiv, xxxix, xl
center of curvature xx, xxii, xxvii, xxix, xxxiii, xxxiv, xxxvii, xxxviii, 234 (n. 15), 236 (n. 23), 239 (n. 35), 240 (nn. 37, 38), 241 (nn. 42, 44, 46), 245 (n. 62), 250 (n. 96), 251 (nn. 101, 102), 253 (n. 111), 257 (n. 130)
center of sight xvii-xxxiv, xxxvi-xxxviii, xli (n. 5), xliv (n. 23), 233 (n. 3), 236 (n. 23), 237 (nn. 24, 25), 238 (nn. 26, 28), 240 (n. 36), 241 (nn. 42, 45-47), 242 (n. 50), 243 (nn. 52, 54, 55), 244 (nn. 60-62), 245 (n. 64), 246 (n. 70), 247 (nn. 74, 75), 248 (n. 85), 251 (nn. 100, 102, 104), 252 (nn. 107, 108), 253 (n. 111), 254 (n. 117), 255-256 (n. 124), 256 (nn. 127, 128, 130), 257 (n. 130), 258 (nn. 138, 140) *See also* **centrum, centrum visus,** *and* **visus** *in Latin-English Index.*

circle of reflection 256 (n. 128)
color xv, xvi, xxv, 233 (n. 3), 245 (n. 65), 250 (n. 92) *See also* **weakening of color** *below.*
common section 241 (nn. 42, 47), 257 (n. 135) *See also* **linea communis, sectio communis** *in Latin-English Index.*
composite image xxxi, 242 (n. 51), 243 (nn. 52, 53)
compound misperceptions 250 (n. 92), 255 (n. 122) *See also* **misperception** *below.*
concave conical mirrors xv, xxxiii, xlviii, 248 (n. 82)
concave cylindrical mirrors xv, xxxi, xxxiii, xlviii, 248 (n. 82), 256 (n. 127), 257 (n. 135)
concave image xxviii-xxxiii, 253 (n. 111), 256 (n. 129), 258 (n. 139)
concave paraboloidal mirrors xxxvi, xliii (n. 21)
concave spherical mirrors xv, xxv-xxvii, xxix-xxxii, xxxv, xxxvi, xxxviii, xxxix, xlii (nn. 12, 16), xliii (nn. 20, 21), xlviii, 233 (n. 2), 256 (n. 127)
conic section xxiv, xlvi, lii (n. 7), 248 (nn. 83, 85), 249 (n. 87), 250 (nn. 88-90) *See also* **sectio piramidalis** *in Latin-English Index.*
convex conical mirrors xv, xxiii-xxv, xxxiv, xlv, xlviii, li (n. 6), 248 (n. 85), 258 (n. 138)
convex cylindrical mirrors xv, xxii-xxiv, xxxi, xxxiv, xlviii, li (n. 6), 248 (n. 85), 250 (n. 91)
convex image xxiii, xxviii-xxxiii, 256 (nn. 124, 129), 258 (n. 139)
convex spherical lenses xxxv, xlii (n. 16), xliii (n. 17) *See also* **lenses** *below.*
convex spherical mirrors xv, xvi, xix-xxii, xxxiv, xl, xlii (n. 12), xlviii, li (n. 6), 239 (n. 35), 240 (n. 41), 249 (n. 87)
cross-section xxiv, xxv, xxxiii, xli (n. 8), 250 (n. 96), 251 (n. 101), 257 (n. 130)
cylindric section *See* **ellipse/elliptical section** *below.*

diameter xvi, xxii, xxvii, xxix, xxxii, xxxv, xxxvi, xxxviii, xlii (n. 16), 238 (n. 30), 252 (nn. 107, 108), 253 (n. 111), 254 (n. 115)
diminution xxxix
Diocles xxxv, xliii (nn. 19, 20)
direct vision xv, xliii (n. 20)
distance-judgment xv, 233 (nn. 1, 3), 250 (n. 92)
Dupré, Sven xxxv, xliii (n. 18), xliv (n. 24)

ellipse/elliptical section xxii, xxiii, xxxii, xli (n. 10), 245 (n. 67), 246 (nn. 70, 72), 248 (n. 83), 256 (n. 127), 257 (nn. 134, 135) *See also* **sectio columpnaris** *in Latin-English Index.*
empirical analysis/determination xxi, 243 (n. 53), 247 (n. 74) *See also* **experiment** *below.*
endpoint xx-xxii, xxix-xxxi, xxxiii, xxxvii, xli (n. 8), 241 (nn. 46, 47), 242 (n. 51), 244 (nn. 58-60), 245 (n. 64), 247 (n. 75), 248 (n. 83), 257 (nn. 130, 135) *See also* **endpoint of tangency** *below.*
endpoint of tangency xvii, xix-xxi, xli (n. 3), 233 (n. 5), 234-235 (n. 15), 240 (n. 36), 241 (n. 44), 242 (nn. 50, 52), 243 (nn. 52, 53), 244 (n. 59), 254 (n. 117)

equal-angles law/principle xlii (n. 16), 254 (n. 117)
Euclid/Euclidean xxxiv, 233 (n. 6), 234 (nn. 10, 11, 13), 235 (nn. 18, 19), 238 (n. 30), 239 (n. 34), 246 (nn. 68, 69), 247 (n. 77), 251 (n. 103), 252 (n. 105), 253 (n. 113), 254 (nn. 116, 117) *See also* **Euclides** *in Latin-English Index.*
experiment xliv (n. 23)
eye xv, xxiv, xxv, xxvii, xli (nn. 8, 10), 243 (n. 55), 244 (n. 60), 245 (n. 63), 247 (n. 74), 250 (nn. 91, 96), 251 (nn. 101, 104), 256 (n. 129), 258 (n. 140) *See also* **oculus, visus** *in Latin-English Index.*
eyeglasses xliii

Falco, Charles xxxv, xliii (n. 17), xliv (n. 25)
focal point xxxv, xxxvii-xxxix, xlii (n. 16), 248 (n. 83)
focal properties xxxv, xxxvi, xxxix
focus/focusing properties xxxv, xxxviii, xxxix, xlii (n. 16), xliii (nn. 20, 21)
form xv, xix, xxiii, xxiv, xxvi, xxviii-xxxiii, xxxvi, xxxvii, xli (n. 3), 233 (n. 8), 236 (nn. 23, 24), 242 (nn. 49, 51, 52), 243 (nn. 57, 58), 244 (n. 61), 246 (n. 70), 247 (n. 80), 250 (n. 92), 251 (n. 97), 253 (nn. 110, 111), 254 (nn. 117-119), 255 (n. 124), 256 (n. 125), 257 (nn. 130, 132), 258 (nn. 137, 138) *See also* **forma, species** *in Latin-English Index.*

geometrical diagram xlii (n. 15)
geometrical explanation xxxiv
Ghent University xliii (n. 17)
great circle xvi, xx, xxvi, xli (n. 3), 239 (n. 33), 241 (nn. 42, 47), 250 (n. 96), 251 (n. 98), 252 (n. 107), 256 (nn. 127, 128), 257 (n. 130) *See also* **circulus, circulus magnus** *in Latin-English Index.*

Hockney, David xxxiv, xxxv, xlii (n. 16), xliii (n. 17), xliv (n. 25)
Hockney-Falco Thesis xliii (n. 17), xliv (n. 25)
hyperbola/hyperbolic section *See* **conic section** *above.*
hypotenuse 233 (n. 7), 247 (n. 78)

Ilardi, Vincent xliii (n. 17)
illumination xv
image-displacement xvi
image-distortion xv, xvi, xxii, xxiii, xxxi, xxxiv, xxxvi, xxxix, xl *See also* **deceptio, fallacia** *in Latin-English Index.*
image-formation xv, xxxi, xxxiv, xxxvi, xxxix, xl
image-inversion xxxv, xxxviii, xxxix, 233 (n. 2), 250 (n. 92)
image-line xxvii, xli (n. 8), 253 (n. 111)
image-location xxv, xxxvi, xxxviii, 239 (n. 35), 246 (nn. 70, 72), 253 (n. 111), 258 (n. 136) *See also* **locus ymaginis** *in Latin-English Index.*
image-orientation xxv, 233 (n. 2) *See also* **situs** *in Latin-English Index.*
image-point/point-image xvii, xx, xxi, xxiii, 234-235 (n. 15), 239 (n. 35), 243 (n. 52), 246 (n. 71), 253 (n. 111), 254 (n. 117) *See also* **punctus ymaginis** *in Latin-English Index.*

image-projection xxxv, xxxix, xliii (n. 17)
image-reversal xvi, xxv, xxxii, xxxiii, 233 (n. 2), 245 (n. 65), 250 (n. 92)
image-size xli (n. 8)
incident ray xlii (n. 16), 237 (n. 25), 238 (n. 27), 251 (n. 102), 258 (n. 138)
inversion point xxxv
isosceles triangle 252 (n. 105), 255 (n. 123)

judgment xv *See also* **distance-judgment** *above and* **misjudgment, perceptual determination of image-location** *below*.

Kepler, Johannes *See* **post-Keplerian** *below.*
Kitāb al-Manāẓir *See* **Arabic manuscripts/sources** *above.*

lemmas xix-xxiv, xlv, li (n. 2), 241 (nn. 44, 45), 248 (n. 85)
lenses xxxv, xlii (n. 16), xliii (nn. 17, 18) *See also* **convex spherical lenses** *above.*
light xv, xxxix, xlii (n. 16), 233 (nn. 1, 3), 245 (n. 65), 250 (n. 92) *See also* **ambient light** *above and* **weakening of light** *below.*
light-ray/solar ray xxxiv, xxxvi, xlii (n. 16), xliv (n. 24)
Lindberg, David xlii (n. 15)
line of incidence xvii, xviii, xxxvi, xxxvii, xliv (n. 23), 236 (n. 24), 244 (n. 60)
line of longitude xxii-xxiv, xxxi-xxxiii, 246 (n. 72), 248 (n. 83), 250 (n. 90), 258 (n. 138) *See also* **linea longitudinis** *in Latin-English Index.*
line of reflection xvii, xviii, xxiii, xxviii, xxix, xxxi, xxxii, xxxvi, xli (nn. 8, 10), xliv (n. 23), 236 (nn. 23, 24), 237 (n. 24), 238 (n. 26), 243 (n. 56), 244 (nn. 59, 60), 246 (nn. 70, 71), 251 (n. 97), 254 (n. 117), 256 (nn. 124, 128), 258 (n. 138) *See also* **linea reflexionis** *in Latin-English Index*.
line-segment xxi, xxiv, xxv
luminous object xlii (n. 16)

magnification xxxviii-xl
major axis 245 (n. 67), 246 (n. 70), 248 (n. 83)
mathematicians xl, 240 (n. 40), 242 (n. 48)
Menelaus of Alexandria 242 (n. 48)
midpoint xxiii, xxv, xxix, xxx, xxxiii, xxxviii, xli (n. 8), xlii (n. 16), 241 (n. 46), 245 (n. 62)
minor axis xxii, xxxii, 245 (n. 67), 246 (n. 70), 257 (n. 135)
mirrors *See* **concave conical, concave cylindrical, concave spherical, convex conical, convex cylindrical,** *and* **convex spherical mirrors** *above and* **plane mirrors** *below.*
misjudgment 233 (n. 1)
misperception xv, xvi, xxv, xlv, li (n. 6), 233 (n. 4), 249 (n. 87), 250 (n. 92), 255 (n. 122) *See also* **compound misperceptions** *above.*

normal xvii, xviii, xxii, xxiv, xxxii, xli (n. 10), lii (n. 7), 241 (n. 47), 243 (nn. 55, 56), 244 (n. 58), 245 (n. 64), 246 (n. 70), 247 (n. 75), 248 (nn. 81, 85), 250 (nn. 89, 90), 251 (n. 102), 255 (n. 124) *See also* **cathetus** *above*.

object *See* **luminous object** *above and* **visible object** *below*.
object-image xxxiii
object-line xxi, xxii, xxiv, xxvi-xxviii, xxxi-xxxiii, xxxvii, xxxviii, xli (nn. 8, 9), 237 (n. 25), 238 (n. 26), 241 (n. 47), 243 (n. 55), 244 (nn. 60, 62), 245 (nn. 62-64), 247 (n. 75), 256 (nn. 129, 130), 257 (n. 130)
object-point/point-object xvii, xix-xxv, xxviii, xxix, xxxvi, xxxvii, xliv (n. 23), 234 (n. 15), 236 (n. 23), 239 (n. 35), 240 (n. 36), 243 (n. 55), 246 (n. 70), 248 (n. 85), 251 (nn. 99, 102), 252 (n. 108), 253 (n. 111), 254 (n. 117), 256 (n. 127), 258 (n. 136) *See also* **punctum/punctus, punctus visus** *in Latin-English Index*.
object-size xli (n. 8)
obliquity xxii, xxviii, 256 (n. 124)
optical devices/instruments xxxv, xl, xlii (n. 16), xliii (nn. 17, 18), xliv (n. 24)
origination point xlii (n. 16)

paraboloidal mirrors *See* **burning mirrors** *above*.
Pecham, John xxxiv, xlii (n. 15)
perceptual determination of image-location xv
Perspectivist optics/tradition xxxv, xxxix
physical images *See* **real images** *below.*
physical optics *See* **catoptrics** *above.*
physical phenomenon xxxix
plane of reflection xviii, xxi-xxiv, xxvi, xxviii, xxxii, 246 (n. 72), 248 (nn. 83, 85), 250 (nn. 88, 90), 253 (n. 110), 255 (n. 124), 256 (n. 126), 257 (n. 135)
plane mirrors xv, xvi, xxii, xxiv, xxv, xxxiv, xlviii, 233 (n. 2), 250 (n. 93)
plane triangle 242 (n. 48)
point of reflection/reflection-point xvii, xix, xxii-xxiv, xxxvi, xli (nn. 3, 10), 233 (n. 5), 234 (n. 15), 236 (n. 24), 238 (n. 28), 240 (n. 36), 242 (n. 49), 243 (n. 56), 246 (n. 70), 248 (nn. 83, 85), 254 (n. 117), 257 (n. 130), 258 (nn. 136, 137) *See also* **punctus conversionis, punctus reflexionis** *in Latin-English Index*.
point-image *See* **image-point** *above*.
polygon 234 (n. 13)
point-object *See* **object-point** *above*.
post-Keplerian xxxix, xl
proportion/proportional/proportionality xvi, xxi, 234 (nn. 10, 13), 235 (nn. 15, 18), 236 (nn. 22, 24), 237 (n. 24), 241 (n. 45), 246 (n. 68), 247 (n. 77), 249 (n. 87), 254 (n. 117) *See also* **proportio** *in Latin-English Index.*
psychological images *See* **virtual images** *below.*
psychological phenomenon xxxix
Ptolemy/Ptolemaic xxxiv, xxxvi, xxxviii, xl, xli-xlii (n. 12), xliv (n. 23)
pupil 251 (n. 101)

Quṣṭā ben Lūqā xliii

radius xxv, xxviii-xxx, xxxvii, xxxviii, xli (n. 9), xlii (n. 16), 236-237 (n. 24), 238 (n. 30), 255 (n. 121)

ratio xix, xx, 234 (nn. 13, 15), 240 (nn. 39, 40), 249 (n. 87) *See also* **proportio** *in Latin-English Index.*
ray-analysis xxxv
ray-couple xlii (n. 16)
real images xxxv, xxxvi, xxxviii, xxxix, xlii-xliii (n. 16)
rectangle 234 (nn. 11, 13), 246 (n. 69)
reference-point xxxviii
reflected ray xxxvi, xxxvii, xlii (n. 16), 237 (n. 25), 238 (nn. 26, 27), 246 (n. 70), 248 (nn. 81, 85), 251 (n. 102), 255-256 (n. 124), 257 (n. 130), 258 (nn. 136, 138)
reflecting surface xv, xxiii, xxv-xxxiii, xxxvi-xxxviii, xli (nn. 8, 9), xliii (n. 16), xliv (n. 23), 250 (n. 92), 251 (nn. 97, 104), 257 (n. 130)
reflective/reflectivity xvi, xxxiv
Risner, Friedrich 246 (n. 73), 248 (n. 82), 250 (n. 90), 255 (nn. 120, 122, 123)

Sabra, A. I. xliii (n. 20), xliv (n. 24), xlvii, li (n. 5), lii (n. 8)
Schechner, Sara J. xliii (n. 17)
separation (of perceived dispositions) 233 (n. 4)
size-distortion xvi, xix, xxv, xxxii, xxxiii
size-judgment *See* **distance-judgment** *above.*
Smith, A. Mark xvii, xix, xxix, xxxvi, xli (nn. 2, 4, 7, 10-12), xlii (nn. 13-15), xliv (nn. 23, 25, 26), li (nn. 1, 4, 5), lii (n. 9), 233 (n. 4), 234 (n. 15), 239 (n. 35), 243 (n. 53), 248 (nn. 83, 84), 250 (n. 95), 251 (n. 102), 252 (nn. 107, 108), 254 (n. 117), 256 (n. 127), 257 (n. 135), 258 (nn. 136, 137)
spatial discrepancy xv
spatial disposition xvi, xxii *See also* **situs** *in Latin-English Index.*
spherical triangle 242 (n. 48)
surface of reflection *See* **reflecting surface** *above.*
syllogism/to syllogise 250 (n. 92)
symmetry/symmetrical xxxii, xxxvii, 237 (n. 24), 242 (n. 51), 246 (n. 72), 253 (n. 110)

telescope/telescopic xxxv, xl, xliii (nn. 17, 18)
threshold condition xv
Toomer, G. J. xliii (nn. 19, 20)
triangle xviii, 233 (n.7), 235 (n. 18), 239 (nn. 33, 34), 241 (n. 47), 242 (n. 48), 247 (nn. 77, 78), 251 (nn. 98, 103), 252 (nn. 105, 109), 253 (n. 113), 254 (n. 115), 255 (n. 123), 256 (n. 125) *See also* **isosceles triangle, plane triangle** *and* **spherical triangle** *above.*

University of Arizona xxxv

veridical vision xv
vertex xxiv, xxv, xxxiii, 239 (n. 32), 254 (n. 116), 258 (n. 138) *See also* **acumen, caput** *in Latin-English Index.*
vertex-designation 249 (n. 87)

viewpoint xvi, xxxviii, xxxix, 233 (n. 2)
virtual images xxxv, xxxix, xliii (n. 16)
visible object xxx, 250 (n. 90), 251 (n. 104), 258 (n. 138) *See also* **object-point** *above. See also* **res visa, visibilia, visibilis,** *and* **visum** *in Latin-English Index.*
visible point 252 (n. 107) *See also* **punctus visus** *in Latin-English Index.*
visual angle 245 (n. 63)
visual anomaly xl
visual axis 244 (n. 61), 245 (n. 63) *See also* **axis visualis, linea visualis** *in Latin-English Index.*
visual perception xli (n. 12), xlii (n. 13)
visual phenomenon xxxix
visual ray xxxiv

weakening of color/of light xv, xxv, 233 (n. 3), 245 (n. 65), 250 (n. 92) *See also* **debilitas, debilitatio** *in Latin-English Index.*
Winter, H. J. J. xliii (n. 21)
Witelo xxxiv