Lengths, Widths, Surfaces: A Portrait of Old Babylonian Algebra and Its Kin by Jens Høyrup

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Not long after the decipherment of cuneiform it was discovered that the Babylonians used a sexagesimal place value number system. Late Babylonian (*ca* 750 BC – AD 100) astronomical texts, in particular the astronomical ephemerides studied by Joseph Epping and Franz Xaver Kugler at the end of the 19th and the beginning of the 20th century, made extensive use of sexagesimal numbers, regularly dealing with numbers having up to seven sexagesimal places. The mathematical methods used in these astronomical texts are not especially complex, although their application to solving the problems of lunar and planetary theory is highly ingenious. All of the essential mathematical tools used in these astronomical computations are found already a millennium and a half earlier in mathematical texts of the Old Babylonian period (*ca* 2000–1500 BC). Indeed, based upon the numbers of mathematical texts that have been identified, it seems that the Old Babylonian period was the heyday of Babylonian mathematics.

One of the most remarkable discoveries in the study of Old Babylonian mathematics was made in the 1920s when Otto Neugebauer and his colleagues found texts containing Babylonian solutions of second degree problems. Furthermore, the Old Babylonian methods of solving these problems were understood to be identical to our modern methods. In short, this meant that the Babylonians possessed a numerical algebra. This view was unchallenged until the late 1980s when Jens Høyrup first proposed an alternate reading of Old Babylonian mathematical problems, one that claimed that the underlying techniques for solving second degree problems were geometrical, not numerical. The book under review represents the culmination of Høyrup's work over the past decade and a half.

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Høyrup's main tool for analyzing Babylonian mathematical texts is what he calls the 'conformal translation'. In a conformal translation, each Akkadian word is consistently translated with a specific English word or phrase, and, as far as possible, the word order of the original text is preserved. Technical expressions are translated with English words that reflect the original, non-mathematical meaning of the Akkadian word. For example, two subtractive operations are distinguished in the conformal translation: Akkadian *nasāhum* is rendered as 'to tear out', whereas *matûm* translates as 'to be(come) small(er)'. The conformal translations inevitably make for uncomfortable reading, employing as they do many obscure English terms; even familiar expressions are used in contexts where it is not at all intuitive what they mean. For example,

> The surfaces of my two confrontations I have accumulated: $21'40''$, and my confrontations I have accumulated: $50'$. The moiety of $21'40''$ you break, $10'50''$ you inscribe. The moiety of $50'$ you break, $25'$ and $25'$ you make hold. [BM 13901, Obv. I.43–46, translated on p. 67]

probably means little more to most readers than the cuneiform transliteration does to a non-Assyriologist. Nevertheless, unwieldy as it may be, Høyrup demonstrates that the conformal translation, being much closer to the sense of the original text, is the only way to get to the heart of Babylonian mathematical texts. Terms such as 'torn out' and 'append' begin to make sense when we think of them as cut-and-pasting to an imaginary geometrical figure.

After setting out the principals of his analytical method in the first couple of chapters of the book, Høyrup works through more than 50 problems from texts published in O. Neugebauer's *Mathematische Keilschrifttexte* [1935–1937], O. Neugebauer and A. Sachs' *Mathematical Cuneiform Texts* [1986], and E. M. Bruins and M. Rutten's *Textes mathématiques de Suse* [1961], which are supplemented on occasion by F. Thureau-Dangin's *Textes mathématiques babyloniens* [1938] and other publications. (Høyrup has made no attempt to collate the original tablets systematically in order to improve on the published transliterations, but this would be a huge undertaking almost certainly producing very meagre results). In every case he is able to show that a geometrical interpretation of the text is possible. Key to this is translating the term *wāsītum* as 'projection' based upon the general meaning ‘something that sticks out’. This word appears frequently in the mathematical problem texts, always accompanying the number 1, but had no place in numerical understanding of the algebra. However, in the geometrical reading it can readily be understood as indicating that a given line is ‘projected’ into a broad line of unit width. This two-dimensional broad line can then be added (‘appended’) to or taken away (‘torn out’) from a two-dimensional surface. Høyrup’s various arguments in support of his reading of Old Babylonian algebra as being geometrical rather than algebraic are totally convincing.

In chapter 7 Høyrup addresses some of the standard questions posed to historians of Babylonian mathematics by other historians of science. For example, is Babylonian ‘algebra’ really an algebra, especially if it is now to be understood as being essentially geometrical, rather than numerical? Questions such as these are, in my opinion at least, not especially interesting since they generally seem to come down to a question of definition. Nevertheless, Høyrup at least shows that if we use any reasonable definition of algebra, then Babylonian algebra does indeed fall into this category.

In the remainder of the book, Høyrup turns his attention to the wider context of mathematics within Old Babylonian culture. Through a detailed and largely philological examination of local variations in Old Babylonian mathematical practice in chapter 9, Høyrup argues, for example, that the division of Mesopotamia into a Sumerian core and a periphery which had only been under Ur III rule for a limited period is also reflected in a similar division among the mathematical texts. Chapter 10 addresses the origin and development of Old Babylonian geometrical algebra, arguing that it arose out of a deliberate melding of the computational methods of the Ur III scribes with the tradition of practical mathematical knowledge known to surveyors. Finally, chapter 11 discusses the relationship of Old Babylonian algebra to Greek and later mathematics. In parts these chapters are somewhat speculative in nature, and the evidence Høyrup adduces in support of his claims is not always fully convincing. In particular, one is left wondering how other mathematical texts—for example, the tables of reciprocals and multiplications which are preserved in far greater numbers than the problem texts—fit into the picture. Nevertheless, there are many interesting and valuable ideas contained within these chapters.

Høyrup has single-handedly transformed our understanding of Babylonian mathematics with the work presented in this book. There can be little doubt that he is correct in his proposal that Old Babylonian algebra was geometrical rather than numerical in nature. It is not an easy read, but it nevertheless needs to be read by everyone who has a serious interest in ancient mathematics.

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