Ägyptische Algorithmen. Eine Untersuchung zu den mittelägyptischen mathematischen Aufgabentexten by Annette Imhausen

Ägyptologische Abhandlungen 65. Wiesbaden: Harrassowitz Verlag, 2003. Pp. xii + 387. ISBN 3–447–04644–9. Paper € 58.00

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With this book, ancient Egyptian mathematics has returned from the dead. Although Egypt is home to one of the world's oldest literate mathematical cultures, it has been the subject of academic study only since the 1870s with the publication of the Rhind papyrus [Eisenlohr 1877]. New sources appeared steadily over the following decades until the Moscow papyrus was edited by Struve [1930]. Then the material dried up and very few new manuscripts have seen the light of day since then. There have been several general overviews in the last few decades: Gillings' *Mathematics of the Pharaohs* [1972], Robins and Shute's *Rhind Mathematical Papyrus* [1987], and Clagett's *Ancient Egyptian Mathematics* [1999] are probably the best known. Less familiar to both Egyptologists and historians of mathematics outside the francophone world are Couchoud's *Mathématiques égyptiennes. Recherches sur les connaissances mathématiques de l'Égypte pharaonique* [1993] and Caveing's *Essai sur le savoir mathématique dans la Mésopotamie et l'Égypte anciennes* [1994]. (Tracking them down for this review, I discovered that neither had been borrowed from Oxford's internationally renowned and heavily used Griffith Institute for Egyptology and Ancient Near Eastern Studies in the decade since their accession.)

On the face of it then, Egyptian mathematics hardly seems a dead subject: there has been steady activity and output over the last 130 years. It has nevertheless been intellectually moribund, as the very titles of these books suggest. They consist, more or less, of the same subject matter presented in the same way: attempts to explicate Egyptian mathematics in terms of modern mathematical thinking and terminology and to compare Egyptian achievements

(often detrimentally) with those of ancient Greece. In the absence of new evidence there has been very little new to say for many decades. Imhausen has almost no new primary source material, but she is bursting with new interpretations because she seeks to understand her subject matter not in contemporary or comparative terms but as what it might have meant to those who wrote and read it nearly four millennia ago. To that end she puts her formidable Egyptological training to use as well as her close familiarity with the latest methodological trends in the history of the neighboring ancient mathematical traditions (Babylonian, Greek, Roman).

A substantial A4-sized publication running to nearly 400 pages, the book is divided into 13 chapters plus introduction, conclusion, a sizable appendix, and the usual indices and bibliography. The introduction [5–32] summarizes the historiography of ancient Egyptian mathematics and outlines the goals and methodology of the book. An important first step is to define the subject of study—incredibly, not a common practice in the study of ancient mathematics—to exclude ancient sources such as administrative accounts which are merely of mathematical interest, leaving only the supra-utilitarian *intellectual* activity of mathematics that is recorded on some ten documents (papyri, ostraca or pottery fragments, wooden writing tablets, and a leather roll, all dating to the second millennium BC). They contain either arithmetical or metrological tables, or worked solutions to mathematical problems, or both. Whereas most introductions to ancient Egyptian mathematics conclude with an overview of arithmetical techniques, Imhausen chooses rather to present her central thesis: that ancient Egyptian mathematics is essentially algorithmic, and that the extant mathematical problems can be classified according to the algorithms and terminology they employ.

The main part of the book [33–175] is thus devoted to the analyses of a 100 individual examples of Egyptian mathematical problems, according to the typology and principles set forth in the introduction. Hieroglyphic representations, alphabetic transcriptions, and German translations of all of these problems can be found in the appendix [193–364], given in the order of their conventional numbering in the sources. (The manuscripts themselves are written in hieratic, or informal cursive script, which is very difficult to read. It is normal Egyptological practice to transcribe hieratic into the more elegant

and formal hieroglyphs, just as historians of more recent periods might type up handwritten sources for increased legibility.)

For purposes of analysis, Imhausen groups her problems into two main categories: basic techniques, and administrative mathematics. There is a much smaller one on construction and the inevitable final 'fragments and miscellaneous' section. Thus, she rightly sees Egyptian mathematical culture as deeply influenced and informed by scribal and administrative practice. That is not to say that the mathematical problems *are* simply typical bureaucratic methods abstracted from their context; rather, it means that that they draw on scenarios, terminology, and techniques from the professional lives of scribes and accountants in their formulation and solution. The mathematics is not fully comprehensible without reference to wider scribal culture, Imhausen contends, and it may well be that the converse is also true.

Within the broad categorizations of 'basic techniques' and 'administrative mathematics', Imhausen's primary sorting principle is lexical. She groups the problems according to key words—not only the already famous $^c\underline{h}^c$ (pronounced '*aha*', literally, heap), which defines problems about finding unknown quantities, long recognized as a native problem classification (and which Imhausen interprets anew [see below]). Some of those key words are subjects of the problems, others are verbs used as technical terms for the crucial operation in a solution. For instance, '*skm*' ('to complete') means to find the complementary aliquot fraction to the one given in the problem (that is, so that they will together sum to 1). In this way Imhausen avoids modern preconceptions about mathematical typology (e.g., arithmetic and geometric progressions, area and volume geometry [Clagett 1999], equations of the first and second degree [Gillings 1972]) and seeks instead the Egyptian scribes' own conceptions of their mathematical world.

Another major innovation, as I have already indicated, is her acknowledgement and analysis of the essentially *algorithmic* nature of the problems, which is often very complex. She shows too that reading the layout of the solution on the page, not only the text as words, is also crucial to a full understanding of the complexities and subtleties of the corpus. The 15 well-known *aha* problems have long been the subject of vigorous debate, for instance: Do they use the method of false position, as first stated by Peet [1923], or not? By

paying close attention to the algorithmic structure of their solutions, Imhausen shows that in fact they fall into three distinct groups, only the first of which uses false position (though the value of that false position is never explicitly stated). The other two use other methods entirely. Thus, it is not enough for Imhausen to group problems together on the basis of their terminology alone: structural analysis often reveals crucial mathematical variations within lexically homogenous groups of problems.

So, in the wake of this comprehensive and convincing study, what can there possibly be left to do in ancient Egyptian mathematics? Has Imhausen closed the field down again as soon as it has been opened up? On the contrary. Most obviously, she has not dealt with the extensive arithmetical tables also known from second millennium Egypt; but there are also three other, perhaps more interesting and uncharted, avenues to explore.

First, there are two mathematical genres closely associated with the problems that have not yet received Imhausen's analytical attention: calculations and diagrams. The majority of the problems in the Rhind papyrus include calculations which are *not* part of the algorithmic solutions though they may be interpolated within them. That is, they ask no questions, make no statements, give no orders to the reader. They are rhetorically distinct from the algorithms and of a different textual texture. There are other manuscripts—for instance, Rhind problem 49 and the fragment from Lahun, UC 32160—which consist *only* of calculations. So does one of the Rhind's most famous 'problems', number 79 [see Table]. There is no algorithm here, no instructions for solution, although one can be inferred from the calculation presented. (Not surprisingly, Imhausen catalogues it under *varia* [89–91]. A suggestive parallel from the fringes of Babylonia, newly identified by Christine Proust [2002], is based on powers of 9, not 7, and has ants and birds in place of mice and cats. It too is a calculation, not a problem.)

The majority of the problems in the Moscow papyrus, by contrast, include no calculations, even when the algorithm they use is otherwise exactly parallel with an example from the Rhind that does include a calculation. What, then, is the textual function of these calculations? Do they play a pedagogical role, for instance, or is it simply a matter of scribal preference? What do they tell us about

A household

.	2801
2	5602
4	11204
Sum	19607

Houses	7
Cats	49
Mice	343
Emmer wheat	2401(text: 2301)
ḥq3t grain	16807
Sum	19607

The Rhind Papyrus: Problem 79

whether the manuscripts are part of a copied tradition (cf. Greek) or a memorized one (cf. Babylonian)? Were they written by teachers or students—as text books or exercise books? In my own work, analysis of calculations has proved central to understanding the pedagogical context of mathematics in early second-millennium Babylonia [see, e.g., Robson 2002]; it has the potential to be equally fruitful in Egypt. Similarly, the role of the visual in early mathematics has been greatly undervalued until recent years. There are some 14 diagrams in the ancient Egyptian mathematical corpus: What are their representational conventions, and how do those conventions relate to other aspects of Egyptian visual culture? Are words and/or numerals integral to the diagrams? Are the problems comprehensible without the diagrams or (as Reviel Netz [1999] has shown for the Euclidean tradition) are they an integral part of the mathematical structure?

Finally, and most speculatively, what if anything can be said about the relationship between the Rhind and the Moscow papyri, the two most extensive sources in the corpus? I have already suggested that the two manuscripts differ significantly in their use of calculations. Jens Høyrup [2002, 317–361] has recently produced stimulating work on lexical, orthographic, and structural variation in Old Babylonian mathematical problems in an attempt to disentangle local traditions within a previously undifferentiated corpus. Is the same sort of study possible for ancient Egypt and if so what

would it tell us? Reading these works as examples of Middle Egyptian literary culture as well as pieces of mathematics might yield all sorts of unexpected insights.

New methods of close reading and source criticism, and new attitudes to ancient material and intellectual culture have opened up new and exciting opportunities to combine linguistic, historical, and archaeological approaches to ancient mathematics. The study of ancient Egyptian mathematics is alive and kicking thanks to Imhausen's seminal and engaging new work. All those interested in the origins of mathematics should read it and will reap both profit and pleasure. But if a full-length Egyptological monograph in German seems too large a commitment to begin with, I can equally recommend Imhausen's recent articles (in English) in *Historia Mathematica* [2003a] and *Science in Context* [2003b] to whet your appetite for this most fascinating and newly stimulating of topics.

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