Ancient Mathematics by Serafina Cuomo

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Ancient Mathematics is an introduction to, and overview of, mathematical sources of various kinds from the Mediterranean region from the fifth century BC to the sixth century AD. Traditionally, the focus of the historiography of ancient mathematics has been on high-level mathematics associated with names like 'Euclid', 'Apollonius', 'Archimedes', and others. Cuomo departs from this restrictive preselection and casts her net much wider—not only does she include 'professional numeracy' such as land-surveying and accounts, she also provides glimpses of mathematics presented in sources from poetry and politics. The result is an impressively rich picture of mathematics within its social and cultural context that conveys the importance and variety of mathematical practices in Greek and Roman culture.

Cuomo's sources can be assigned to one of the following three groups: 1) classical, high-level mathematical texts which are presented in excerpts and introduced with questions about their authenticity, transmission, and origin; 2) evidence from the practical uses of mathematics in daily life such as land-surveying, accounts, and others; 3) passages from literary and political documents reflecting on the perception of mathematics. In addition to textual evidence, material and pictorial evidence such as mathematical instruments, e.g., surveying instruments [68, 155], abaci [12, 147], and plans [7, 64, 156] is used to help the reader access and appreciate ancient mathematics.

The period covered in just under 300 pages stretches over more than 1000 years. It is divided into four sections: 'Early Greek Mathematics', 'Hellenistic Mathematics', 'Graeco-Roman Mathematics' and 'Late Ancient Mathematics'. Each of these sections is assigned two chapters, one for 'the evidence' and the other for 'the questions'.

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The first chapter begins by pointing to the difficulties posed by the available source material: 'fragmentary, scattered over time and place, or so concentrated in one place (Athens) as to make any generalization dangerous' [5]. This sounds only too familiar to me, having worked on Egyptian material for some time, where we face the same problem. This situation has led to speculative interpretations that have since become 'truths', and Cuomo's book goes a long way to correcting them by presenting the available sources and explaining the problems attached to them. There is no contemporary evidence of the mathematical achievements of Pythagoras and later statements, although made by 'the Greeks themselves', are proven to be unreliable. As in Egypt, Greek culture covers a long period of time; and Greek mathematics too should not be seen as homogeneous. How then, the reader of some older accounts of Greek mathematics may wonder, did previous authors gain so much knowledge about early Greek mathematics? The answer—Cuomo cites and discusses the original sources for these studies in chapter 2—is that there are passages about early Greek mathematicians and philosophers by authors like Diogenes Laertius, Proclus, and others. It is necessary to point out that these authors lived at a time when the early Greek mathematicians had already reached mythical status. There is no contemporary evidence of early Greek mathematics, nor is there a tradition of their knowledge and histories that was handed down faithfully. Rather, as Cuomo points out throughout the book, earlier material—when available—was subjected to reorganization and improvement according to the interests of later mathematicians. And it is with this caveat in mind that we have to study the Greek accounts of their predecessors. Chapter 2 starts by looking into the use of mathematics within politics as represented by land-division, commercial arithmetic, and accounts. Cuomo analyzes mathematical practices as an expression of democracy [40–41], but is also quick to point out that this is by no means the only form of state to use mathematics for its purposes.

In chapters 3 and 4 Cuomo then discusses Hellenistic mathematics. The first paragraph of chapter 3 [62] sketches the geographical and temporal setting, and outlines the history of the empire of Alexander the Great in the third and second centuries BC. The evidence for this period (and all following periods) mostly comes from Egypt, which included, in Alexandria, an intellectual center where many scholars visited, lived, and worked, and which, by virtue of its comparatively dry climate, provided good conditions for the preservation of papyrus. Egypt at that time presents us with a variety of sources coming from at least two traditions, the native Egyptian one and the Greek tradition. In addition, the Persian rule of Egypt has left some influences on the Egyptian culture as well, e.g., on mathematical techniques (see below). Sources of the Egyptian tradition are written in Demotic, the stage of the Egyptian language before Coptic (its last stage). Sources of the Greek tradition are written in Greek. While the existence of these two traditions has been recognized for some time, the Demotic side of this period in Egypt has only now begun to be included. The main reason for the previous focus on Greek sources is the late start of Demotic studies within Egyptology due to the extremely cursive form of script which renders it intrinsically difficult to understand. However, recent publications have made it obvious that a focus on only one of the two traditions will result in an incomplete picture.

Cuomo includes a couple of problems from the Demotic mathematical papyrus Cairo JE 89127–30 [71–72] cited in the translation of Richard Parker. This source is particularly interesting not only because of its mathematical content, but also because the other side of this papyrus contains a collection of laws. Based on paleographic criteria, it can be assumed that both texts were written around the same time (third century BC); and the combination of these two subjects on one papyrus is remarkable.[1] Apart from the monograph on Demotic mathematical papyri which is cited in the bibliography, several other publications of Demotic mathematical texts deserve to be mentioned, e.g., the edition of another mathematical papyrus containing a group of problems about trapezoid shaped fields [Parker 1975], as well as the publication of Demotic mathematical *ostraca*.[2] Cuomo outlines the features of the Demotic mathematical papyri:

> The problems in the demotic papyrus are solved not generally, but for specific causes, and, rather than a deductive proof, they contain a verification, or check step, introduced by the expression 'to cause that you know it'. [72]

[1] For an edition of the legal text, see Donker van Heel 1990.

[2] See the list of mathematical *ostraca* compiled by Jim Ritter [2000, 134n27].

This is in accordance with the classic Egyptian tradition, as we can see in the earlier hieratic mathematical papyri, e.g., the Rhind, Moscow, and Lahun papyri. These date from more than 1000 years earlier than the Demotic sources. However, there are also changes to be noted in comparison to these earlier texts. Some problem types are 'new' for Egypt, but have been known from Mesopotamian sources for a long time [see Høyrup 2002]. Remarkable also is the appearance of multiplication tables, e.g., a multiplication table for 64 from 1 to 16 in P. British Museum 10520 [see Parker 1972, 64–65] which we do not find in the earlier sources. Indeed, the Egyptian technique of carrying out a multiplication documented in the Rhind papyrus, for example, makes multiplication tables like these obsolete.

The questions Cuomo chose in chapter 4 for this period focus on the Greek side of the picture. In the first section 'The problem of the real Euclid' [126–135], Cuomo indicates the starting points for attempts to determine evidence for earlier Greek mathematics and the difficulties attached to them. Her scheme of the transmission of Euclid's elements [127] illustrates well the many traditions involved. The second section 'The problem of the birth of a mathematical community' [135–141] discusses the situation of patronage and collaboration resulting in the emergence of a mathematical community. To this group belonged famous mathematicians like Euclid, Archimedes, and Apollonius and their close acquaintances whom we find mentioned in their works. Another question that could be raised in this chapter, resulting from the evidence Cuomo presented in the previous chapter, is that of the relation between Greek and Egyptian (i.e., Demotic) mathematics. To attempt to answer this question, a thorough analysis of the Demotic material is needed, and some of these sources have only become accessible after Cuomo's book was published [see, e.g., Manning 2003].

Chapters 5 and 6 give an overview of mathematics in Greco-Roman times. Again Cuomo introduces the reader to a wealth of sources, e.g., mathematical papyri, financial documents, metrological texts, and planetary tables. Included also are mathematical instruments such as abaci [147], sundials [154], and sighting instruments like the *groma* [155]. Following the material evidence, the second part of chapter 5 introduces the reader to Vitruvius and Hero of Alexandria, followed by numerous 'other Romans' (Julius Sextus Frontinus, Hyginus Gromaticus, Marcus Junius Nipsus, Balbus, Celsus, Lucius

Volusius Maecianus, Columella, Pliny the Younger, and his uncle Pliny the elder) and 'other Greeks' (Strabo, Philo of Alexandria, Nicomachus of Gerasa, Ptolemy, Sextus Empiricus, Alcinous, Theon of Smyrna, and Galen). Consequently, the two issues raised in chapter 6 are 'the problem of Greek *versus* Roman mathematics' [193–201] and 'the problem of pure *versus* applied mathematics' [201–210]. Cuomo concludes that

> the divides... were much more complicated than simple Greek/Roman or pure/applied dichotomies. Those divides had a political significance, not just in a cross-national, but also in a cross-social-strata sense. [201]

The two final chapters introduce the reader to Late Ancient mathematics (third to sixth century AD). The material evidence cited here includes accounts demonstrating the use of mathematics in everyday administrative practices, and school texts [214] showing the teaching of mathematical techniques used in calculating interest on loans or leases of land. Again, the second half of the 'evidence chapter' (chapter 7) presents mathematicians of this era and their works, e.g., Diophantus and his *Arithmetic* [218–223]; Pappus with his *Mathematical Collection*, commentaries, and *Geography* [223–231]; Eutocius [231–234]; and others. The last chapter focuses on two problems, those of 'divine mathematics' and of ancient histories of mathematics. Cuomo discusses the use of mathematics in Christianity, e.g., in time-keeping to regulate daily prayer and to establish the date of the Easter festival. The second section then analyzes the work of Pappus, Proclus, and Eutocius who 'classified, defined and systematized' [256] earlier works using 'the past as it suited them and their present concerns.' [261].

Throughout, this book is engagingly written; and it is a pleasure to entrust oneself to Cuomo's choices as one is led through the various periods. There are many illustrations, all of good quality, which help one understand what the sources actually look like. The abundance of references given makes this book not only perfect for a beginner but offers valuable guidance into further reading. It is obvious that the author worked carefully, and the readers benefit from her hard work. I was rather frustrated, however, by the table of contents [v], which simply gives the skeleton outline of the division according to periods and evidence vs. problems. The table of contents given at the end of this review was established by including also the subheadings one finds in the individual chapters; maybe it will be helpful for some readers. Likewise, in addition to the figures of places mentioned [ix–xi] and the glossary [263–266], I would have welcomed an overview of all the people mentioned throughout the book. Some of them, but not all, can be found in the index [287–290]; so maybe a separate index of persons would be appropriate.

The success of this book relies mainly on two elements. First, Cuomo has included both the ‘classic’ high-level mathematical texts as well as lesser known works. Thus, we find a variety in this book that may well represent the breadth of ancient Greek mathematical culture. It is only by taking into account the lower-level mathematics and the many uses that mathematics was put to, that one can appreciate Greek mathematics. It is thus deeply satisfying to read the account of classic mathematical texts within their historical and social context. The second outstanding achievement of this book is the extended use that Cuomo makes of her philological training. Previous accounts of Greek mathematics might not even mention the sources that the claims of their authors were based on. Cuomo goes a long way in rectifying this, by not only indicating the available source material but also by giving the reader a detailed introduction to the problems attached to them. This hopefully serves to help mathematicians understand what sorts of problems historians of mathematics face in their work. It also teaches a certain scepticism towards ‘long established truths’ about Greek mathematics which may well be ancient myths rather than realistic accounts. At the same time Cuomo does not discard all later Greek accounts about earlier mathematicians, but indicates that some of them—if read carefully—prove to be quite illuminating.

I believe this book is apt to serve many types of readers, from the total beginner to those who have already read the classics; and with the detailed explanation given in each argument, it is usable by mathematicians, historians of mathematics, and general historians alike. Cuomo has clearly demonstrated that there is a wealth of sources available to provide information about ancient mathematics, a wealth that has not traditionally been included in historical studies.

It left me with the wish that many of the aspects Cuomo touched upon will be explored in more detail in (her) further publications.

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