Classics in the History of Greek Mathematics edited by Jean Christianidis

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The volume reprints 19 essays, distributed in six parts. These are the 'Classics' referred to in the title, namely, contributions standing as fundamental in the development of the field. The selection of the essays to be reprinted has been made by six distinguished experts; every part is preceded by an introduction that puts the selected essays into their contexts, often providing useful additional information. The parts, the respective editors, and the reprinted essays are:

Part 1. The beginnings of Greek mathematics (H.-J. Waschkies)

J. Mittelstrass. 1962–1966. 'Die Entdeckung der Möglichkeit von Wissenschaft'. *Archive for History of Exact Sciences* 2:410–435.

Á. Szabó. 1956. 'Wie ist die Mathematik zu einer deduktiven Wissenschaft geworden?' *Acta Antiqua Academiae Scientiarum Hungaricae* 4:109–151.

W. R. Knorr. 1981. 'On the Early History of Axiomatic: The Interaction of Mathematics and Philosophy in Greek Antiquity'. Pp. 145–186 in J. Hintikka, D. Gruender, and E. Agazzi edd. *Theory Change, Ancient Axiomatics, and Galileo's Methodology: Proceedings of the 1978 Pisa Conference on the History and Philosophy of Science.* Dordrecht/Boston.

Part 2. Studies on Greek geometry (R. Netz)

W. R. Knorr. 1983. 'Construction as Existence Proof in Ancient Geometry'. *Ancient Philosophy* 3:125–148.

K. Saito. 1985. 'Book II of Euclid's *Elements* in the Light of the Theory of Conic Sections'. *Historia Scientiarum* 28:31–60.

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G. E. R. Lloyd. 1992. 'The *Meno* and the Mysteries of Mathematics'. *Phronesis* 37:166–183.

Part 3. Studies on proportion theory and incommensurability (K. Saito)

O. Becker. 1932–1933. 'Eudoxos-Studien I. Eine voreudoxische Proportionenlehre und ihre Spuren bei Aristoteles und Euklid'. *Quellen und Studien zur Geschichte der Mathematik, Astronomie, und Physik* B2:311–333.

K. von Fritz. 1945. 'The Discovery of Incommensurability by Hippasus of Metapontum'. *Annals of Mathematics* 46:242–263.

H. Freudenthal. 1966. 'Y avait-il une crise des fondements des mathématiques dans l'antiquité?' *Bulletin de la Société mathématique de Belgique* 18:43–55.

W. R. Knorr. 2001. 'The Impact of Modern Mathematics on Ancient Mathematics'. *Revue d'histoire des mathématiques* 7:121–135.

Part 4. Studies on Greek algebra (J. Sesiano)

K. Vogel. 1933. 'Zur Berechnung der quadratischen Gleichungen bei den Babyloniern'. *Unterrichtsblätter für Mathematik und Naturwissenschaften* 39:76–81.

G. J. Toomer. 1984. 'Lost Greek Mathematical Works in Arabic Translation'. *Mathematical Intelligencer* 6:32–38.

T. L. Heath. 1910. 'Diophantus' Methods of Solution'. Chapter 4 in *Diophantus of Alexandria: A Study in the History of Greek Algebra.* Cambridge.

Part 5. Did the Greeks have the notion of common fraction? Did they use it? (J. Christianidis)

W. R. Knorr. 1982. 'Techniques of Fractions in Ancient Egypt and Greece'. *Historia Mathematica* 9:133–171.

D. H. Fowler. 1992. 'Logistic and Fractions in Early Greek Mathematics: A New Interpretation'. Pp. 133–147 in P. Benoit, K. Chemla, J. Ritter edd. *Histoire de fractions, fractions d'histoire.* Basel.

Part 6. Methodological issues in the historiography of Greek mathematics (S. Unguru)

S. Unguru. 1975–1976. 'On the Need to Rewrite the History of Greek Mathematics'. *Archive for History of Exact Sciences* 15:67–113.

B. L. van der Waerden. 1975–1976. 'Defence of a "Shocking" Point of View'. *Archive for History of Exact Sciences* 15:199–210.

A. Weil. 1978. 'Who Betrayed Euclid? (Extract from a Letter to the Editor)'. *Archive for History of Exact Sciences* 19:91–93.

S. Unguru. 1979. 'History of Ancient Mathematics: Some Reflections on the State of the Art'. *Isis* 70:555–565.

A collection of studies such as this is a tool for the working historian not only because it reprints essays often difficult to find, but also because it offers a cross-section of the main historiographical currents that represents well the evolution of the field in the last decades. As is clear from the list, in fact, while several technical articles are presented (especially in parts 4 and 5), the majority of them is concerned with methodological issues, thereby greatly enlarging the boundaries of part 6.

In this respect, the collection testifies to the essentially historiographic character of the scholarly production in the last three decades. The domain of research portrayed in the book underwent a phase transition with the pivotal article by S. Unguru 'On the Need to Rewrite the History of Greek Mathematics'. This article reacted against the interpretation in algebraic terms of certain portions of the ancient Greek corpus, the so-called 'geometrical algebra' invented by P. Tannery and championed by H. G. Zeuthen and, after him, by B. L. van der Waerden. The ideology, expressed or unexpressed, underlying the 'geometrical algebra' interpretation was that, after all, proved mathematical statements are necessarily true; as a consequence, an allegedly invariant 'mathematical core' is independent from the language in which it is formulated. After a series of more or less rude reactions, partly represented in part 6, Unguru's article entailed a whole recalibration of the historiographical attitude towards mathematics as done in the past. An approach in which modern symbols and notions were employed as a matter of course to explain Greek mathematics was replaced by one in which efforts to understand it 'in its own terms' and attention to the cultural context seem to have finally become a common historiographical practice. (To be sure, not everything that was written before Unguru's paper was algebraically-dressed, as von Fritz' and most notably Becker's seminal papers attest.) Such a renewed attitude produced a wide-ranging spectrum of contributions. These range from strictly technical papers such as Saito's (recall that the algebraic interpretation of book 2 of the *Element* was the stronghold of the 'geometrical algebra') to studies in which a respectable amount of historical data coming from a variety of sources is collected and given a consistent interpretation, such as Knorr's fourth essay here presented. However, the main outcome was the conception of studies in which making some methodological point is among the main goals, if not the main goal. A confirmation of the fact that this is the current historiographical stance comes from the very volume under review: among W. R. Knorr's massive and often technically overwhelming production, three of the four articles selected focus on methodological issues.

What makes the book even more valuable, despite the radical changes just outlined, is that one finds in it essays, such as Vogel's or Heath's, which are representative of the 'algebraic' approach. This is not the mark of a schizophrenic attitude of the editors, but should more properly be taken to suggest that accounts made in the ancient fashion can still prove valuable in guiding an algebraically-minded reader through such difficult texts as Diophantus' *Arithmetica*. As remarked above, such accounts can even be taken, still today, to say something *true*, although in a wrong historiographical perspective, on certain portions of Greek mathematics.

Interestingly enough, the renewed attention to contextual issues produced a new type of *a priori* arguments, less patently unsound than the algebraic interpretations and thereby much more difficult to uncover. Such are for instance impossibility arguments. In them, from the mere fact that certain mathematical steps are not attested in the Greek mathematical corpus (e.g., the usual operations on common fractions), a *blocage mental* of sorts is inferred on the side of the Greek mathematicians (the lack of the *notion* of a common fraction). Of this kind is David Fowler's paper reproduced in part 5—and there confuted. It is obvious that such arguments are unmethodical; and from a factual point of view, they simply ignore that Greek mathematics has not been transmitted to us in its entirety.

Investigations on the interactions of mathematics with other branches of Greek thought, most notably philosophy, got greatly enhanced in the enlarged view created by the renewed historiographical perspective, despite Knorr's effort to defend a strictly internalist position in the first essays of his among those selected. Curiously enough, the book under review gives more prominence to papers in which an alleged connection between mathematics and philosophical issues is shown not to exist. A case in point is the foundational crisis following the invention (or discovery, using a term that appears to be nearer to the underlying ideology of Greek mathematics) of irrationality. That such a crisis was nothing but a historiographical figment was first shown in Hans Freudenthal's paper. Knorr returned to this issue with his usual effectiveness, pointing out, in the third essay of his here reproduced, that the figment was in fact almost a necessary outcome of a cultural *milieu* such as the one of Weimar Germany. The same *milieu* could explain the purposes of most of Becker's contributions to the study of Greek mathematics, the one here reproduced included. Another case of a demonstrably false link is dealt with in the third of Knorr's contributions selected, where it is shown that Zeuthen's thesis that constructions were intended as existence proofs in Greek mathematics is not supported by the actual evidence. (Actually, Knorr endorses an interpretation of Zeuthen's thesis that, while being the current one, is far stronger than the one borne out by an equanimous reading of his original article: at least judging from the number of self-references, Knorr's paper seems to have been more a self-invited essay-review of his own, forthcoming book *The Ancient Tradition of Geometric Problems,* rather than an unbiased assessment of Zeuthen's position.) The renewed emphasis on the cultural environment in which Greek mathematicians moved has opened the field of ancient mathematics to the fresh, and at times rather unconventional, views of scholars coming from other domains, most notably historians of ancient thought. It is disappointing that such contributions are represented in the volume by Lloyd's article only, in which pointless speculations seem to be the only content of any discernible originality.

It is absurd to question the choices of the editors, but two remarks should be made. First, part 5 seems, frankly speaking, much too specific. One is led to suspect that the editor of the part, and of the whole volume, chose the argument in order to show that the authors of the two articles presented are actually wrong in contending that Greek mathematicians had no notion of a common fraction. In fact, the introductory essay is uniquely concerned with presenting examples from Diophantus' *Arithmetica* that falsify such a contention, especially as advocated by David Fowler. Yet the reader had already at his disposal the same set of examples, devised to make the same point, in Knorr 1991.

Second, articles dealing with issues of textual tradition are totally absent. Recent studies, however, have permitted a better, if only provisional, assessment of the relationships between the Greek and the Arabic tradition of the *Elements*: this domain of research deserved more attention. The short paper by Gerald Toomer on the Arabic tradition of Greek mathematical treatises presents a state of affairs that has greatly evolved since then. Maybe an article showing how actual textual issues are treated by the working historian of mathematics would have served the interests of the reader better.

Essays are reproduced that date back more than 40 years, and written in French or German. This is a very important feature of the book, especially because a large portion of contemporary scholarship appears to resort almost exclusively to the most recent secondary literature and to contributions written in English.

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