The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook edited by Victor J. Katz

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It is no understatement that the arrival of this unique reference has been eagerly anticipated by the community of historians of mathematics. Never before has a single work delivered to scholars such a rich and comprehensive guide to the history of non-western mathematics. Victor Katz was the perfect candidate to initiate and oversee this project and, as the editor, his vision of a single sourcebook in which each cultural area was prepared by a renowned specialist in their field has been fully accomplished. The resulting product is a thorough and insightful coverage of five key centers in non-western mathematics: Egypt, Mesopotamia, China, India, and Islam, each of which is allotted a single chapter with its own reference section and bibliography. Indeed, the selected authors epitomize the new trends in the history of mathematics: in effect, they show that to produce well-rounded, critical, and perceptive accounts of mathematics past, you must be fluent in the requisite languages and that you must have familiarity with the primary sources, awareness of the broader issues in historiography, as well as mathematical facility. As expected, the authors show historical and mathematical sympathy, always with a notable respect for preceding generations of scholarship when they disagree; and they display an impressive (even daunting) knowledge of other intellectual fields, including anthropology, archaeology, linguistics, material culture, paleography, philology, philosophy, and sociology, to name but a few, which they draw upon where appropriate to deepen and enrich their accounts. Far from being disjointed, as can be the case with a multi-author collection, this work is highly cohesive. In fact, one highly valuable, possibly unanticipated, consequence of this book is the presentation of five distinct methodologies by top professionals who each tackle the history of mathematics

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differently—sometimes subtly, sometimes substantially—but always coherently, in a way that you would never find in a monograph.

The work opens with a modest introduction: Katz politely and quite properly lets the authors speak for their own fields. Each chapter is self-contained and its subject is introduced and illustrated by means of primary sources, translations, and mathematical and historical commentaries. Excerpts are carefully selected to give an overview of the breadth of the field and, where appropriate, pictures, diagrams, metrological tables, and transcription guides clarify the main body of the text. All accounts give the reader a reassuring taste of a much bigger field.

What is immediately distinctive, particularly in the first three chapters, is the self-conscious, revisionary tone in the scholarship and the identification of the inadequacies of earlier accounts. As the authors observe, practices such as casting past mathematics directly into its 'modern' equivalent, or comparing and evaluating these non-western traditions with respect to their 'western' counterpart, do little to help modern audiences appreciate these mathematical cultures. Annette Imhausen stresses the need to keep mathematical algorithms in their original layout and format, and not to decontextualize them by translating them directly with modern notation. Eleanor Robson laments the lack of attention to vitally relevant details such as provenance and chronology, in previous scholarship 'when [cuneiform] tablets were considered not as archaeological artifacts but rather as bearers of text' [92]. Joseph Dauben questions the assessment of the Chinese mathematical tradition as 'authoritarian' as potentially trivializing, and suggests that it needlessly polarizes it with the Greek tradition. Furthermore, there are many anachronisms and much idle speculation circulating in histories of mathematics, particularly, it seems, in those bearing on non-western mathematics, where our knowledge is sketchy and many details are still to be filled in. These are immediately and firmly dispelled at appropriate times throughout the book. For example, Kim Plofker tactfully questions the notion of the 'ritual origins of geometry' [387] as well as claims which associate Vedic mathematics with various modern-day computational algorithms.

One delightful feature of the various accounts is the inclusion of exchanges between members within these early mathematical communities. Imhausen opens her section with details of a competitive squabble between two Egyptian scribes, each trying to outdo the other in mathematical prowess [10ff]. Robson illustrates mathematical pedagogy by a humorous dialogue [80] between a supervisor issuing to a younger trainee scribe advice that had been passed down to him. Plofker gives us a glimpse into the divisions and disagreements in the Indian mathematical community by including an outline of one of the most famous rivalries recorded, that between Āryabhaṭa and his successor Brahmagupta [419]—despite the fact we only have occasional references to this rivalry in texts, they suffice to show that the relationship was far from collegial! Berggren includes a debate between two medieval Islamic scientists who argue over optimal solutions to various problems and dispute the validity of approximation used in mathematics [568ff]. They draw not only from their own mathematical tradition but invoke mathematical precedents set by others such as Archimedes, Aristotle, Galen, Hipparchus, and Ptolemy. These excerpts remind the reader that such historical texts have immediate human appeal, and also give a sense of the sociology of the individuals who were responsible for them and of the ways in which those individuals interacted professionally.

One important feature of the history of mathematics is the transmission of ideas from culture to culture, particularly in the ancient and medieval periods. Reflecting upon the importation of new ideas into a pre-existing culture can lead to valuable insights. An idea, technique, or concept may be adopted; but it may be also be changed, misunderstood, or rejected. The authors have each included aspects of transmission as they are able where appropriate, and the references stir the reader's curiosity for seeing it covered more systematically.

Imhausen was given the task of covering the Egyptian mathematical tradition from roughly the Archaic period (*ca* 3000 BC) to the Graeco-Roman Period (ending *ca* AD 395). At the outset, she gives a personal insight into the highs and lows experienced by any historian of mathematics in this field. Among the particular frustrations for an Egyptologist in this area is the lack of sources. Nonetheless, she expertly selects a number of wide-ranging texts which reveal the mathematical sophistication of that culture at various times. Imhausen not only details the mathematics, but also highlights linguistic and grammatical features and offers comments of a more anthropological, archaeological, and paleographical nature as well. She introduces the reader to some simple features of Egyptian hieroglyphs and the various transcriptional conventions used by modern Egyptologists. Texts of a more technical nature are always given with their hieroglyphic transcription and mathematical interpretation, and, on occasion, a photo image of the particular papyrus. Given this careful tutelage and the inclusion of the appropriate sources right there, the reader feels as though they could actually read the original themselves! Accompanying every example is a thorough commentary which consciously attempts to stay as 'literal' as possible. Lack of sources compels historians to be more versatile and resourceful, and indeed Imhausen draws from the progress within the wider field of Egyptology to deepen her analyses.

Imhausen notes the striking similarity between the solutions of similar sorts of problems and suggests that there may have been some general algorithmic-type approach that was understood but never explicitly expressed. She hints at a more general typology of features within individual texts, although notes that there was no standardized practice—the format of each text was to a large degree a product of the tastes and predilections of the individual who was writing it. She introduces three useful typologies—rhetorical, numeric, and algorithmic [24]—which highlight other various aspects in the texts. She offers a fascinating insight into the state of technical terminology in Egyptian mathematics [25]: Egyptian scholars, unlike those in Mesopotamia, seem to have used different but closely related technical terms to express mathematical nuance and the exact significance of this is still to be determined. She shows that different stages of a mathematical problem were consistently presented in accordance with various grammatical markers [25]; for example, the title was expressed in the infinitive construction, the working in second person, the results in the third person (*sḏm.ḫr.f*), and so on. She makes brief mention of the role of diagrams and their uses within the Egyptian tradition. Among some of the mathematical features likely to be of interest are the technique of false position [28], rules for the area of a circle [29] and the object referred to as a *nb.t* [31], bread-and-beer problems [38ff], and various ratio problems from the Graeco-Roman period [48–50].

Robson, in her usual dynamic and definitive approach, reminds the reader that Mesopotamian mathematics is so much more than the nine-times table and the 'Pythagorean triples' of Plimpton 322.

Her account revolves around the following three themes: first, the ways in which Western views of Mesopotamian mathematics have changed over the last two millennia; second, the who, why, and how of this mathematical tradition; and third, the rationale behind the selection and production of her translations [58]. She gives an overview of the scholarly tradition and shows the need to revise it, citing such deficiencies in earlier approaches as the lack of any sustained questioning of authorship, context, and function [60]. She definitively distances Mesopotamian mathematics from the so-called 'infancy of the western tradition', revealing a picture far more complex and rich than ever previously described. She upholds that doing a proper job as a historian of mathematics is more than just reading the numbers and shows how much more we can know about the mathematical aspects when we expand our lines of inquiry beyond the texts' contents alone. She highlights the interconnections between mathematics and other aspects of social and culture enterprises, and describes features hitherto overlooked such as social context and financing.

She carefully outlines the 'multistage' operation for the preparation and publication of cuneiform tablets [66]. She notes translation worries, establishing her preference in the conformal *versus* modernizing [67] debate,[1] and raises issues concerning the translation of technical terminology. She informs the reader of editorial conventions and not only standardizes best practice, but epitomizes it herself throughout the chapter. The reader is treated to photos of tablets, expertly executed transcriptions, thoughtful translations, mathematical commentaries, and even reflections on the physical state of the antiquities themselves when appropriate.

Robson conveys to us the nature of mathematics as a human enterprise, reminding us that no scribe was simply a mathematician. She outlines scribal education, carefully details scribal errors, and describes the situations and circumstances of particular scribal families such as the Shangû-Ninurta family from the late fifth century BC [161] and the Sîn-leqi-unninni family which flourished around 200 BC

[1] That is, whether to translate the text as literally as possible so that the English is made to 'conform' to the original Akkadian as far as the translator is able (following Friberg and Høyrup for example), or to 'modernize' it by rendering it in language and symbolism that is instantly recognizable to modern readers (following Neugebauer and others).

in Uruk [174]. She concludes from the frequency of occurrence of particular mathematical examples certain trends about the reception, the audience, and the popularity of particular areas in mathematics. Importantly, for future generations, she highlights the challenges of unprovenanced tablets and she draws tentative conclusions about regional differences in mathematical practice, something that she can do because of her interest in archaeological provenance.

She selects a breadth of mathematical material from the nuances of sexagesimal arithmetic to 'geometrical algebra', arithmetic progressions, geometry, and various practical applications of mathematics in their multifarious formats (which include problem texts, trainee scribes' rough-working, and reference lists). She presents previously published as well as unpublished tablets; the famous tablet concerning the square root of 2 is highlighted with its previously unpublished reverse. She illustrates some of the difficulties that the mathematical Assyriologist [130ff] encounters by including a challenging tablet (BM85194) made difficult by its numerical errors, rare words, and accidental omissions and additions, and by detailing the various attempts to make sense of it.

Yet further east, the three millennia or so that span the Chinese mathematical tradition are covered by Dauben. In his opening words, he invokes the 'standard' view of Chinese mathematics as 'utilitarian, authoritarian, and basically conservative' [187] and challenges this characterization; his selections thereafter are very much made with this sentiment in mind as he presents both techniques and problems that are especially typical of Chinese mathematics in addition to those that are distinctly innovative. He superbly conveys the difficulties of working with the Chinese language and the perils of translation particular to it by his illustration of the ways in which scholars have disagreed quite significantly about how to translate even a title. For example, the classic Chinese mathematical text *Jiu zhang suan shu* [227ff] has been translated as 'Arithmetic in Nine Sections', 'Nine Chapters on the Mathematical Art', 'Computational Prescriptions in Nine Chapters' and 'Nine Categories of Mathematical Methods', among others. Indeed, Dauben refers to it simply as 'Nine Chapters'! Other philological delights are littered throughout his chapter, notably the astonishing fact that there was no word for triangle within the Chinese tradition [232].

Dauben incorporates archaeological finds from as late as two decades ago and draws from a variety of media to highlight features of Chinese mathematical industry. He includes illustrations from a stone relief from a Han dynasty tomb, a drawing on silk from 350 BC, and a bronze standard measure, as well as a mathematical text written on bamboo strips. He gives the actual Chinese characters where appropriate and uses a hand-rendered font to demonstrate the various workings of the 'counting rods'—effectively demonstrating how this system for depicting numerals lent itself readily to efficient arithmetic algorithms [194ff]. He carefully describes the ingenious procedure known as the 'out-in' principle [199ff], a technique invoked usually in the context of geometrical proofs for demonstrating equivalencies. He also highlights the innovative use of colors in proofs [251, and elsewhere].

He notes that Chinese mathematical texts served two primary purposes, one research-directed and the other educational [193]. He gives insight into the motivations of mathematicians, quoting the Chinese author Zhao Shuang who stated 'my sincere hope was to demolish the high walls and reveal the mysteries of the halls and chambers within' [194]. This sentiment is perhaps atypical in the history of mathematics—it is sometimes speculated, for example, that Sanskrit Paṇḍits deliberately obscured their material for the specific purpose of keeping it esoteric and esteemed!

Dauben reveals his command of the broader Chinese intellectual tradition by noting methodological similarities with Chinese philosophy [213]. He shows that Chinese mathematics was not just a practical offshoot from the various needs of the empire, but also firmly an 'art' in its own right [213]. He gives glimpses into the various challenges for a practicing mathematician, including a quote from Liu Hui who admits that the complete solution of a problem is beyond his abilities [249]; and he outlines the broader interests of mathematicians in philosophical or metaphysical issues [301] as well as details of their position in the social hierarchies [303]. He reminds the reader of the fact that the status of mathematics in any society is not assured: he shows how mathematics fell in and out of favour [308] at various times throughout Chinese history and notes the arrival of the Jesuits and the impact that this had on Chinese mathematics [366]. He describes the reception of Euclid by Chinese scholars, who, paradoxically enough, were more interested in the results than his axiomatic method. He notes that Chinese scholars remained puzzled by the reception and status that this work had in other cultures, since they found it repetitive and needlessly complicated, giving them no new mathematical detail than already existed in their own tradition!

Importantly, Dauben covers the *goug-gu* (better known as the Pythagorean) theorem [215] and the more controversial threads of scholarship surrounding it.[2] As is well documented in this book, there are other early instances of this numerical relationship: Plofker notes [387–390] its first appearance in Sanskrit sources in about 800 BC and an indirect appreciation of it can be found can be found in Egyptian [49–50] and Mesopotamian sources [140–141]. Most scholars are now firmly of the conviction that instances of particularly useful mathematical facts can appear independently in different cultures, without the need for far-flung speculation about intellectual appropriation.

Among some of the other interesting mathematical aspects of Dauben's account are square- and cube-root algorithms, calculation of volumes [259], the double difference method [288], the representation of big numbers [297] and links therein to Archimedes, conceptions of infinity and the endless cycle of numbers [301], the Chinese remainder theorem as well as Chinese 'algebra' [324, 345], the binomial coefficients [330]—which very nicely illustrated by a reprint from the actual manuscript—and various applications in mathematical astronomy and time-keeping, for example [213ff]. Furthermore, Dauben gives us insights into counting boards [447] and the ways in which they can keep track of the coefficients of various combinations of unknowns of arbitrary power, so that elimination becomes a mechanical process.

Oft quoted is al-Bīrūnī's assessment of Indian mathematics as being 'a mixture of costly crystals and common pebbles' [435]. As has been shown by Plofker through the excerpts that she presents, this metaphor is completely inappropriate. More importantly though, she shows us how al-Bīrūnī misunderstood the circumstances in which Indian mathematics was practiced. As she points out, Indian mathematics, like most other intellectual disciplines in India, were carried out for the most part in an oral environment, which meant that mathematicians had quite different pressures on them as they engaged in

[2] E.g., 'Was Pythagoras Chinese?' Indeed, compare Dauben's reference to Liu Hui as the Chinese Euclid!

mathematical activity. Plofker documents mathematical highlights drawn from a staggering time period—from the emergence of literate intellectual cultures until it was 'westernized'—and details its assimilation into modern global mathematics with excerpts from just half a century ago.

In this chronological span, she illustrates the various manifestations of mathematical activity, be it mathematics proper, its various applications, instrumentation, mathematics education, or the various early 'ethnomathematical' expressions of mathematical knowledge [386]. She speculates on the reasons for the commissioning and copying of mathematical texts in their thousands and she covers the standard favorite authors, including excerpts from the Śulbasūtras and the Bakhshālī manuscript, Āryabhaṭa, Bhāskara I, Lalla, Mahāvīra and so on; much is presented here for the first time. She includes many excerpts which until recently have escaped the notice of historians of mathematics because they are not directly in mathematical sources but appear in other intellectual traditions. For example, an interesting technique for the computation of 2^n is found in an early work on prosody—not only obscure in location but laconic in expression—the mathematical content of which she teases out expertly and seemingly without effort.

Plofker emphasizes the importance of the relationships between mathematicians and their successive generations, an aspect critical to a discipline carried out in an oral environment. She ironically notes [400] that in fact the more detailed texts and explanatory diagrams were reserved for the 'dull-witted', and that the brilliant student was one who could untangle the terse abbreviated metrical verses to make both a linguistically and mathematically consistent interpretation. She observes the challenges in this by citing the example of one of India's most gifted mathematicians, Bhāskara, as he struggled to make sense of a particular rule given by Brahmagupta concerning cyclic quadrilaterals [462]. It would seem that he simply misunderstood the 'cyclic' prerequisite of the rule—one can hardly blame him, for it was never originally mentioned by Brahmagupta in the first place!

Plofker quickly reveals her versatility and breadth as a scholar by frequently noting details of transmission. India has been called the 'recipient and remodeler of foreign traditions' [Pingree 1978] and she knowledgeably and frequently comments on issues concerning the transmission of ideas into and out of the Indian mathematical tradition. Perhaps because of the reputation that Sanskrit has for being notoriously difficult, she has been careful not to dwell on technical terminology. Plofker gives insight into the role of the commentary [400ff]; and, slipping naturally into the role of commentator herself, she provides a hyper-commentary to an excerpt from Bhāskara, who is himself commenting on a work by Āryabhaṭa. She relates the importance of colophons [441] and the development of different schools of thought. She details the various social structures in India, situating the audience of these texts and defining the typical status of the mathematician. She remarks on the hereditary nature of mathematics education as well as the role of women within the mathematical context. She also makes the remarkable observation that many mathematical techniques which are used and described in mathematical applications are never seen in general mathematical works. She wraps up her coverage of this area by describing encounters with modern western mathematics [507], the details of mathematical education in British India, and the struggle between indigenous knowledge and the implementation of the modern European curriculum.

Excerpts of particular mathematical interest are abundant and include her description of the number systems and numerals [395–398], the 'circulature' of the square [392], the *karaṇīs* [407], the computation of sines [408–409, and elsewhere], the 'pulverizer' [416], cyclic quadrilaterals [424–425], computation with seeds or 'algebra' [467], infinite series—which include the Mādhava-Newton series and various manifestations and approximations of π [481ff]—sequences, combinatorics, and magic squares [493ff], as well as various applications in mathematical astronomy.

Berggren's assignment concerns a geographical region previously covered in this book, but far removed in time, circumstances, population, language, and society. Robson's account of mathematics in the Ancient Near East ended in the first century AD; Berggren picks up some 700 years later with the emergence of Islam. He notes at the outset that the designation 'Islamic' refers to those regions of the world where Islam was the dominant religious and cultural tradition, but that this term can be overly exclusive since many of the notable practitioners belonged to other religious traditions and cultural groups. The other appellation commonly invoked in this field is 'Arabic', to convey the dominant language in which these texts were composed;

but this designation too has its drawbacks. Berggren himself prefers the former designation.

Berggren deliberately chooses to arrange his material differently than the other contributors. His organization is primarily thematic, though the material within a given theme is then arranged chronologically—a wise decision that enables him to manage the overwhelming number of sources available. He includes a satisfying explanation of Arabic names [520], a topic often quite daunting to new readers. He refrains from too much detail on the Arabic language and paleography, perhaps because the script and orientation is so different to what the majority of his readers are comfortable with. At one point, he directs the reader to 'spotting' features in an original manuscript [533] but with little assistance for those unfamiliar with Arabic paleography. A small table outlining number systems and numerals might have been useful at this point, particularly as they are the origin of our present notation.

Berggren perfectly characterizes Islamic mathematics as *heuristic*, that is, as an enterprise of solving geometrical problems; and he identifies it as being inspired from three traditions principally—Greek mathematics and geometry, the numerical solutions of indeterminate problems from Diophantus, and the practical manuals of Heron. He illustrates a tight relationship between mathematical theory and practice [519, and elsewhere], and the excerpts that he selects reveal the various audiences that these texts might have been composed for—audiences that include practitioners and artisans, students, and other colleagues [585]. Berggren pays particular attention to the details of the translations that he provides, warning against the allure of 'false-friends'[3] [519] in mathematical texts and gives considered accounts of some difficult terms.[4] He has done an excellent job of covering and describing some quite thorny passages and unraveling excerpts densely packed with sophisticated mathematical ideas.

Of particular delight to those interested in mathematical details will be the various construction problems, the extraction of a fifth root [538], 'algebra' [542ff], stereographic projection [573], the description and function of the rusty compass [577ff] and the perfect

[3] That is, when the name of a mathematical object or concept in that context is distinctly different from what we associate it with in modern mathematics.

[4] See, e.g., 594n102 and his description of *manshūr*.

compass [595], volumes of revolutions [587ff], trigonometry and non-linear interpolations of sine tables [621ff, esp. 626–627], determination of the directions (the *qibla*) to Mecca [635], combinatorics [658], and especially that theory as applied to eclipse possibilities [659].

This reference work will not only be of constant use to the professional researcher, but also the interested amateur and the teacher of both secondary and tertiary mathematics. It will prove ideal not only for the purpose of injecting history into the regular mathematics curriculum, but also for teaching the history of mathematics. As this book will serve as a reference guide for consultation on particular topics and themes, there is a need for a comprehensive and exhaustive index. The index at present provides a basic coverage of the content, but could be greatly expanded. For example, some old favorites do not appear as such in the index ('Plimpton 322', the 'Rhind Mathematical Papyrus', 'Rusty compass', to name a few). Some less obvious entries are indexed but more prominent ones are left out; for example, particular Mesopotamian scribes are indexed but the scribal families on which Robson spends several pages[5] are left out. Many other entries could be amplified as not all instances are listed: for example, 'False Position' could have added to it pages 148 and 550; 'Zero' and 'Errors in Calculation', to name a few, could be similarly expanded. Furthermore, this book is unique because it contains so many excerpts from primary sources, many of which are definitively translated and published here for the first time. To aid the reader, it would have been beneficial to compile an *index locorum* cataloging the passages translated by author for quick reference. This would allow both professional and amateur quick access to relevant passages by author as well as by theme or chronological period.

All in all, this sourcebook does something tremendously important for the field. By means of carefully selected examples and adequate guidance, the authors have consciously given the reader a chance to work with and interpret the primary sources themselves. Thanks to their groundwork, explanations, and reference tables, readers get to feel something of the exhilaration and empowering experience of penetrating aspects of these texts. It is through such an

[5] That is, the Shangû-Ninurta family [161ff] and the Sîn-leqi-unninni family [174ff].

accessible exposition that the next generation of historians of mathematics will be inspired and motivated to continue the tradition as it should be practiced.

BIBLIOGRAPHY

Pingree, D. 1978. 'History of Mathematical Astronomy in India'. Pp. 533–633 in vol. 15 of C. C. Gillispie ed. *Dictionary of Scientific Biography.* New York.