Unexpected Links between Egyptian and Babylonian Mathematics by Jöran Friberg

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Amazing Traces of a Babylonian Origin in Greek Mathematics by Jöran Friberg

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In these two books, Jöran Friberg, an expert on the mathematical texts of ancient Mesopotamia, revisits old questions concerning the transmission of mathematical ideas and methods between the various ancient cultures situated in the Middle East and around the Mediterranean. Although the overall structure of the two books is different, the comparative approach is rather similar. The basic strategy is to examine some Egyptian or Greek mathematical text which has been translated and interpreted mathematically by methods developed or adopted by Friberg, and then to follow this by some selection of Mesopotamian texts that are similar in various ways. There has been much development in our understanding of the mathematical texts of the ancient Mesopotamian cultures in recent years and a comparative study of these texts with those of other ancient cultures is most welcome.

Except for some fairly brief remarks, however, there is little to guide the non-specialist reader through the argument, and there is almost no discussion of the social or intellectual context in which these texts were produced. Indeed, some chapters simply consist in translations of the texts followed by Friberg's mathematical interpretation, seemingly implying that mathematics speaks for itself. This is a dubious assumption under the best of circumstances; but in the

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case of mathematical cultures so far removed from our own, it is particularly precarious. Furthermore, because Friberg's stated aim is to find similarities between the texts he examines, he often overlooks key differences or transforms the mathematical presentation in the text into his own idiom, which serves to highlight the mathematical, or structural, similarity, but often at the expense of ignoring the historical, or practical, differences.

Unexpected Links explores similarities in the structure and content of the mathematical papyri from Egypt and mathematical cuneiform tablets. While in certain specific cases, we may question the historical significance, or doubt the relevance, of particular similarities, on the whole this book does indeed demonstrate the usefulness of the comparative approach for generating new interpretations of these sources, especially the Egyptian papyri, of which we have so few.

According to Friberg, the opening chapter gathers together and examines the texts that formed his personal point of departure in comparing the mathematics of ancient Egypt and Mesopotamia, and is, as a subtitle suggests, somewhat 'fanciful' [2]. It treats a number of texts that discuss ascending and descending geometric series and their sums, which are mathematically, and sometimes thematically, related to the nursery rhyme:

> As I was going to St. Ives,
> I met a man with seven wives.
> Each wife had seven sacs,
> each sack had seven cats,
> each cat had seven kits.
> Kits, cats, sacks and wives,
> how many were going to St. Ives? [14]

This chapter presents a rather striking example of the phenomena of closely related problems cropping up in different mathematical cultures, which Høyrup [1989] has called 'sub-scientific mathematics'. Whereas Høyrup, however, generally believes that these sorts of problems circulated, and were transmitted, through oral traditions, Friberg, on the other hand, seems to believe that the transmission took place through a 'supposed chain of related texts' [23].

After a brief introductory chapter, the book is divided into three sections organized by texts written in very different periods of Egypt's history and in three languages. The first chapter treats Hieratic texts, of which there are two larger papyri, P. Rhind (P. BM 10057/8) and P. Moscow E 4676, and some fragments. Friberg shows that there are a fair number of similarities between these Egyptian papyri and certain Babylonian tablets, both in terms of the overall structure and in terms of the types of problems addressed. Using the comparison of a few key examples, he argues against the opinion that, in the early part of the second millennium BC, Egyptian mathematics was much inferior to Babylonian mathematics. Nevertheless, despite Friberg's high opinion of Hieratic Egyptian mathematics, this approach, because it largely ignores the social and intellectual contexts, still involves a supposed ability to rate the mathematics of one culture against that of another. If such a rating is to be carried out fruitfully, however, the scale upon which this rating is done must be made fully explicit.

The second chapter compares texts that were written much later, and in Demotic Egyptian, with Babylonian sources. The core argument of this section centers on a papyrus of the third century BC, P. Cairo J. E. 89127–30, 89137–43 (*verso*). Friberg convincingly argues that there is a marked similarity between the types of mathematics found in P. Cairo and those found in late Babylonian texts. He shows, for example, that many of the problems of P. Cairo can be fruitfully explained by the style of Babylonian mathematics that scholars have recently dubbed metrical algebra; and that the method of solving certain problems, such as calculating the area of a circle, is the same in the Demotic Egyptian and Babylonian mathematical texts. Although Friberg is not the first to have argued for the transmission of Babylonian mathematics into Demotic sources [Parker 1972, 5–6; Høyrup 2002, 405–406], he brings a wide array of evidence to bear on the issue. In this regard, Friberg claims that these texts show that in Egypt during the time of Euclid, or slightly thereafter, there were individuals familiar with solving problems using Babylonian metrical algebra [191]. The influence of this assumed familiarity forms the main topic of *Amazing Traces*.

The third chapter of *Unexpected Links* discusses Greek mathematical papyri of Egyptian provenance. Friberg's main findings are that this Greek material is essentially similar to the Demotic material and, hence, likewise shows evidence of influence from late Babylonian sources. These texts span a long period but many of them are contemporary with Greek astronomical papyri containing methods which have been shown to originate in Babylonian sources [Jones 1999]. This comparison with the astronomical sources, however, may be taken as a cautionary tale. In the case of astronomy, it is now clear that the theoretical tradition as represented by works such as the *Almagest* and the practical tradition as represented in the papyri co-existed for long periods of time, despite being based on a different set of theoretical assumptions, employing different mathematical methods, and being practiced by individuals from different cultural groups. So we should be wary of assuming that all the mathematical texts written in Hellenistic or Imperial Egypt were of interest to all who were practicing mathematics in that region.

Whereas the final chapter of *Unexpected Links* discusses Greek papyri that were written in what we may call the practical tradition, *Amazing Traces* investigates selections of texts from the more theoretical traditions that we generally think of as constituting the core of Greek mathematics. In fact, over half of the book is devoted to comparisons of Euclidean texts with Babylonian texts. This is followed by comparisons of Babylonian texts with other Greek authors, either directly or indirectly reported, such as Heron, Diophantus, Hippocrates, or Theodorus. Unlike *Unexpected Links*, which largely proceeds chronologically and is divided linguistically, *Amazing Traces* is organized into many small chapters treating specific mathematical topics, such as '*Elements* X and Babylonian Metrical Algebra,' 'Hippocrates' Lunes and Babylonian Figures with Curved Boundaries' or 'Theodorus of Cyrene's Irrationality Proof and Descending Infinite Chains of Birectangles'.

As a collection of Babylonian texts that are mathematically related to Greek texts, *Amazing Traces* will be a valuable resource for historians of Greek mathematics; but as a reading of the Greek texts themselves, this work is beset with a number of difficulties. The orientation of the scholarship is much more mathematical than historical and Friberg often allows similarities that can be extracted through mathematical analysis of the text to guide his views, with much less regard for the historical circumstances. A few examples may serve to make this point.

In order to compare the *demonstrations* in *Elements* 2 with the *calculations* in Babylonian tablets, Friberg is compelled to address the difference in presentation between these two types of text. He does this by imagining what would happen to the Greek lettered diagrams of *Elements* 2 'if the letters are removed and instead lengths and areas with their numerical values are explicitly indicated in the Babylonian style' [4–5]. This supposedly simple transformation, however, completely changes the underlying nature of *Elements* 2 from drawing diagrams and making arguments about them to laying the theoretical foundations for the transformations of certain equations, which although geometric in some sense are meant to represent numeric values.

Following this mode of interpretation, Friberg reads *Elem.* 2.5–6 as demonstrations that certain Babylonian style *rectangular-linear systems of equations* have certain solutions [12–13]. There is still, however, no evidence that Greek geometers working in the Euclidean tradition were concerned with solving such equations. There is, on the other hand, considerable evidence that they were interested in using the geometric theorems provided by *Elem.* 2.5–6 to solve problems that arose in their geometrical investigations, that is, in the course of drawing diagrams and making arguments about them. Saito [1985] has argued for a purely geometric reading of *Elem.* 2.5–6 on the basis of the role of these theorems in Greek conic theory. A similar argument for the fundamentally geometric nature of these theorems could be based on Apollonius' *Cutting off a Ratio*, a text which shows at great length how to draw a line through a given point, falling upon two given lines and cutting from them a given ratio, and which makes extensive use of *Elem.* 2.5–6.

The fact that, as *Unexpected Links* makes clear, there were individuals in Egypt roughly contemporaneous with Euclid and Apollonius who were using Babylonian style metrical algebra to solve equations and make computations only serves to highlight the differences between these two traditions. It is in exploring such differences that it would be useful to consider the cultural contexts of these different mathematical traditions and social positions of the practitioners.

By focusing on the similarities between Greek geometry and Babylonian sources, Friberg often interprets Greek mathematical methods as being essentially similar to our own or to those of the

Babylonians and offers readings that are fairly far from a straightforward geometrical reading of the text. For example, he reads a number of theorems of Euclid's *Data* as providing justifications for 'the steps of an algorithmic computation' whereas there is no indication in the *Data* that computations are at issue [232]. In fact, in authors such as Heron and Ptolemy, *Data* style arguments are certainly used to give generalized expressions of algorisms; but it remains to be shown that this practice goes back to Euclid and certainly there are early authors, such as Apollonius and Archimedes, who use the theorems of the *Data* in purely geometric ways.

Because he often does not provide any discussion of the texts beyond a mathematical analysis, it is sometimes not clear what link Friberg sees between the Greek and Babylonian sources. Thus, chapter 6, '*Elements* IV and Old Babylonian Figures within Figures,' gives a brief discussion of the construction of a regular pentagon from *Elem.* 4 and then a list of problems that involve the calculation of the properties of regular figures in Babylonian sources. Since the Euclidian text has no interest in calculation, however, and the Babylonian texts have no interest in construction, the only connection is the appearance of regular figures.

Despite these reservations about Friberg's approach, historians of mathematics will be thankful that he has brought together such a large number of sources and thus laid the groundwork for other comparisons of these different traditions of ancient mathematics.

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