From Summetria to Symmetry: The Making of a Revolutionary Scientific Concept by Giora Hon and Bernard R. Goldstein

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Reviewed by
Joseph W. Dauben
City University of New York
jdauben@att.net

The basic idea of this fascinating book is that while symmetry has often been regarded as an innate concept of the human mind, there is no historical evidence to support this; and that in fact, the understanding of symmetry is basically a product of the 18th century. As the authors argue, there are two major aspects to this matter, one aesthetic, the other mathematical, both converging on the figures of Adrien-Marie Legendre, who was the first to formulate an exact mathematical definition of symmetry in terms of what he called 'incongruent counterparts', and Gaspard Monge, who was the first to use the term 'symmetry' in a textbook on statics written for students in the French naval academy (wherein symmetry was applied to the problem of determining the center of gravity of ships). In their consideration of the aesthetic aspects of the history of symmetry, the authors consider such thinkers as Plato and Archimedes, Galen, Vitruvius, Alberti, Dürer, Perrault, Montesquieu, and Diderot; whereas the mathematical side of the story includes the works of (again) Plato and Aristotle, Euclid, Archimedes, Boethius, Oresme, Kepler, Galileo, Barrow, and Newton, among others. Noteworthy is the authors' attention to such matters as the subject of harmony and its relations to symmetry in studies of the impact of Vitruvius on Copernicus and the architectural conception of a planetary system, Galileo and the significance of harmony in music, Kepler and Descartes on the structure of snowflakes, and the extent to which both Kepler and Leibniz regarded harmony as a fundamental concept in astronomy and metaphysics. The authors also consider the appearance of symmetry in natural history, specifically in the contexts of botany, crystallography, and zoology.

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Undoubtedly the one aspect of this book that will cause the greatest concern among historians of mathematics is the extent to which (or whether) the concept of symmetry can be considered as truly 'revolutionary'; and if so, in what sense we are to understand the concept of 'revolution'. This problem has been treated extensively since the work of Thomas Kuhn, whose analysis of *The Structure of Scientific Revolutions* seemed to suggest that revolutions could not occur in mathematics due to its cumulative nature.[1] After Goldstein and Hon consider the 'revolutions' that did not happen in the works of Euler and Kant with respect to symmetry, they devote an entire chapter to Legendre's 'revolutionary definition of symmetry as a scientific concept', whereby he regards it as a relation, not a property, by drawing on Robert Simson's critical commentary on Euclid. Here the problems of symmetrical polyhedra and mirror images in optics played their parts in generating Legendre's thinking about symmetry. Early responses to Legendre's definition of symmetrical solids by Lacroix, Garnier, Hirsch, and Cauchy bring the book to an end, with a final chapter dealing with applications of symmetry in mathematics and physics in the period 1788–1815, where the book concludes with considerations of bilateral symmetry and its significance as an abstract concept in terms of events (probability) and functions (algebra).

The authors maintain [49] that Legendre's definition deserves to be regarded as 'revolutionary' because the 'pace of usages of symmetry accelerates: new applications of symmetry appear in a variety of scientific domains'. They argue that with his 'explicit definition of equality by symmetry which he embedded in the proof structure of his *Éléments de géométrie* (1794)', subsequent systematic application of the concept of symmetry in diverse areas of science 'took a dramatic turn'. Because there was no evolution of concepts from the past that led to Legendre's novel concept, they consider Legendre's definition as a break with past tradition regarding symmetry, and therein lies its revolutionary character in mathematical terms. Understanding symmetry as a transformation which leaves something invariant, the authors again stress the meaning of symmetry in the sense of relation rather than property.

[1] See the extensive consideration of this matter in the collection of essays edited by Donald Gillies [1992].

This is a book full of technical detail, but with plenty of interesting information to engage readers well beyond the circles of mathematicians and historians sure to be interested in this account of symmetry.

BIBLIOGRAPHY

Gillies, D. 1992. ed. *Revolutions in Mathematics*. Oxford. Repr. 1995.