Omar al-Khayyam, Algebra wa al-Muqabala*: An Essay by the Uniquely Wise ᶜAbel Fath Omar bin al-Khayyam on Algebra and Equations* translated by Roshdi Khalil

Great Books of Islamic Civilization. Reading, UK: Garnet Publishing, 2008. Pp. xiv + 59. ISBN 978–1–85964–181–1. Paper $29.95

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The volume under review consists of an English translation of Omar Khayyam's 12th-century classic, *Algebra*, which is devoted to the enumeration of all types of cubic equations and the solution of such types as have a positive root. Despite what one might think from Omar's title, 'Algebra', his methods depend heavily on three classic geometric works: the *Elements* and the *Data* of Euclid (the latter a treatise on given magnitudes) and the *Conics* of Apollonius. (The latter two, especially, are not easy going for even mathematically trained readers, ancient or modern.) Accordingly, Omar's solutions to cubic equations are expressed as line segments determined by the intersection of conic sections. Although Omar explicitly states that he tried to find numeric solutions for such equations (like the ones he knew for the roots of quadratic equations), he admits frankly that he was unable to do so and expresses the hope that a later mathematician will succeed where he has failed. (Jerome Cardan realized this hope with the publication of his *Ars Magna* in 1545.)

Despite the importance of its contents, Omar's *Algebra* was not one of the many Arabic works that contributed so importantly to the European Renaissance. Indeed, it was only in 1742, with Gerard Meerman's *Specimen calculi fluxionalis*, that the attention of Western scholars was drawn to a copy of Omar's treatise in the Warner collection in Leiden. The eminent historian of mathematics F. Woepcke first published the Arabic text with a French translation of the *Algebra* in 1851; but it was only in 1931 that Dr. Daoud S. Kasir published an English translation, one based on an Arabic manuscript of the work in the possession of Professor D. E. Smith of Columbia

ISSN 1549–4497 (online) ISSN 1549–4470 (print) ISSN 1549–4489 (CD-ROM)
Aestimatio 7 (2010) 54–59

University. Kasir made considerable use of previous scholarly studies relevant to the topic, especially of Woepcke's French translation and the valuable mathematical and historical notes that Woepcke included in his work. Kasir does not present an Arabic text but remarks [1931, 9] that the text of the manuscript which he used is 'substantially identical' to that of MS 14 in the Warner collection in Leiden. In 1950, H. J. J. Winter and W. ᶜArafat published another English translation of Omar's work and a Russian translation was published in Moscow in 1961.

In 1981, there appeared an edition of the text based on all known manuscripts of Khayyam's work with a French translation by A. Djebbar and R. Rashed, which was republished in *Al-Khayyam mathématicien* in 1999. More recently, an English version of this has appeared [see Rashed and Vahabzadeh 2003].[1]

We now have, therefore, four English translations of Omar Khayyam's *Algebra*. The work under review, the one of these four most recently published, is difficult to relate to previous publications since Khalil says only that he translated a copy of the book that 'is in Aleppo'. Djebbar and Rashed [1999] make no reference to a copy in Aleppo; so one assumes that Khalil got a microfilm of a manuscript copy of the book from the archives of the Institute for the History of Arabic Science in Aleppo.

I shall now compare a few passages of the version under review with those in Kasir's book. (I have used Djebbar and Rashed's Arabic text as a check on both.) First, from the beginning of the work:

Khalil

> One of the educational notions needed in the branch of philosophy known as mathematics is the art of algebra and equations, invented to determine unknown numbers and areas. [1]

Djebbar and Rashed[2]

> One of the mathematical notions that one needs in the part of knowledge known as mathematics is the art of algebra and *al-muqābala*, intended to determine numerical and geometrical unknowns. [1999, 11]

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[1] I have not seen the editions of 1999 or 2003, and have relied on the edition of 1981.

[2] In quoting Djebbar and Rashed [1999], I have translated their French.

Kasir

> One of the branches of knowledge needed in that division of philosophy known as mathematics is the science of completion and reduction, which aims at the determination of numerical and geometrical unknowns. [1931, 43]

Khalil has taken the modern usage of *ta^{c}limiyya*, namely, 'educational'; but the sense of that word in medieval mathematical texts was, as Kasir renders it, 'mathematical'. On the other hand, Khalil's translation of the last part reflects the Arabic text more closely, since the Arabic text plainly says'unknowns relating to areas', though Omar probably intended to include other types of unknown geometrical magnitudes such as lines and volumes.

From the solution of the first species of trinomial cubic equations ('cube plus some sides are equal to a number'), I have underlined some of the main differences between Kasir's and Khalil's translations [1931, 77–78, and 18, respectively].

Kasir begins, 'Let the line AB be the side of a square equal to the given number of roots'. Khalil begins, 'We set AB to be the side of a square <u>whose length</u> equals the given number of the roots'. Kasir explains in a footnote that it is the area of the square on AB that is equal to the given number of the roots, whereas Khalil's addition of the words 'whose length' misleads the reader into thinking that it is the length of AB, not its square, that is equal to the number of roots. (Khalil has also dropped the 'a' from the diagram.)

Kasir continues,

> Construct a solid whose base is equal to the square on AB, equal in volume to the given number. The construction has been shown previously. Let BC be the height of the solid.

Khalil's version renders this as

> We construct a <u>parallelepiped</u> with a square base whose side is ab, and its height is bc, which we assume is equal to the given number. The construction is similar to what we have done before. We make bc perpendicular to ab.

Kasir brings in a reference to 'volume' and Khalil brings in one to 'parallelepiped', both of which are doubtless helpful to the modern reader, though each is an addition to the Arabic text which simply says that the solid is to be equal to the given number.

In their translations of Khayyam's solution of his 'first type' of cubic equation, Kasir and Khalil more or less agree on their translations of Omar's explanation of the phrase 'solid number', although Khalil's decision to call it 'numerical parallelepiped' loses the clear reference of Khayyam's terminology to that of book 6 of Euclid's *Elements*.

Kasir then translates Khayyam's construction of a circle and a parabola by

> Produce AB to Z and construct a parabola whose vertex is the point B, axis BZ, and <u>parameter</u> AB. Describe on BC a semicircle. It necessarily intersects the <u>conic</u>. Let the point of intersection be D.

Khalil renders the same passage as

> We extend ab to z, then construct the parabola mbd, with vertex b, axis bz, and its perpendicular side ab, so the parabola mbd is known, as we have shown previously, and it is tangent to the line bc. We construct a semicircle on bc which must intersect the (conic) section, say, at d.

Khalil's translation correctly reflects the medieval terminology 'perpendicular side' for the modern term 'parameter' (though the modern reader might appreciate an explanation), as well as Khayyam's reference to intersecting 'the section' (not the 'conic') and his decision not to name the parabola until he has constructed it.

Khayyam concludes his construction by dropping a perpendicular DZ from the point D onto the axis of the parabola at Z. Kasir refers to DZ as an 'ordinate', whereas Khalil's literal translation of the Arabic as 'one of the lines of order' may well leave some of his readers puzzled.

A number of nuances in the Arabic text are lost in this translation. For example, in the introductory part of his work [2], Khalil translates an admittedly difficult passage as:

> By quantities we mean continuous quantities, and they are of four types: line, surface, solid and time.... Some (researchers) consider place to be a continuous quantity of the same type as surface. This is not the case, as one can verify. The truth is: space is a surface with conditions, whose verification is not part of our goal in this book.

One may compare this with the rendering of the same passage in Djebbar and Rashed:

> By magnitudes I mean continuous quantity, and they are four: line, surface, body and time.. . . Some people think that place is a species subdividing surface under the genus of the continuous, but exact acquaintance overthrows this opinion. We will thus correct: Place is a surface in a certain state, whose exact knowledge does not stem from the subject occupying us here. [1999, 2]

The word 'magnitudes' is the standard translation of the Arabic plural 'maqādīr', and using the same term, 'quantities', for conceptually different words blurs an important distinction between the broader term 'quantity' (which includes the discrete quantities, numbers) and 'magnitude' (which is limited to continuous quantity). Then, near the end of this passage, Khalil translates

> Euclid proved certain equations to find the required rational measurable quantities in chapter five of his book (the *Elements*). . . [3]

The Arabic of this passage is, admittedly, somewhat loose; but one acquainted with the history of ancient mathematics will recognize immediately that Omar is simply referring to the fact that Euclid proved certain propositions relating to proportions of magnitudes in his fifth book. It has nothing to do with equations or rational measurable quantities.

A welcome feature of this edition is its inclusion [44–57] of a short treatise by Omar on solving a problem of dividing the arc of a quadrant of a circle, AB, with center H and radius HB, into two parts at a point Z so that when a perpendicular is dropped from Z onto the radius HB the result is that $BH{:}ZM :: HM{:}MB$. That this short treatise, highly relevant to Khayyam's work on cubics, is not in Kasir's edition, which is widely available in college and university libraries, is unfortunate; so its inclusion in the book under review was a good decision. Unfortunately, there are places here, too, where the translation is either loose or inaccurate. For example, Khalil writes of dividing the arc into 'two halves' (though the text clearly says 'two parts') and he refers to the circle's 'diagonal' BD, rather than the text's 'diameter'BD. Later in the demonstration Khalil refers to 'reflecting figure eight of the second article of the book of sections'. An accurate translation (such as that of Djebbar and Rashed) would be 'the converse of theorem eight of the second book of the Conics <of Apollonius>'. Another unfortunate mistake in translation is '. . . this triangle cannot be an equilateral triangle' [47], where the text reads 'this triangle cannot be isosceles'.

On the whole, however, the translation is competent and the book serves the useful purpose of making available to English readers the algebraic work of one of the great figures in the history of mathematics in a short and inexpensive version. If certain nuances are lost in the translation, it is still the case that one reading the book will understand that Omar solved a difficult problem and will come away with a good sense of how, in terms of the mathematics of his own time, he did it.

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