*Gaṇitasārakaumudī: The Moonlight of the Essence of Mathematics by Ṭhakkura Pherū. Edited with Introduction, Translation and Mathematics Commentary* by SaKHYa

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This book is the very happy result of the collaborative work of four excellent historians of Indian mathematics and astronomy, completed during Sreeramula Rajeswara Sarma's six-month sojourn in Kyoto. In his preface, Sarma describes his three Japanese colleagues and their team as 'the largest group working on Sanskrit texts on astronomy and mathematics today'. Together, they chose a common name made of the initials of their surnames ('Sa' for S. R. Sarma, 'K' for Takanori Kusuba, 'H' for Takao Hayashi, and 'Ya' for Michio Yano), SaKHYa, which is also a Sanskrit word for friendship or fellowship.

The author of the *Gaṇitasārakaumudī* (*GSK*), Ṭhakkura Pherū, was born in Kannāṇapura (modern Kaliyana, in the Bhiwani district of the Haryana state) in the second half of the 13th century AD. Belonging to a Jain family, he had a wide-ranging education, reading Jain but also Sanskrit and Prakrit texts on astronomy, astrology, mathematics, and architecture. Since his family was traditionally associated with the trade of luxury goods, banking, and money exchange, he became acquainted with these subjects and found an appointment at the treasury of the Khaljī Sultans of Delhi. This must have happened sometime before 1315, since Pherū's *Ratnaparīkṣā*, composed in 1315, states that he had already 'seen with his own eyes the vast ocean-like collection of gems in ᶜAlā' al-Dīn's treasury'. Pherū's name 'Ṭhakkura', which was already his father's (but not his grandfather's) name, could have been a title enjoyed by the Jains associated with the court of the Sultans in Delhi. As a matter of fact, contacts existed between Jains and Muslims on the west coast of India even before the establishment of the Delhi Sultanate in the 12th


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century; and the Sultans sought for conducting banking and minting activities the cooperation of the Jains who controlled minting in the Gujarat-Rajasthan-Delhi region. Pherū was one of those Jains who served as mediators between the Islamic and Sanskritic traditions of learning. This is the principal reason why he was such a versatile author, writing on astronomy (*Jyotiṣasāra*, 1315), architecture (*Vāstusāra*, 1315), coins (*Dravyaparīkṣā*,[1] 1318) when he occupied a high position in the mint of Quṭb al-Dīn Mubārak Shāh (1316–1320), minerals (*Dhātūtpatti*[2]), and mathematics (*Gaṇitasārakaumudī* or *Gaṇitasāra*). This last work is not dated; but SaKHYa observe that the division proposed by Pherū of the silver *ṭaṁka* into 50 *drammas*[3] was no longer in use after ᶜAlā' al-Dīn Muḥammad (1296–1316) issued a silver *ṭaṁka* of 60 *drammas* (according to the *Dravyaparīkṣā*), a rate that was continued until Quṭb al-Dīn Mubārak. Therefore, the *Gaṇitasārakaumudī* could not have been composed after 1316.

In all his works, Pherū's aim was to provide professionals such as bankers, traders, accountants, and masons, with a practical and useful manual. This explains why his works were not composed in Sanskrit (titles excepted) but in a Middle Indic Apabhraṁṣa with many vernacular terms and phonetic variations of Persians terms—e.g., 'goṁmaṭa' from Persian 'gumbad' ('dome'), 'munāraya' from Persian 'mīnār' ('minaret').

The first three chapters of the *Gaṇitasārakaumudī* are well structured like the mathematical texts which he consulted, principally Śrīdhāra's *Pāṭīgaṇita* (eighth century AD) or *Triśatikā* and Mahāvīra's *Gaṇitasāsaṅgraha* (*GSS*: *ca* 850 AD). These texts are good examples of what the Indian mathematicians called arithmetic (*pāṭīgaṇita*), usually also involving mensuration. In his *Brāhmasphuṭasiddānta* (628 AD), Brahmagupta rather spoke of 'dust work' (*dhūlikarma*), alluding to the writing and operations with figures drawn in the sand spread on a plank or soil. Crude as it may appear, this method of calculating could involve algebra in the sense that problems which we describe today with the help of equations (and thus consider as belonging to algebra) were solved by algorithms manipulating figures

[1] *Examination of the Metal Content in the Coins.*

[2] *Origin of Minerals.*

[3] Cf. *GSK* 1.4a: 'dammas' in the text.

arranged into columns drawn in the sand (see the so-called 'Diophantine' equations below).

These first three chapters describe 25 basic operations, eight classes of reductions of fractions, and eight types of procedures. Pherū does not take into consideration the simplest and elementary but often begins from a higher level of difficulty. For instance, in *GSK* 1.16a, he does not recall the addition but, as the *Pāṭīgaṇita* and *Gaṇitasāsaṅgraha* do, deals directly with the addition of successive integers up to $n$ (the 'desired' (*icchā*) number) and the resultant sum (*saṁkalita*):

> Add unity to the requisite (*icchā*) and halve it. Multiply it by the requisite. This is the summation of the natural series (*saṁkaliya*).

Bizarrely, this formula

$$1 + 2 + \ldots + n = \left(\frac{n+1}{2}\right) \times n$$

is extended to

$$\frac{nx + x}{2x} \times n,$$

where $x$ is called *paṇh-akkhara* (translated by SaKHYa as '<the number of> the letters in question'). The Patan Manuscript (PM), an anonymous and undated manuscript providing in Sanskrit the solution of several problems stated in the *Gaṇitasārakaumudī*, refers to $x$ as *śabda* (word) [PM B8], as if, in order to calculate a number of words, one adds up not the words but theirs letters, i.e., signs.

But what leads one to consider the *Gaṇitasārakaumudī* as an original and even a difficult work is contained in its last two chapters. According to SaKHYa, the fourth and fifth chapters of the *Gaṇitasārakaumudī* are made of what Pherū learned from his own experience and from that of his contemporaries. They involve mechanical shortcuts in commercial arithmetic, mathematical riddles (examples of both below), as well as rules for converting calendars and constructing magic squares. Classical Indian solid geometry is also applied to such new shapes as square or circular towers with spiral stairways, minarets with fluted columns, piles of angular and circular pilasters (e.g., the famous Qutub Minar), and especially to domes (see discussion below) and arches, which were employed successfully for the first time in 1311 by ᶜAlā' al-Dīn Muḥammad. The practical aim here was to calculate the number of bricks needed to build walls having these shapes. According to SaKHYa [xix], Pherū, in his section entitled 'Computation of Bricks', proposed such calculations for nine types of walls as opposed to one type only in other works. Even productions of grains, sugar canes, melted butter, camel prices according to their age, salaries of sawyers according to the breadth and length of the wood pieces, and so on, are taken into consideration.

It is not exactly true that

> the fourth chapter of the *Gaṇitasārakaumudī*, entitled 'Four (Special) Topics', may be characterized as a supplement to the traditional *pāṭī* mathematics treated in the first three chapters [xxix]

for the following reasons. Pherū does not include in his first three chapters such traditional *pāṭī* mathematics as the procedures for solving 'Diophantine' equations by the *kuṭṭaka*.[4] This omission could most probably be explained by the fact that Pherū was not an expert in mathematics or in astronomy, while these problems occurred especially in the context of mathematical astronomy. Nevertheless, problems of this kind were often also proposed in disguise as recreational problems. This form of presentation is certainly due to their easiness of exposition, since their solution usually involves great difficulties. Fermat's last theorem—there are no integer solutions $x, y, z$ to the equation $x^n + y^n = z^n$, when $n > 2$—to which many great mathematicians devoted more than three centuries of hard work,[5] deals with the most famous of these 'Diophantine' equations. Pherū makes no exception when he proposes [*GSK* 4.46] to compute the number of flowers obtained after doubling and adding three, respectively, a certain (not specified) number of times, or seeks [*GSK* 4.47] to find the number of flowers that a devotee had before he entered each of the four doors of a temple, giving a flower to the doorkeeper (*jakkha* = Sanskrit *yakṣa*) each time that he crosses his door and the half of his bouquet to the image of the god each time that he enters the temple,

[4] See, by instance, Mahāvīra, *GSS* 115 ½ for the explanation of such procedures, and 116 ½ ff. for many 'Diophantine' problems.

[5] The theorem was enunciated in 1647 but proved only in 1995.

knowing that, at the end, he will have 20 flowers. These problems are easy and no definite procedure is given by Pherū or by the PM;[6] but *GSK*, 4.51, which follows a similar pattern, is a trifle more difficult and necessitates a procedure that is described in 4.50: to find the number ($x_0$) of varisolas (a kind of sweetmeat) that a mother-in-law has put on a plate, knowing that she has given the same quantity of them ($y$) to each of her five sons-in-law, but also that, after each son has taken his share from the plate, she has multiplied the remaining varisolas by the rank of the next son ($2x_1$ presented to the second, $3x_2$ to the third, and so on), until the last son takes his share ($y$) and empties the plate.

This problem yields the equations

$$\begin{aligned} x_1 &= x_0 - y, \\ x_2 &= 2x_1 - y, \\ x_3 &= 3x_2 - y, \\ x_4 &= 4x_3 - y, \text{and} \\ x_5 &= 5x_4 - y = 0. \end{aligned}$$

Replacing each $x_i\,(i = 1, 2, 3, 4)$ in the following equation by its expression in the previous one, one finds a 'simple', although 'Diophantine', equation linking $x_0$ to $y$:

$$5 \times 4 \times 3 \times 2x_0 = (5 \times 4 \times 3 \times 2 + 5 \times 4 \times 3 + 5 \times 4 + 5 + 1) \times y,$$

of which $y = 120$, $x_0 = 206$ is a solution. But this is not the least positive solution: $y = 60$, $x_0 = 103$ is another solution, as can be easily verified by following the wording of the problem. In their explanation, SaKHYa give a more general system $x_i = a_i x_{i-1} - y\,(i = 1 \ldots n$, with $x_n = 0)$ and, applying the algorithm described in *Gaṇitasārakaumudī* 4.50 to the example, as does the PM [see A11–12], finds $y = 120, x_0 = 206$.

The algorithm is as follows: the successive coefficients ($a_i$ in the general procedure) are written one below each other.

[6] See PM A7–8, where the same problems are proposed in Sanskrit.

| | |
|---|---|
| | The second coefficient ($a_2$) is to be multiplied by |
| $a_1 = 1$ | the following one ($a_2 \times a_3$) and the result added |
| $a_2 = 2$ | to it ($a_2 \times a_3 + a_3$). The result, multiplied by the |
| $a_3 = 3$ | following coefficient $((a_2 \times a_3 + a_3) \times a_4)$, is added |
| $a_4 = 4$ | to it $((a_2 \times a_3 + a_3) \times a_4 + a_4)$, and so on until one |
| $a_5 = 5$ | obtains $((a_2 \times a_3 + a_3) \times a_4 + a_4) \times a_5 + a_5$, which |
| $\ldots$ | increased by 1, gives the solution for $y$, while the solution for $x_0$ is simply the product of all the $a_i$. In the example, this yields 206 and 120. Remarkable is the fact that the process could be continued if one adds more $a_i$. |

The inability of this algorithm to give a solution free of common factors is partially explained by the fact that, for the sake of simplicity, it does not take into consideration the common factors of the coefficients $a_i$.

The same kind of procedure is also used, in *GSS* 116 ½ for instance, to solve a more general type of 'Diophantine' equation such as $ax + by = c$, which is equivalent to $ax \equiv c[b]$[7] and means that the remainder of $ax$ when divided by $b$ is $c$. In that case, the column is made of the successive remainders of the Euclidian algorithm (*kuṭṭaka* in Sanskrit texts) applied to $a$ and $b$, completed by a certain 'clever' number called *mati*, a number already introduced by Āryabhaṭa in *Āryabhaṭīya* 2.32–33 (499 AD). This algorithm is called *vallikā-kuṭṭīkāra*—'vallikā' is diminutive of 'vallī' ('creeper')—because the algorithm begins at the bottom of two adjacent columns of numbers and proceeds through the numbers as does a creeper.

This algorithm is also applied to the resolution of two (or more) 'modulo' equations, $x \equiv c_1[a_1]$, $x \equiv c_2[a_2], \ldots$, as in *GSS* 121 ½: $x \equiv 7[8]$ and $x \equiv 3[13]$, of which the positive solutions are $55 + 104 \times k$ ($k$ an integer). In that case, as in the simple case, the least positive solution for $x$ is never explicitly stated by a formula but obtained through a 'creeper' procedure. Related to this kind of problem is the 'think a number' problem in *GSK* 4.58 (also in PM A18, with two important mistakes this time). The questioner has to 'guess' a number ($x$) chosen by an interlocutor, knowing only the remainders

[7] Read: '$ax$ congruent to $c$, modulo $b$'.

$r_1 \equiv x[3]$, $r_2 \equiv x[5]$ and $r_3 \equiv x[7]$, or alternately, asking the interlocutor to compute $p = 70r_1 + 21r_2 + 15r_3 + 105$, after which the questioner reveals $x$ as though by magic. Pherū does not chose any of the two options: he simply states the property of $p$ which is to reveal $x$ (rather, the least positive solution of the three equations $x \equiv r_1[3]$, $x \equiv r_2[5]$, $x \equiv r_3[7]$) when reduced modulo 105. In their explanation, SaKHYa choose the last option, which, in our opinion, is not as 'magical' as the first one, when the numbers 70, 21 and 15 are not revealed to the participant. At least, this is the classical way of exposing that trick [see Beiler 1966, 31]. Now, as we have already noted, to give the expression $70r_1 + 21r_2 + 15r_3$ as the solution (modulo 105) of the equations is not *pāṭī* at all. Indeed, this very same problem, with its solution so expressed, occurred for the first time in the *Sunzi Suanjing* (fourth/fifth centuries AD) according to Martzloff [1988, 296]. This is the reason why the multiple 'modulo' equations problem is called 'the Chinese remainders problem' in modern handbooks of algebra [see, e.g., Bland 2002].

To come now to mensuration in 'the traditional *pāṭī* mathematics (allegedly) treated in the first three chapters', SaKHYa note that Pherū is sometimes very original. For instance, he expresses (of course not with formulas) the area ($S$) and the volume ($V$) of a sphere as:

$$S = \frac{C}{4} \times C \times \left(1 + \frac{1}{9}\right) \qquad [\textit{GSK}\ 3.65]$$

and

$$\begin{aligned} V &= \frac{d^3}{2} \times \left(1 + \frac{1}{9}\right) \\ &= d^3 \times \left(1 - \frac{1}{4}\right) \times \left(1 - \frac{1}{4}\right) \qquad [\textit{GSK}\ 3.65] \end{aligned}$$

The last is equivalent to Mahāvīra's

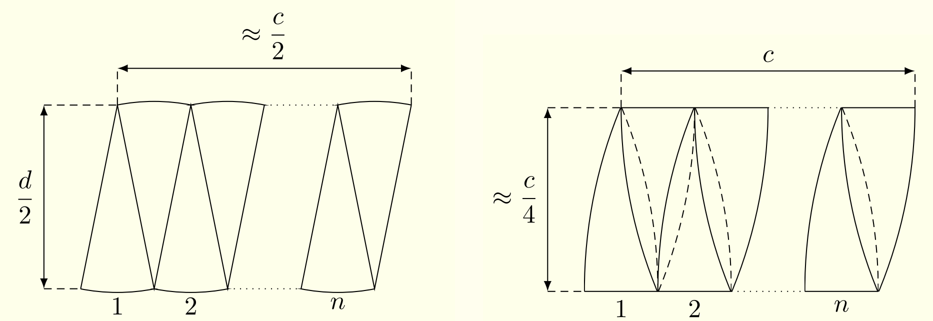
$$V = \left(\frac{d}{2}\right)^3 \times \frac{1}{2}.9 \qquad [\text{GSS}\ 8.28b],$$

but the correcting factor $1 + 1/9$ in the other two formulas is specific to Pherū. According to SaKHYa, the $S$ could have been obtained from the formula

$$S_D = \frac{C}{2} \times \frac{d}{2}$$

for the disk's area, which was well-known in India since Āyabhaṭa [*Āryabhaṭīya* 2.7] but already found by Archimedes [287–212]. As usual in Greek mathematics, Archimedes proved it by the method of exhaustion (a double *reductio ad absurdum*) in his *Measurement of a Circle*; but he could have 'guessed' what was to be proved by the following method: one cuts the disk into $2n$ identical triangles with their apexes at the center, where $n$ an integer $\geq 2$. Disposing $n$ triangles with apexes downward and $n$ triangles with apexes upwards so that they fit perfectly into each other, one gets a 'parallelogram' with waved bottom and top, its base being $C/2$ in length and its height $d/2$. When $n$ increases, the 'parallelogram' tends to a rectangle of area

$$\frac{C}{2} \times \frac{d}{2}.$$




For the sphere, SaKHYa suggest cutting 'narrow barleycorn figures' from the north pole to the south pole, which, when cut by the equator, yield $2n$ identical triangles. These $2n$ triangles could, as for the disk, be placed head-to-tail, thus forming a rectangle of length $C$ and height $C/4$ but their property of being spherical forces them to overlap (besides the fact that they cannot really be flattened). Consequently, the area $C/4 \times C$ must be somewhat reduced by a factor which Pherū evaluates—'by experiment?', ask SaKHYa—to $1 + 1/9$.

To corroborate SaKHYa's reconstruction of the Pherū's area formula for the sphere, let us remark that Pherū himself declares:

> This has been told according to experience. There is no doubt. It should be known thus. [*GSK* 3.76b]

He notes this not about the area of the sphere but about the area of a dome or hemisphere, just after ‘the circumference multiplied by half the diameter and increased by one-ninth <of itself> is <the volume of> the empty space in a dome’ [*GSK* 3.76a]. In SaKHYa’s translation, ‘<the volume of>’ should be replaced by ‘<the area of>’, since the text effectively describes

$$C \times \frac{d}{2} \times \left(1 + \frac{1}{9}\right),$$

which is an area. There seems to be a confusion between the area of a dome or hemisphere, [*GSK* 3.65] and the area of a disk.[8] In fact, *GSK* 3.74–76 is rather confused. So, 3.75a, which concerns computing the piling of a dome, asks: ‘The inner circumference of a wall is nineteen and its breadth six. What is its piling?’. With these data—SaKHYa conclude that 6 is not the dome’s width but its diameter, since *GSK* 3.58 gives 19 for the circumference of a circle of diameter 6—one cannot compute anything but the volume under the dome:

$$V\neg\dagger = \frac{1}{2} \times \frac{6^3}{2} \times \left(1 + \frac{1}{9}\right) = 60 \qquad \text{[see } \mathit{GSK}\text{ 3.65].}$$

The rest of the text gives the outer circumference (and, therefore, the width) of the dome: ‘The outer circumference is, O learned man, twenty-four. What will be the area?’ [*GSK* 3.75b], after which follows the text about <the area of> the dome quoted above. If one applies to the data the formula of *GSK* 3.76a, one gets

$$S = 19 \times 3 \times \left(1 + \frac{1}{9}\right) = 63\frac{1}{3}$$

(57 without the $1 + {}^{1}\!/_{9}$ correction). Note that the two computed values are close to the exact value of the wall’s width (computed as the volume of the outer hemisphere minus the volume of the inner one), i.e., $\approx 58.8$. Let us note also that a part of *GSK* 3.74 suggests computing the area of the intermediary hemisphere: ‘<When the circumference is measured> at the middle of the outside (?), it is the area.’ The intermediary circumference is

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[8] Note that $C \times {}^{d}\!/_{2} = 2\pi \times r^2$ is also the modern formula for the area of the hemisphere.

$$\frac{19+24}{2} = 21.5$$

in length, and the corresponding dome is

$$\frac{1}{2} \times \frac{21.5}{4} \times 21.5 \times \left(1 + \frac{1}{9}\right) \approx 64.2,$$

which is close to the expected value, once again.

To conclude, one cannot help but think that, in this very case, Pherū tried several formulas empirically in order to find the volume of the dome and, finally, the number of bricks, which, after all, is the purpose of this part of chapter 3, entitled 'Procedure for Piling'. Strangely enough, he got the number of bricks for only one of the nine types of wall described. Another quick method, based on the area instead of the volumes could have been used: multiply the area of the intermediary hemisphere by the width of the wall, i.e.,

$$\frac{24-19}{2\pi} \times 64 \times 2 \approx 51.$$

In the fifth chapter of *Gaṇitasārakaumudī*, which SaKHYa [xi] describe as 'presumably added as a supplement', since '[t]his part is not so well structured as the preceding parts' but still 'explicitly mentions Pherū as the author', the formula

$$V = d^3 \times \left(1 - \frac{1}{4}\right) \times \left(1 - \frac{1}{4}\right)$$

is repeated [*GSK* 5.25] and used to calculate the volume of a sphere having a diameter 6. The result is written '120' in the edition, but SaKHYa reconstruct it as 121||, that is $121\tfrac{2}{4}$—'|' is Pherū's shorthand for a quarter—and notes that 120 is exact according to the second volume formula of 3.65. The area also is given as

$$\frac{C}{4} \times C \times \left(1 + \frac{1}{9}\right) = 90 < | > +10 < \frac{1}{36} >= 100 < \mathrm{SS6} > .$$

Strangely, SaKHYa put 'SS6' into angular brackets $< \; >$, as if it were an addition to the translation but discuss it in their mathematical commentary as if it were present in the text. 'SS' being Pherū's shorthand notation for separating units from twentieths, '100SS6' means $100 + \tfrac{6}{20}$, which is not exactly $90\tfrac{1}{4} + 10\tfrac{1}{36} = 100\tfrac{5}{18}$. As SaKHYa remark, quoting Strauch, 100 (written 100|1) would be a better approximate value than $100 + \tfrac{6}{20}$, but the reason for this choice must be the very common use of 20 as a conversion factor in the *Gaṇitasārakaumudī* and in Pherū's time. The twentieth of any basic measure is usually called *visuva/visova(ga)* (Sanskrit *viṁśopa(ka)*), and is itself divided into twentieths, called *visuvaṁsaga/vissaṁsa* (Sanskrit *viṁśopa-aṁśaka-/viṁśa-aṁśa*). For instance, in *GSK* 1.3–4a, the monetary units are described as:

> *damma*,
> 1*visova* $= \frac{1}{20}$ *damma*,
> 1 *vissaṁsa* $= \frac{1}{20}$ *visova*,
> 1 *paḍ ivissaṁsa* $= \frac{1}{20}$ *vissaṁsa*,
> 1 *kāiṇi* $= \frac{1}{20}$ *paḍ ivissaṁsa* and
> 1 *paḍ kāiṇi* $= \frac{1}{20}$ *kāiṇi*

In spite of this visagesimal division, the numeration remains decimal and Pherū gives [*GSK* 4.6–10] very simple rules or mechanical short-cuts for converting an amount of *dammas* that is to be shared by 10, 20, or 100 persons, or simply to be divided by 1000, 10000, or 100000. In *GSK* 4.8, 209534 *dammas* are given to 100 persons, each receiving 2095 *dammas* $+ 3 \times 2$ *visovas* (for 3 tenths $= 3 \times 2$ twentieths) $+ 4 \times 2^2$ *vissaṁsa* (for 4 hundredths = 4 tenths of tenths $= 4 \times 2 \times 2$ twentieths of twentieths $= 4 \times 2^2$ four-hundredths). The other divisions by $10^n$ convert the last $n$ digits into *visovas*, *vissaṁsa*, and so on by multiplying them successively by 2, $2^2$, …, and the largest division factor is $100000 = 10^5$ for there are only five submultiples of the *damma*. Strangely, *GSK* 4.9–10 (division by 10 and 20) are introduced with the words, 'Now, on the regional method of accountancy', for which SaKHYa give no explanation.

Again, in regard to chapter 5, SaKHYa deduce [192] the value of the length unit *kaṁviya* (also *kaṁbiya* or *kaṁvī*) from the value given by Srinivasan to the weight unit *sera*: 600 g. < 1 sera < 850 g., by using the table of 'specific gravities' given in *GSK* 5.28 for different culinary substances. So, sesamum oil (named *tilla* by Pherū, Sanskrit *tila*) weighs 10 *maṇas/kaṁviya*$^3$. According to SaKHYa, sesamum oil has a specific gravity of 0.92 kg/dm$^3$, so that 240 kg < 10 *maṅas* = 400 *seras* < 340 kg $\Longleftrightarrow {}^{240}\!/_{0.92}$ dm$^3$ < *kaṁviya*$^3$ < ${}^{340}\!/_{0.92}$ dm$^3$, wherefrom 63 cm < *kaṁviya* < 72 cm (SaKHYa), by extracting the cubic root. This derivation is probably overconfident on the exact value of the sesamum oil's density. One could have checked it by using, for instance, one of the stones' 'specific gravities' listed in another table [*GSK* 3.67–68], perhaps more reliable than oil's.

Pherū's works were discovered in 1946 in a single manuscript and edited in 1961 by Agar Chand Nahata and Bhanwar Lal Nahata. The present work is based on this edition and on the edition of the Patan Manuscript (by T. Hayashi with the Bakhshālī Manuscript). After Sarma's preface, it contains an introduction (on Pherū's life and works, on the mathematics of the *Gaṇitasārakaumudī*) with a very useful mathematical glossary (English-Sanskrit/Prakrit) and a table comparing the fundamental operations of the *Gaṇitasārakaumudī* with 10 other mathematical works. The edition, revised according to the language, the verses, and the mathematical content (the original text being pushed away in notes), is followed by a literal translation and a very substantial mathematical commentary. Four appendices (concordance of *Gaṇitasārakaumudī* with other works, type problems, index to the numbers, glossary-index), a bibliography and two indices (mathematical terms, Sanskrit/Prakrit authors and titles) close this study. Its quality and its presentation will make of it an essential reading for every researcher not only in the history of Indian mathematics but also in Middle Indic languages, as well as in the monetary and economic history of the Delhi region in the early 14th century.

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