*A Passage to Infinity: Medieval Indian Mathematics from Kerala and Its Impact* by George Gheverghese Joseph

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This ambitious work undertakes to address in just over 200 pages a very extensive set of topics concerning the so-called Kerala school, which first became known to European historians in the mid-19th century [2–3]. Its members were a remarkably brilliant and innovative group of mathematicians and astronomers active in the mid-second millennium AD in southwest India. They were responsible for, among other things, important results on infinite series and infinitesimal methods that were later rediscovered by European mathematicians investigating the 'new analysis' or calculus.

The book's chief goals are the following: to investigate and describe the mathematical genesis, technical practices, and major discoveries of the Kerala school; to explore its social origins and context as well as its relation to traditional knowledge systems in the region; and to analyze historiographic problems concerning modern historical views of second-millennium Indian mathematics in general and the Kerala school in particular, including recent hypotheses about possible scientific transmissions from Kerala to early modern Europe. The formidable task of covering this extensive ground is shared by several researchers, mostly collaborators in the UK Arts and Humanities Research Board's Research Project on Medieval Kerala Mathematics, whom the author credits in the acknowledgements and in the individual chapters where their contributions appear.

The first chapter is a short introduction outlining historiographic issues in the history of mathematics and the book's objectives. The second chapter, 'Social Origins of the Kerala School', includes research by M. Vijaylakshmy and V. M. Mallayya in a historical survey of intellectual traditions in medieval Kerala and biographical summaries of some central figures. Chapter 3, 'Mathematical Origins of the Kerala School', is chiefly focused on recapitulating the work


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of the early sixth-century astronomer/mathematician Āryabhaṭa I, among whose followers in astronomy the Kerala scientists are usually counted. 'Highlights of Kerala Mathematics and Astronomy' in chapter 4 briefly outlines some of their seminal discoveries, while chapter 5, 'Indian Trigonometry from Ancient Beginnings to Nīlakaṇṭha', which is based on research by V. M. Mallayya, summarizes trigonometric findings by medieval Indian mathematicians before the Kerala school and culminates in an extensive discussion of trigonometry in the works of the Kerala scholar Nīlakaṇṭha. This theme is continued in the next and longest chapter, 'Squaring the Circle', based on the work of Dr. Mallayya and the late K. V. Sarma, which elegantly outlines what is widely considered the crown jewel of Kerala mathematics, namely, the derivation of the Mādhava-Leibniz infinite series for the circumference of a circle and associated methods for the computation of arc-length and $\pi$. Chapter 7, 'Reaching for the Stars', surveys Kerala school work on power series for sine and cosine functions. The next two chapters, 'Changing Perspectives on Indian Mathematics' and 'Exploring Transmissions: A Case Study of Kerala Mathematics' are largely devoted to exploring the possibility of transmission of Kerala mathematics to Europe before the 19th century. They incorporate work by D. Almeida, U. Baldini, and A. Bala. Finally, a brief conclusion extends the investigation to general historiographic questions concerning transmission and innovation in mathematics, and their dependence on cultural context.

The compression of so much material into such a limited space has understandably produced some elisions and ambiguities. The frequent use of transliterated Sanskrit is a well-chosen compromise between reproducing Sanskrit in *nāgarī* script and using only translated technical terms; but it would have been more helpful to use a consistent transliteration scheme with a full range of diacritical marks. For instance, on pages 94–95, the name 'Vaṭeśvara' is spelled sometimes with an underdot indicating the retroflex 'ṭ' and sometimes without, but never rendered precisely with both accents. The alphanumeric encoding named after the Sanskrit consonants 'ka', 'ṭa', 'pa', and 'ya' is identified [e.g., 36, 217] as 'Katapyadi' instead of the more standard and intuitive 'kaṭapayādi'. The word 'śāstra' ('science', 'treatise') is spelled 'shastra' when separate but 'sastra' when compounded in, e.g., 'jyotisastra' ('astral science') [201: more precisely, 'jyotiḥśāstra'].

More confusing than these minor typographical glitches are the frequent allusions and assertions carelessly expressed or insufficiently explained. Readers unfamiliar with Roman Catholic religious orders, for example, may not immediately understand that the passing reference to the French scholar Marin Mersenne as a 'minim [*sic*] monk' [164] means that Mersenne was a member of the Minim Friars. The above-mentioned Vaṭeśvara does not appear in the book's rather hit-or-miss index or in the list of 'Major Personalities and Texts in Indian Mathematics' on page 12, although a section of chapter 5 is devoted to Vaṭeśvara's trigonometric work, described [95] as 'one of the most comprehensive and innovative achievements of early Indian trigonometry'. The preeminent sixth-century scientist Āryabhaṭa I is briefly stated to have 'attended the University of Nalanda' [42], i.e., the renowned medieval center of Buddhist learning in the Bihar region. This is an oft-repeated but ill-supported legend based on Āryabhaṭa's description of 'knowledge honored in Kusumapura', referring probably to the medieval urban center that is now Patna, close to but not identical with the Buddhist institution of Nalanda. His chief work, the *Āryabhaṭīya*, is called 'the premier Indian text to be read and commented on for at least another thousand years' in the realm of Indian mathematics [54], which oddly ignores the immense popularity and canonical status of the 12th-century *Līlāvatī* of Bhāskara II. Likewise, it is by no means certain that 'at the time of Āryabhaṭa, mathematics was rarely treated outside its astronomical context' [62]: the lack of surviving texts from this period makes it impossible to pronounce conclusively on the nature of textual genres in the Sanskrit exact sciences. Moreover, the author surely does not intend to claim that Āryabhaṭa was the first Indian mathematician to solve the problem of computing decimal place-value square roots, but that is the impression he produces by the claim that 'ever since Aryabhata devised a method to calculate square roots, Indian mathematicians could approximate' a trigonometric quantity by a rational number [66]. It is similarly confusing to assert that Indian mathematicians after Āryabhaṭa 'calculated sine values for any angle in radians' [59], when the units of length in question were actually equivalent to arc-minutes rather than radians. Other puzzling and potentially misleading remarks of this nature can be found throughout the book; most seem to spring from a hasty or clumsy attempt to squeeze rather complicated historical and mathematical information into an expository framework too small for it.

These flaws are regrettable because they risk obscuring the many valuable contributions contained in the volume. The detailed explanations in modern mathematical notation of various significant results found by Kerala mathematicians, particularly in chapters 5, 6 and 7, are especially helpful. So are the surveys of current research that tie in the work of the volume's contributors with that of fellow scholars. (To their detriment, however, the bibliography and notes omit any mention of the published research of the late David Pingree.) The discussion in chapter 2 of the social context within which the Kerala scholars worked is also commendably detailed, although much of the exposition in both the chapter's text and the notes suffers from a lack of specific supporting citations—a brief footnote at the start of the chapter does invoke recent joint articles by Joseph and other contributors as its general basis. The reader intrigued by the interesting descriptions of, for example, the family-run *Gurukula* educational institutions in Kerala [33] finds no sources cited there to guide the quest for more information. Despite these limitations, this material covers important ground and is well worth reading.

The topic that ultimately inspired the book's genesis, as the author notes on page 1, is a question of cross-cultural transmission: namely, 'the conjecture of the transmission of Kerala mathematics to Europe, with a view to informing the wider history of mathematics' [3]. To investigate this issue, the author and other members of the above-mentioned Project on Medieval Kerala Mathematics examined correspondence, reports, and Indian manuscripts in European archives with known or possible connections to 16th- and 17th-century Jesuit missionaries in South India who were rightly deemed the most likely candidates to supply a conduit for translation and transmission of scientific texts [179–185]. The inspection of this under-studied and historically important corpus is a laudable achievement, especially in light of the neglect of much of this material (some of it hitherto not even catalogued) by institutions and scholars in the lands where it currently resides.

Since a historically validated narrative of early modern European mathematicians borrowing core concepts of calculus from predecessors in Kerala would have made headlines in scholarship on the history of mathematics and beyond (while doubtless inspiring a surge of interest in the Indian mathematical tradition which is both well deserved and long overdue), it is hard to help feeling disappointed that this hypothesis ultimately came to nothing. As Joseph candidly observes, the sifting of the various archives 'has yielded no direct evidence of the conjectured transmission' [186]. He quotes the summing-up by fellow researcher Ugo Baldini in greater detail:

> Thus, unless new evidence is found and some basically new circumstance is established, the only possible deduction seems to be that not only no information exists on a Jesuit mathematician having managed to study some advanced Indian text (not to say to transmit it, or its content, to Europe), but no serious clue appears of a scientific interchange not purely superficial and more than occasional. [191]

Joseph, following the lead of another contributor (Arun Bala), then raises the question [192–193] whether a different type of transmission might have taken place without leaving documentary evidence:

> '...the Indian mathematical discoveries may have reached Europe as a set of practical computing rules rather than a body of mathematical discoveries'...if there was transmission of knowledge of infinite series to Europe, it was done indirectly through practical uses, with a truncated version being passed on from local craftsmen to their foreign counterparts (such as navigators) and then being reconstructed in Europe by the mathematically knowledgeable without being aware of its provenance.

This is certainly a very vague and speculative conjecture, as the author acknowledges. He proposes it for consideration not entirely on its own (still undetermined) merits but as part of a larger historiographic claim, namely, that the assumption 'of independent European discovery of some of the Kerala mathematics...as a default solution by most historians is debatable' [193]. In other words, he suggests that most historians discount the possibility of Indian influence on the early modern invention of calculus more on the basis of Eurocentric bias than as part of a consistent historiographic outlook. Noting that the renowned historian of ancient science Otto Neugebauer accepted certain combinations of plausible circumstantial evidence in the absence of direct evidence for inferring scientific transmission from one culture to another [162], Joseph argues that requiring documentary evidence to support the conjecture of a transmission of calculus concepts from Kerala to Europe is somewhat capricious and unfair:

> O'Leary uses an admixture of the Neugebauer and the van der Waerden paradigm to claim the Greek origin of Indian astronomy and mathematics....In these circumstances priority, communication routes and methodological similarities appear to establish a socially acceptable case for transmission from West to East. Despite these elements being in place, the case for transmission of Kerala mathematics to Europe seems to require stronger evidence. [163]

This implied accusation relies on some exaggerated or distorted arguments as well as some valid criticisms. It is certainly true that there was a great deal of Eurocentric bias in much 19th- and 20th-century scholarship and speculation concerning cross-cultural transmission of science. Moreover, it is also true that Indian mathematics remains much more under-studied and much more incompletely treated in scholarship on the history of mathematics than other mathematical traditions. We cannot assume from these facts, however, that Eurocentric bias is still dictating modern historians' attitudes towards conjectures about scientific transmission involving India. It is not true, for example, that such speculations as those of O'Leary in 1948 (much less those of Sédillot in 1875 or Bentley in 1823, justly deplored on pages 157–158) would be widely regarded as 'a socially acceptable case for transmission from West to East' among historians of science today.

Furthermore, the 'Neugebauer paradigm' for weighing circumstantial evidence of transmission obviously cannot apply in exactly the same way to well-documented historical developments in mathematics and science as it does to poorly documented ones. It is one thing for Neugebauer to argue, for example, that Euclid's so-called 'geometrical algebra', which has left no clear record of independent discovery in extant Greek sources, was probably ultimately influenced by related ideas in earlier Babylonian mathematics. It is quite another to argue that infinitesimal calculus, whose various stages of development in the hands of European mathematicians are very well attested in surviving texts, was probably influenced by related ideas in earlier Kerala works, despite the complete absence (so far) of detectable traces of Kerala material in the abundant textual record of early modern European mathematics. Both these examples involve the hypothesis of a scientific transmission from 'East' to 'West': Mesopotamia to Greece in the first case and Kerala to Europe in the second. The crucial difference between them is not a matter of Eurocentric bias but rather that in the former case there is virtually no documentary evidence supporting the alternative hypothesis of a completely independent rediscovery by the 'Western' mathematicians, whereas in the latter case there is a great deal of such evidence.

That said, it must be acknowledged that Joseph makes a very good point about the need for this sort of direct discussion of historiographic assumptions: 'The methodology underlying the testing of such claims and assessing the relevant evidence remains relatively undeveloped' [199]. Different historians will inevitably sometimes come to different conclusions about what qualifies as historically probable or historiographically sound. What matters more than unanimity is clarity about the reasoning and criteria employed to reach the different conclusions. In foregrounding this issue within the comparative history of mathematics, as well as in the contributions described above, *A Passage to Infinity* has performed a valuable service.

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