
Jonathan Barnes et al., *Eleatica 2008. Zenone e l'infinito* edited by Livio Rossetti and Massimo Pulpito

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Reviewed by
Walter Leszl
University of Florence
leszlwalter@gmail.com

The book arises from the Eleatica lectures of 2008 delivered in Italian by Professor Barnes at the Fondazione Alario per Elea-Velia in Ascea (province of Salerno) on the topic 'Zeno and Infinity'. It includes the Italian text of the lectures (the English original was translated by M. Pulpito), together with the responses submitted by eight scholars, the rejoinder (in English) to each respondent by Barnes, and an introduction by the editors. The introduction is preceded by a shorter (anonymous) overview in English, which in part overlaps the introduction. The latter includes a short survey of modern and contemporary studies on Zeno (starting with Renouvier in about 1860), which would be more useful if less selective (for instance, the contributions by Gregory Vlastos are ignored) and if, in mentioning controversies, gave more information as to what the controversies were about. At the end of the introduction we are told who is Jonathan Barnes, including the information that (after the lectures) he obtained the eagerly desired (*ambita*) distinction of being elected an honorary citizen of Elea-Velia, thus becoming a fellow-citizen of the Zeno who was the object of his lessons. This is apparently taken very seriously, for Livio Rossetti also mentions in his curriculum the same distinction as one of the most important facts. Evidently, there is the conviction at work that the committee (presumably of citizens of Ascea, a little town close to the ruins) which elects these honorary citizens are worthy direct descendants of the citizens of ancient Elea. Concerning the ancient town, Barnes himself remarks that such a little town gave a greater contribution to philosophy than the big metropolis of Rome.

In his lectures, Barnes concentrates on Zeno's fragment B1 [Diels and Kranz 1951, ch. 29] of which I reproduce the translation given in the overview:

But if they [things generally] exist, it is necessary that each has a certain size and thickness, and that the one bit of it is distant from the other. And the same remark goes for the projecting bit, for it too will have a size, and a bit of it will project. Now it is all the same to say that once and to say it forever; for no bit of it is last in such a way that there will not be one bit in front of another. Thus if several things exist, it is necessary that they be both small and large—so small as to have no size, so large as to be infinite.

In fact, Barnes declaredly concentrates not on the whole apparently contradictory conclusion but on its second part, that involving infinite greatness. (This explains the title of the lectures, which would not be equally applicable if also the first part were considered.) As he tells us, assuming Proclus' testimony that Zeno produced 40 arguments, he is concerned with one 80th of the philosopher's production. Thus, though he occasionally makes reference to some other Zenonian arguments, he is not concerned with offering 'a full and rounded account of Zeno' [186]. Further, he declares that he

decided that the lectures would tackle some philosophical questions and not touch—save incidentally—on the philological and the historical [185],

without intending to suggest by this that these other two sorts of question are not important. What he maintains is that, at least in the case of Zeno, a philosophical analysis can be conducted without having to presuppose that his paradoxes 'can only be grasped after some historico-philological effort' [186].

In his exposition, in the first chapter (entitled 'Zenone paradossologo'), Barnes does say something about Zeno in general. He dismisses as mere fiction not only the story of Zeno's visit to Athens together with Parmenides but also the presentation of his position in the first part of Plato's *Parmenides*, where it is suggested that he elaborated his arguments against 'the many things' with the intent of defending Parmenides' monism. Against this the suggestion is given that Zeno was only interested in elaborating paradoxes in the modern sense of the word, i.e., arguments that are seemingly flawless in logic and yet reach absurd conclusions. Thus, Barnes is induced to define the Eleatic thinker as 'a philosopher without philosophy' inasmuch as the conclusions reached by means of paradoxes give rise to no point of doctrine.

In chapter 2, Barnes proceeds (I rely on the overview) to point out that the notion of infinity is neither difficult nor technical, and certainly not incoherent or contradictory; and, therefore, that it is not in infinity *per se* that we should expect to locate the primary source of paradoxality. Not

difficult, because the fact that it is hard to picture an infinite magnitude does not mean that the concept itself is hard to grasp. Not technical, because, although there are some technical notions in mathematics involving the infinite, these technicalities do not apply to the ordinary concept of the infinite, which is the only one that is at play in the paradoxes of Zeno (and which alone is germane to their solution). Not intrinsically paradoxical, because the undeniable existence of the paradoxes of infinity does not mean that to think of infinity necessarily involves us in contradictions. In the main, to use that concept is just to recognize that we have to do with sequences (e.g., the sequence of cardinal numbers) that can be prolonged without ever having to stop before a limit.

In chapter 3, Barnes examines the Zenonian argument quoted above, which, in the form it has come down to us, clearly leaves out some steps. He affirms that he agrees with the classical reconstruction of the argument, according to which every body, inasmuch as it has a certain size, is potentially divisible into an infinite number of bits, each of these bits in turn having a certain size. As the initial size of the body at issue is the sum of the sizes of its bits, and these bits are infinite in number, then the whole body will have infinite size. Barnes calls this argument the Dichotomy, since it is assumed that the bits in question are a sequence of halves starting with the first half into which the original magnitude is divided.

In the sequel Barnes points out that Zeno's argument requires some additional premises, concerning first of all whether the bits of a magnitude are such as to be both exhaustive and exclusive. Leaving out some details, however interesting, attention should be given to the crucial assumption made by Zeno, namely, that the sum of an infinite number of quantities is infinite. This assumption appeared plausible to various Greek philosophers after Zeno (as an example Barnes quotes Epicurus, *Epistula ad Herodotum* §57). However, this appearance of plausibility is not sufficient. What emerges in this analysis is that the Dichotomy is exposed to a well-known mathematical objection. In the case of the so-called 'convergent series', i.e., series whose elements converge to a finite number, it is not true that the sum of an infinite number of magnitudes is equal to an infinite magnitude. This is precisely the case of the sequence that is involved in Zeno's argument: the successive addition of the elements of the sequence does yield a convergent series.

Barnes concedes that the mathematics with which Zeno is likely to have been acquainted was not so advanced. He also points out that there is another way of understanding the Dichotomy paradox. (This interpretation was advanced by W. E. Abraham [1972], though this is not pointed out either by Barnes or by Rossetti and Pulpito in their introduction. Notice that the text of fr. 1 does not specify the way in which the partition is made.) This is suggested by the version of the paradox which is given by Porphyry: what is contemplated is not a succession of halves of ever decreasing magnitudes but rather that *all* halves are divided into their sub-halves, creating a top-down hierarchy of increasingly dense partitions. In this case, at all levels the series is divergent, not convergent. And yet, Barnes argues, these partitions will continue to produce a number of bits, whose sum is equal to a finite number, for they will always be identical to the size of the original magnitude, whatever the number of bits for any one partition may be.

With the exclusion of this alternative, we come back to the objection to the validity of the argument from a mathematical point of view. It is remarked that the argument is not properly refuted by adding the elements of a convergent series, for this operation cannot be completed: we only have an approximation to a finite number. But this consideration does not, of course, show that Zeno is right. Any reply, including his, that is given to the question 'What is the sum of a series such as $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \dots$ equal to?' cannot be correct. If we change the example and ask what the sum is of an infinite sequence of units, it is the same as asking what the sum is of many units; and this makes as little sense as asking how long is a piece of rope. If an arithmetician does give a reply to the first question, it is because he makes certain stipulations which are convenient for doing mathematics.

In what follows, I will not make any mention of the responses, with a partial exception for two of them, for (as it often happens in these meetings) they are more statements of their authors' interpretation of Zeno's arguments than an attempt to come to grips with Barnes' suggestions. On the latter, I make the following comments. The first is that admitting that Zeno was elaborating paradoxes does not oblige us to regard Plato's testimony as wholly ungrounded. Zeno's arguments are paradoxical (as Barnes seems to concede) also in the sense that they go against common assumptions about the existence of a plurality of bodily entities and their movement and this can be

seen as a way of supporting Eleatic monism.¹ From Plato's testimony itself one gets the impression that Zeno's intentions in his writing were not explicit. Thus, one can only speculate about them and it is quite possible that he had more than one intention, for it is rare that a thinker be wholly single-minded. Zeno's taste for paradoxes certainly led him beyond what would have been needed for a wholly serious defense of an Eleatic point of view. One cannot suppose that he was really convinced, for instance, that Achilles would never overcome the tortoise. He belongs to a period (as rightly stressed by Rossetti in his comment) in which a number of thinkers showed themselves more interested in making intellectual experiments than in putting forward views to be accepted as true.² One can detect some affinities with Gorgias, who, in addition to claiming paradoxically that Helen was innocent because she could not resist the seduction exercised on her by Paris (by means of enticing words and so on), offered a demonstration that nothing exists which, in his case, is a reversal of Eleaticism rather than its defense but still has a relation to it.

The second comment is that Barnes' suggestion that what is at issue for Zeno is an ordinary concept of the infinite that presents no difficulties seems to make it too a-problematic. This can appear to be so because attention is given in an exclusive way to the process, exemplified by the intellectual division of continuous magnitudes (and by the opposite operation of their summation), of traversing a sequence of magnitudes which has no end.³ This case was paradigmatic for Aristotle in propounding his conception of the infinite as only potential. But Aristotle himself, when dealing with time (which for him is eternal, as the world is eternal), had to admit, rather paradoxically, that events which recur forever, like a day, are both potential and actual [cf. *Phys.* 3.6.206b12 ff.]. A further and proper paradox is stated in Kant's *Critique of Pure Reason*, in the thesis of the first antinomy of pure reason: the world cannot be eternal (as conceived, e.g., by Aristotle) since,

¹ There is the complication that Barnes is the author of a paper in which he maintains that monism is an invention of Melissus, not of Parmenides. This would be too long to discuss: I can only say that I am not convinced. Further, in his *The Presocratic Philosophers* [1982], he questions the prevailing view that Zeno's arguments are *reductiones ad absurdum*; but this not unimportant point is left out in the present lectures.

² For this idea and for a survey, see [Solmsen 1975](#).

³ Cf. 49: 'Qualcosa è infinito se non ha limiti, confini, frontiere; se continua sempre, non si ferma mai, non giunge ad una sosta.'

to reach the present, there should have passed a series of moments which cannot be completed. The same point however is not so paradoxical when applied to space, for Lucretius could claim that the extension of the universe is such that a thunderbolt could not only cross it, even if its motion were everlasting, but not even make smaller the extension of space that there is to traverse [cf. *De rer. nat.* 1.1002 ff.]. The example serves to show that the space of the universe is not just immense: it is truly and positively infinite. Concerning time, he argued (at the end of book 3) that, since the condition of death to which we are destined is everlasting, it does not matter how long we live, for not one bit can be subtracted from that everlastingness. Clearly all these considerations concern an infinite which is regarded as actual and not as merely potential (thus implicitly rejecting Aristotle's approach). It is possible that this intuition, at least in its application to space, goes back to the first atomists, who were more or less contemporary of Zeno.

Does all this make a difference to our understanding of Zeno's argument? It does, for it was remarked by some scholars [see, e.g., [Vlastos 1967, 372](#)] that Zeno appears to be assuming that the division is completed and, thus, that an (actually) infinite number of bits is obtained. It was also remarked that Zeno, by making this assumption, is not consistent, because he clearly assumes in some of his arguments (like Achilles and the tortoise) that the series cannot be traversed because there always remains some extension (before Achilles), and because in the very argument under discussion he states:

Now it is all the same to say that once and to say it forever; for no bit of it is last in such a way that there will not be one bit in front of another.

It can be added that the first half of another Zenonian argument against plurality goes as follows:

if there are many, it is necessary that they be as many as they are, neither more nor fewer. But if they are as many as they are, they must be finite[ly many].

Here it is manifestly assumed that any number that is given to existing things is a finite one. However, these inconsistencies tend to show, in my view, that Zeno was ready to use any means he had at hand to reach his paradoxical conclusions, confirming that he was not a serious thinker like Parmenides (who presumed to be inspired by a goddess). He was not quite a philosopher either, since he does not show that he reflected on the concept of infinity of which he made use.

Barnes in his discussion tacitly excludes this interpretation of the argument but not on grounds of consistency. The reason for his approach becomes evident in his reply to the observations by Pulpito. This scholar remarked that Zeno could defend his position at least in the Porphyrian version of the argument by admitting that the infinitieth partition can be reached, which, he also remarks, is what the Eleatic seems to be assuming. In a note, he raises the question: 'Is there such a thing as the infinitieth partition?' [cf. 167–168 and n13]. Barnes in his reply draws attention to this note and makes the following comment:

The phrase 'the infinitieth partition' has no sense.... The sequence of partitions is infinite: each element in the sequence has a succeeding element, and each element is of course the n th element in the sequence (for some natural number n). The expression 'the infinitieth element' is nonsense. The sequence of natural numbers is infinite: each number has a successor, and every number is the n th number (for some finite natural number n). The expression 'the infinitieth number' is nonsense. The adjective 'infinitieth' is nonsense. [204]

Repetition does not yield persuasion. Barnes is committed to an Aristotelian view of the infinite and is confident that a sequence of numbers and of other quantifiables cannot ever be completed. I do not think that in matters of infinity one can be so confident of this (or of the opposite). One need not assume (and Zeno does not appear to have assumed in this case) that the sequence of divisions be intellectually traversed step by step, instead of imagining that it is completed, thus obtaining an (actually) infinite number of parts or bits from the given magnitude. Further, Barnes manifestly has in mind some definition of natural number according to which each number has a successor. But he himself remarks, in another connexion, that the results we obtain depend on the conventions we adopt. So why not modify that definition in the sense that each number, except the infinitieth one, has a successor?

Even if Barnes were right in thinking that from the point of view of modern mathematics and logic the infinitieth number is nonsense, he seems to concede that some ancient thinkers, not influenced by Aristotle, thought otherwise. In fact, it can be remarked that Plato treated as nonsense the proposition that the worlds are infinitely many [*Tim.* 55c–d] but that the atomists who put it forward clearly did not think they were talking nonsense.

This leads to the question of what the task of the scholar should be. Barnes insists that he is interested in the philosophical dimension of Zeno's paradoxes.

Now, it is, of course, quite possible to discuss the paradoxes even without knowing who Zeno was. (Barnes himself is persuaded that Zeno was a *paradoxologos* but Barnes could have added that it does not matter whether the Eleatic really was one or not, since he is considering his arguments in any case as paradoxes.) Bertrand Russell (in his *Principles of Mathematics*) and other modern thinkers have tended to discuss the paradoxes in this way. But a historian of philosophy cannot do the same. Before the misgivings of some of his interlocutors, of which he shows awareness, Barnes does not say in a definite manner whether he wants to proceed as Bertrand Russell or as a historian of philosophy [186]. I would not say (as he apparently wants to) that a historian of philosophy does not confront philosophical questions. The contributions that history and philology have to make are instrumental to the aim of understanding the thought of the ancient philosopher and are not all that distinguishes the historian of philosophy and the pure philosopher. Indeed, if one admits—as Barnes has admitted elsewhere [see 2000, 2007]—that the historian of philosophy aims at that understanding, it must be a philosophical understanding. Moreover, the question of truth or whether the thinker examined is right or wrong, while it especially concerns the philosopher, must indeed not be wholly ignored by the historian but kept in suspension. The difference between the two emerges with sufficient clarity just in the case of the notion of the infinite. If one says that certain assertions about the infinite are nonsense, one is proceeding as Bertrand Russell does, claiming that so and so is true (independently of the question whether Russell would have agreed with Barnes). If one does not start from a preconceived view on the matter and admits that certain assertions about the infinite did make sense to certain ancient thinkers and tries to understand how and why, one is proceeding as a historian of philosophy. As this review shows, I consider myself a historian of philosophy. However Barnes considers himself, it remains that his treatment of Zeno is very instructive, if perhaps a little too longwinded: bringing to light all the tacit assumptions in Zeno's arguments is important but it goes too far when the obvious is labored.

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