
The History of Mathematical Proof in Ancient Traditions edited by Karine Chemla

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This book is the offspring of a working group on ancient mathematical reasoning that met in Paris in the spring of 2002. The lengthy gestation and delays in production mean that not all the articles are up to date bibliographically, especially for work coming from outside the circle of participants. The individual papers comprising the volume are all written by experts and repay close reading. There are 16 chapters, divided into two parts, on the historiography of mathematical proof and on the history of mathematical proof in ancient traditions. These chapters follow a lengthy prologue by Karine Chemla framing the entire project, ‘Historiography and History of Mathematical Proof: A Research Programme’ [1–68].

Chemla opens her prologue with something of a straw man position, declaring that ‘the standard history of mathematical proof in ancient traditions’ [1] asserts that the only valid form of mathematical reasoning is that of the Greeks. It is not clear who exactly is still supposed to subscribe to such a ‘standard view’. The closest Chemla comes to offering a witness is a passing reference to Morris Kline, backed up by Eduard Biot.

Passing on from this somewhat shaky rhetorical opening, Chemla then provides a useful thumbnail sketch of the development of the historiography of mathematical proof from the earliest Western encounters with ancient texts and an overview of the interconnections between the more specialist contents of the subsequent chapters. Below is offered a very brief summary of each of those chapters.

Part 1, ‘Views on the Historiography of Mathematical Proof’, opens with a chapter by Bernard Vitrac entitled ‘The Euclidean Ideal of Proof in *The Elements* and Philological Uncertainties of Heiberg’s Edition of the Text’

[69–134]. Heiberg’s edition of Euclid stands as a monumental testament to his philological capabilities but some of his editorial decisions have been questioned, most notably by Wilbur Knorr in his article on ‘The Wrong Text of Euclid’ [1996]. As no early manuscript of Euclid is known, the question is how to evaluate the changes that have doubtless occurred in the course of transmission and, in particular, how to compare the manuscripts preserved in Greek, what Vitrac refers to as the ‘direct tradition’, with an indirect tradition incorporating commentaries, quotations, and translations into other languages. Vitrac provides a useful summary of the transmission and transformation of Greek writings and an inventory of Euclidean manuscripts. His main text is then an engagement with the ‘recent criticism’ [70] by Knorr. Vitrac was very much on Knorr’s mind in the 1990s, engaged as Vitrac was then on his four-volume translation of *The Elements* into French. The first two volumes had appeared at the time of Knorr’s paper with the remaining two volumes appearing in 1998 and 2001. It is welcome to have Vitrac’s detailed and judicious response, albeit somewhat delayed. Vitrac (and Knorr) place more reliance on the indirect tradition than did Heiberg and Vitrac provides a wealth of detail on editorial variation and the questions that confronted Heiberg and, with more sources available a century later, himself. Knorr carries Vitrac with him on many of his points but there are some divergences. Vitrac ends by arguing that attempts at reconstructing a pure original Euclid are doomed—‘the conception of a new critical edition of the Greek text seems useless to me for the moment’—and calls instead for further effort on the indirect tradition, ‘critical editions of the various identified Arabic, Arabo-Latin and Arabo-Hebrew versions would be preferable’ [122].

Chapter 2, ‘Diagrams and Arguments in Ancient Greek Mathematics: Lessons Drawn from Comparisons of the Manuscript Diagrams with those in Modern Critical Editions’ [135–162], is by Ken Saito and Nathan Sidoli. The role of diagrams in Greek mathematics has received increased attention recently, especially since the pioneering work of Reviel Netz. Heiberg, for example, almost completely ignored manuscript evidence when constructing the diagrams for his critical editions, a point not addressed by Vitrac in the preceding chapter. In this chapter, the authors investigate the manuscripts and offer a comparison between ancient diagrams and those in modern editions. The main differentiating characteristics that they identify are ‘overspecification’, that is, ‘the tendency to represent more regularity among the geometric objects than is demanded by the argument’ [140], e.g., by using rectangles or

squares for quadrilaterals, and ‘graphical indifference’, by which they mean ‘diagrams that are not graphically accurate depictions of the mathematical objects discussed in the text,’ [143] as when unequal lines are depicted as equal or *vice versa*. An important consequence that they draw is that diagrams in the medieval manuscripts were not in themselves ‘meant to convey an idea of the level of generality discussed in the text’ [157], arguing that verbal description or supplementary constructions would be used for this purpose.

A concern for diagrams naturally makes a re-appearance in chapter 3, ‘The Texture of Archimedes’ Writings: Through Heiberg’s Veil’ by Reviel Netz [163–205]. Netz divides his chapter into two parts, on diagrams and on text. In his analysis of the diagrams, he declares that Heiberg goes ‘metrical’, ‘three-dimensional’, and ‘iconic’. That is, in comparison to Netz’ reconstruction of the early, and possibly Archimedean, diagrams, Heiberg’s diagrams present more relevant metrical information of comparative objects, better three-dimensional representation of solids, and more accurate depiction of geometric objects. His analysis of Heiberg’s textual alterations and choices is summarized as ‘textually explicit, non-accessible and consistent’ [202]. Of these, the issue of consistency exercises him the most for here, as in his other writings on Archimedes, Netz stresses the variety of Archimedes’ work in both content and presentation.

Chapter 4, ‘John Philoponus and the Conformity of Mathematical Proofs to Aristotelian Demonstrations’, by Orna Harari [206–227] turns away from Heiberg to consider why Philoponus and Proclus were untroubled by the evident failure of mathematical proofs to satisfy Aristotle’s prescriptions for valid demonstration. Harari’s argument is detailed and technical, and a brief summary does not do her argument justice. However, her main point revolves around the ontological question of whether mathematical objects are immaterial or material. For Philoponus, they were immaterial and so questions of essential relations and grounding conclusions in the cause were irrelevant.

The next two chapters concern the interaction of Western mathematicians and Indian mathematics. Dhruv Raina, in ‘Contextualizing Playfair and Colebrooke on Proof and Demonstration in the Indian Mathematical Tradition (1780–1820)’ [228–259], considers the early British understanding of Indian Mathematics. John Playfair, in an influential address, argued that Indian astronomy as then practiced involved little more than following computa-

tional rules without insight into their origins. He suggested that Indologists should search for background texts on Hindu geometry, which he felt must have underlain the astronomical calculation procedures. Henry Thomas Colebrooke gave the first translation from Sanskrit of a selection of mathematical works. Colebrooke's selection criteria, emphasizing an 'algebraic analysis', strongly influenced subsequent European, and especially British, conceptions of Indian mathematics and astronomy.

Next, Agathe Keller tackles Georg Friedrich Wilhelm Thibaut (1848–1914) in 'Overlooking Mathematical Justifications in the Sanskrit Tradition: The Nuanced Case of G. F. W. Thibaut' [260–273]. Thibaut was a philologist with a fine sense of textual and grammatical detail, and a specialist in the *mīmāṃsā* school of philosophy. This led him to an interest in mathematics and astronomy and he published the oldest known works of Sanskrit geometry. The verses of the oldest texts, the *śulvasūtras*, are difficult to understand and are accompanied by commentaries, often written much later, that explain these difficult, dense, and aphoristic texts, and provide justifications. As a historian, Thibaut was wary of the extent to which later commentaries could be taken to reflect authorial intent accurately. He was also troubled by the way in which Sanskrit sources did not reflect his own sense that mathematical propositions should be stated logically and clearly, and properly demonstrated. Keller unravels these interesting contradictions.

Rounding out part 1 is a chapter by François Charette, 'The Logical Greek versus the Imaginative Oriental: On the Historiography of "Non-Western" Mathematics during the Period 1820–1920' [274–293]. Charette is principally concerned with the views of Hermann Hankel (1839–1873), Moritz Cantor (1829–1920), and Hieronymus Georg Zeuthen (1839–1920) on the comparison of Greek mathematics with Indian, Chinese, and Islamic mathematics. Hankel's book on ancient and medieval mathematics was published posthumously but proved to have a lasting impact on the next generation of historians of mathematics. Cantor was dismissive of Indian mathematics and his analysis of Islamic mathematics was characterized by a search for external influences. Zeuthen regarded the solution of third-degree equations as the decisive step away from medieval mathematics and saw the rapid development of mathematics after that as a result of study of Greek mathematics, and so his periodization of the history of mathematics led him to foreground the Greeks. Charette's analysis shows some of the complexities

underlying the common notion of a Greek exceptionalism reflected in a specific racial or national cast of mind that was current in the 19th century. Moving on from the historiography of early European historians of mathematics, part 2, ‘History of Mathematical Proof in Ancient Traditions: The Other Evidence’ opens with a chapter by G. E. R. Lloyd, ‘The Pluralism of Greek “Mathematics”’ [294–310]. Lloyd raises questions concerning the ‘heterogeneity of the Greek mathematical experience’ [307], arguing that it derived from the competitive nature of Greek intellectual discourse and the tensions between discovery and proof inherent in the privileging of the axiomatic-deductive method of argument.

In chapter 9, Ian Mueller considers ‘Generalizing about Polygonal Numbers in Ancient Greek Mathematics’ [311–326]. Mueller picks up two aspects of Greek reasoning on polygonal numbers. The first, treating Nicomachus, is that much information about the arithmetic and geometric properties of polygonal numbers is carried by the specific configurations of dots (or alphas) that would be destroyed by a Euclidean treatment of numbers as straight lines. The second considers Diophantus’ arguments, ‘cumbersome and roundabout’ [319] but essentially correct. In Diophantus, the geometrical configurations are suppressed in preference to a purely arithmetical presentation. Mueller argues that ‘within the limits of Greek mathematics there can be no mathematical demonstration of an arithmetical characterization of configurationally conceived polygonal numbers’ [325].

Diophantus is also the subject of the next chapter, ‘Reasoning and Symbolism in Diophantus: Preliminary Observations’ [327–361], by Reviel Netz. Netz argues that Diophantus’ use of symbolism ‘has a functional role in his reasoning’ [328] and, specifically, that it is intended that the reader ‘systematically read the sign both verbally and visually’ [341]. Next, he considers the specific modes of reasoning and content presented by Diophantus, concluding that ‘Diophantus set himself the task of presenting lay and school algebra within the format—and expectations—of Greek geometrical analysis’ [359]. His specific rhetorical decisions were designed to facilitate that goal.

From Greece, we turn to Mesopotamia for the next two chapters. In chapter 11, ‘Mathematical Justification as Non-Conceptualized Practice: The Babylonian Example’ [362–383], Jens Høyrup tackles the questions of demonstration and critique in the case of Old Babylonian mathematics. Arguing for a concrete geometrical reading of the steps of the numerical algorithms involved

in many examples, he maintains that the correctness of the procedure is immediately obvious to the user:

one who follows the procedure on the diagram and keeps the exact (geometrical) meaning and use of all terms in mind will feel no more need for an explicit demonstration than when confronted with a modern step-by-step solution of an algebraic equation. [367]

The force of the demonstration is in the procedures. His second point is about the absence in most of Old Babylonian mathematics of a discussion of the conditions under which procedures remain valid. Here, he emphasizes that the bulk of our sources come from an educational locus where training in following correct procedures was more important than discussion of why and under exactly what conditions such procedures were correct:

the social *raison d'être* of Old Babylonian mathematics was the training of future scribes in practical computation, and not deeper insight into the principles and metaphysics of mathematics. [381]

While it is certainly true that the bulk of known Old Babylonian mathematical tablets are connected with the education of trainee scribes, it is now also clear that this is not the case for all of them. Christine Proust analyses one such tablet, CBS 1215, and some related texts in chapter 12, 'Interpretation of Reverse Algorithms in Several Mesopotamian Texts' [384–422]. She argues that, despite containing only numbers, 'this text contains an original mathematical contemplation' [384]. The text in question contains a series of computations of reciprocals of successive doublings of the number 2,5. Crucially, in each case, the reciprocal of the reciprocal is then computed to return to the original number. However, the reverse algorithm is not the first algorithm with the steps reversed but a second iteration of the initial procedure. Further, the numbers are laid out in a precise and unusual manner: 'the principles of the spatial arrangement of numbers [has] a precise meaning in relation to the execution of the reciprocal algorithm' [395]. Hence,

the relationship between Tablet A [CBS 1215] and the school exercises is exactly the opposite of what is usually believed. Tablet A does not seem to be the source of school exercises: rather it seems derived from the school materials with which the scribes of the Old Babylonian period were familiar. [409–410]

Proust's conclusion is that the tablet 'bears witness to the reflection of the ancient Mesopotamian scribes on some of the fundamental principles of

numeric calculation' [417]. Proust's reading of CBS 1215 is thus very similar to Netz' interpretation of Diophantus.

Karine Chemla considers Chinese proof techniques in chapter 13, 'Reading Proofs in Chinese Commentaries: Algebraic Proofs in an Algorithmic Context' [423–486]. As in India, core mathematical texts come with commentaries. In particular, Chemla notes that in the case of the *Nine Chapters on Mathematical Procedures*,

no ancient edition of *The Nine Chapters* has survived that does not contain the commentary completed by Liu Hui in 263 and the explanations added to it by a group of scholars under the supervision of Li Chunfeng. [424]

In order to understand how the original text was approached by Chinese scholars, it is necessary to treat the *Nine Chapters* and its accompanying commentary as a unit. In this deep and penetrating paper, Chemla explores two aspects of the work of the commentary. A problem in the *Nine Chapters* is stated with particular numbers and solved by particular computations. The commentaries unpack this particularity in a couple of directions. They show *why* the computations are the way they are, what Chemla refers to as the 'meaning' of a calculation, in the course of which they show how the specific problems can be generalized. The commentaries also show that the solution algorithms are correct by showing how the procedures can be obtained from known correct algorithms by a sequence of valid transformations, that is, 'algebraic proofs in an algorithmic context'. An important part of Chemla's argument rests upon a detailed analysis of the interplay between computational layout on the counting surface and arithmetical transformations, tying together abstract reasoning and material culture in an intimate fashion. The sequence of transformations on the computing surface provides a significant layer of the 'meaning' of computations.

As was explained by Agathe Keller in her chapter on Thibaut, early Indian mathematical texts are dense and allusive, and require commentary. In chapter 14, 'Dispelling Mathematical Doubts: Assessing Mathematical Correctness of Algorithms in Bhāskara's Commentary on the Mathematical Chapter of the *Āryabhaṭīya*' [487–508], she returns to this topic in an analysis of Bhāskara's commentary on Āryabhaṭa's mathematical treatise. Bhāskara's task is to convince the (hostile) reader that Āryabhaṭa's verses do in fact contain justifiable mathematical procedures. Bhāskara utilizes two main editorial devices. After teasing out the mathematical details of an

interpretation, he then gives a ‘reinterpretation’ investing the verse with an additional layer of meaning. His other technique for showing the validity of a procedure is to provide a separate, independent, procedure showing that it arrives at the same conclusion. Keller works through a series of detailed examples to illustrate her analysis.

Chapter 15, ‘Argumentation for State Examinations: Demonstration in Traditional Chinese and Vietnamese Mathematics’ [509–551] is by Alexei Volkov. Volkov’s contention is that (most) Chinese mathematics treatises of the first millennium functioned as textbooks for their users. Unfortunately, there is little direct contemporary evidence to support this point. However, after a review of Chinese mathematical education and an elucidation of examination procedures, Volkov turns to a Vietnamese witness, a ‘model’ examination paper published by Phan Huy Khuông at the end of his book *Chi minh lập thành toán pháp* (*Guidance for Understanding the ‘Ready-Made Computational Models’*), published in 1820. Arguing that Vietnamese mathematics education closely followed the Chinese system, and that the mathematics curriculum displayed longterm stability, Volkov suggests that the essay given in the model examination paper reflects the style of response that classical Chinese education expected of its students. In particular, he contends that the commentaries such as those of Li Chunfeng were taken as exemplars for examination essays.

Rounding out the volume is Tian Miao’s chapter, ‘A Formal System of the *Gougu* Method: A Study on Li Rui’s *Detailed Outline of Mathematical Procedures for the Right-Angled Triangle*’ [552–574]. The text in question, published in 1806, presents a systematic treatise on methods of solving a right-angled triangle given two of a list of 13 related variables (lengths of sides, sums and differences of sides, and so forth). The 78 problems so generated are carefully organized, both internally (each problem is stated and solved in a similar fashion) and externally (the problems are developed systematically). Tian shows that Li Rui ‘consciously developed his system’ [565] and that his aim was a systematization of a body of traditional knowledge rather than the production of new knowledge.

The book offers a wealth of insights both into the history of Western engagement with the mathematics of a wide variety of ancient cultures and the current state of the art. It is a valuable addition to the scholarly literature, showing the current, very active, struggle of scholars to enter more fully

into the conceptual worlds of ancient mathematical practice from the scant traces left to us.

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