Research in the History and Philosophy of Mathematics: The CSHPM 2014 Annual Meeting in St. Catherine's, Ontario edited by Maria Zack and Elaine Landry

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The Canadian Society for the History and Philosophy of Mathematics, also known as La Société Canadienne d'Histoire et de Philosophie des Mathématiques, was founded in 1974 simultaneously with the launch of its first flagship journal, *Historia Mathematica*. Kenneth O. May, the first editor, must have realized that in order to survive, the new journal would need to be more than a single person's project. With the sponsorship of CSHPM, *Historia Mathematica*, later joined by *Philosophia Mathematica*, continues to thrive.

Over the years, the Society has grown to include mathematicians and historians of mathematics throughout North America and beyond; but, faithful to its historical roots, it has retained its 'Canadian' name. The articles from the CSHPM Annual Meeting have been collected into volumes of proceedings since 1988, but up to now these have been privately circulated to members only. Recently, the Society has entered into an agreement to publish these proceedings as a new series of volumes, inevitably entitled *Proceedings of the Canadian Society for the History and Philosophy of Mathematics / La Société Canadienne d'Histoire et de Philosophie des Mathématiques*. The book under review, containing papers from the meeting in 2014, inaugurates the new series. As is the case with all books from Birkhäuser, it is available in both printed and electronic form, and individual articles may be purchased (or accessed *via* an institutional *SpringerLink* subscription).

Like most proceedings, this is a mixed bag; but there do seem to be a couple of overarching themes. Several of the articles deal with what may be described as analog computers: devices designed to solve mathematical problems. Two articles (by Silverberg and Ackerberg-Hastings) focus on sectors and scales, one (by Crackel, Rickey, and Silverberg) on precise diagrams

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as a computational tool, two others (by Bennett and Abeles) on attempts to create logical diagrams that would allow for visual proofs. Even the mostly straightforward account of the Toronto ICM in 1924 includes a discussion of Peano's attempt to create a logical language and notation that would realize Leibniz's dream of reducing thought to calculation.

A second theme that gets some attention here is mathematical publishing. One article (by Nickerson) focuses on Macmillan and Co. as publishers of textbooks for the English-speaking world, one (by Godard) deals with the pedagogical work of Borel and Lebesgue in France, and one (by Thomas) proposes a judicial analogy to account for the role of publication in mathematics.

Though a few of the papers in this collection deal with early modern mathematics, most of the focus is on the 18th century or later. Many of the articles collected here could be described as 'out-takes' in the sense that they follow up on previously published work or report on work in progress. One often gets the feeling that the whole story is yet to be told. This, I suppose, is fairly typical of conference papers in general.

Most readers will not be reading this book as a reviewer must, from cover to cover, but will rather choose the article or articles that interest them. In what follows, I comment on my favorites.

'Reassembling Humpty-Dumpty' is the title that Theodore J. Crackel, V. Frederick Rickey, and Joel S. Silverberg chose for their paper on George Washington's cyphering book. Since textbooks were scarce and mostly owned by the teacher, 18th-century students often prepared their own elaborate 'books'. These were often kept and used as references in later life. In the case of George Washington's cyphering book, the bulk of the manuscript is to be found in the Library of Congress; but several leaves are missing. The authors recount the story of the missing leaves and manage to find and identify some of them. Unfortunately, in the printed book the reproduction of the images (which are crucial in this article) is less than ideal. The images in the PDF version are much better.

While this article is clearly a report on work in progress, it is interesting and informative. In the process of their investigation, the authors show that surveyors were taught to make accurate scale-diagrams that could be used as *computational* devices. Rather than use Euclidean geometry and trigonometry to determine the length of some line, as one would teach students today,

Washington was taught to create careful scale-diagrams. From the diagram and the knowledge of the scale, it was possible to determine lengths simply by measuring. This opens up a whole new way to think about mechanical drawing, a standard school topic in older days which has now completely vanished from sight.

Also interesting is the paper 'The Eighteenth-Century Origins of the Concept of Mixed-Strategy Equilibrium in Game Theory' by Nicolas Fillion. This is a kind of complement to a paper written by D. R. Bellhouse and Fillion which deals with some probability-problems discussed by Waldegrave, Montmort, and Nicolaus Bernoulli. The questions refer to a card game called Le Her. While the paper with Bellhouse dealt with most of the historical aspects, in this paper Fillion analyzes the game in modern terms, then uses this analysis to think about the solutions proposed in the 18th century. One of his conclusions is pithily expressed: 'concepts and methods are typically older than their foundations'. Fillion argues that while formal game-theory and the precise notion of a mixed strategy were still very much in the future, Montmort, Waldegrave, and Bernoulli do create such a strategy, then disagree as to whether having it amounts to 'solving' the game. They even enter into a discussion of the difference between public advice on how to play the game and private advice on how to proceed in specific cases, for example, when one of the players is weak.

The articles discussing mathematical publishing are all interesting, though in all cases I felt that there was more to say. Sylvia Marie Nickerson's article highlights the role of Isaac Todhunter as an advisor to Macmillan and Company. She points out both the weaknesses and the strengths of Todhunter's view of mathematics, which was largely that of Cambridge University at the time, and how it influenced the way in which mathematics was learned and taught in much of the English-speaking world. In particular, she quotes J. C. Fields' opinion that the calculus had been taught to him 'falsely, irremediably and fundamentally falsely'. (One wonders what he would say about today's textbooks.) The data Nickerson presents on the print-runs of Todhunter's mathematics books are striking, with Euclidean geometry and elementary algebra in the hundreds of thousands, while his books on the history of the theory of probability, the calculus of variations, and the problem of the shape of the Earth—the only books of his I have ever looked at—were printed in runs of 1,000 copies or fewer.

Roger Godard's article on Émile Borel and Henri Lebesgue as authors of textbooks is also quite interesting. The focus is mostly on a series of books published by Gauthier-Villars beginning in 1898. Comprising 50 or so books, the series provided short modern accounts of precise topics. The idea was to bring the student to the research frontier quickly and efficiently. Ten of the books were written by Borel, but the authors form quite an impressive list: Borel, Baire, Lebesgue, de la Vallée Poussin, Volterra, Bernstein, Montel, and Riesz, among others. Godard provides brief comments on the pedagogical approach in these books, focusing on Borel and Lebesgue. There is clearly much more to be said on this topic, however.

Finally, there is R. S. D. Thomas' 'The Judicial Analogy for Mathematical Publication', the only article in the book to treat the philosophy (rather than history) of mathematics. Thomas opens the article by noting that in the past he has written articles criticizing proposed analogies between mathematics and fiction and between mathematics and games. Winningly, he writes that in this article he will 'offer what I view as an improvement on the fiction analogy with my own undue enthusiasm', offering it up for criticism by others. The analogy he proposes is between mathematical proof and a legal argument: in both cases, there is someone to be persuaded (a jury, other mathematicians), there are standards (laws, axioms, rules of evidence, rules of deduction), and there is testimony (a witness tells a story, a mathematician outlines an argument). The result is interesting and thought-provoking, though Thomas explicitly disavows drawing any philosophical conclusions.

As noted with respect to the photographs in the article about George Washington, there are some production issues. Some articles have more typos than others, which suggests to me that the authors were responsible for getting them into publishable shape. The editors' preface does not indicate whether the papers were peer-reviewed before publication.

Publication of this book is a further sign of CSHPM's growing importance as a sponsor of new scholarship in both the history and the philosophy of mathematics. I hope to see many more volumes in the series.