Menelaus' 'Spherics': Early Translation and al-Māhānī/al-Harawī's Version by Roshdi Rashed and Athanase Papadopoulos

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Menelaus' *Spherics*, composed in the 2nd century AD, uses earlier work in spherical geometry, particularly Theodosius' *Spherics*, to develop a theory of the spherical triangle as the basis of a new approach to spherical geometry, trigonometry, and astronomy—that is, to the ancient mathematical discipline called *spherics*.[1] Despite the originality, and applicability of this work,[2] there is no evidence that it was ever studied seriously in its entirety in the ancient period, and only fragments of the Greek text, which are preserved as quotations in later texts and scholia, survive.[3] Indeed, it is not even certain that Ptolemy used this text when he was developing his approach to spherical astronomy in *Alm*. 1.13–2.13 and 8.5–6.[4]

Menelaus' *Spherics* can be divided into three sections. The first treats the geometrical properties of spherical triangles by developing analogies between these and the properties of plane triangles that are developed in Euclid's *Elements*. The second shows how certain arcs of spherical triangles can be related to the lengths of chords related to them and, using a theorem

[1] This review is an expansion of my review of the same book for *Bryn Mawr Classical Review* [Sidoli 2019], which had a strict word limit.

[2] The potential usefulness of this work to spherical astronomy, which was not exploited in any surviving text in Greek, is explained by Nadal, Taha, and Pinel 2004.

[3] The Greek fragments are collected and studied in Bjørnbo 1902, 22–24 and Acerbi 2015. It is possible that Menelaus himself applied the methods developed in his *Spherics* in a lost work on spherical astronomy. But, if so, this approach was not adopted by Ptolemy or any other known Greco-Roman author.

[4] Bjørnbo 1902, 92 raises doubts that Menelaus wrote the so-called Menelaus (Sector) Theorem, a line of thinking that I have developed more fully in giving a number of further arguments [Sidoli 2006]. If Menelaus did not write the Sector Theorem, then nothing else compels us to believe that Ptolemy used his text at all.

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known as the Sector Theorem (Menelaus Theorem), provides a method for the metrical treatment of the arcs of great-circles. The third section develops these methods for application to problems in spherical astronomy, a field that investigated issues such as the length of daylight and nighttime, and the rising times of stars or arcs of the ecliptic.

The book under review is a valuable contribution to our understanding of the history of Menelaus' *Spherics* in the medieval period, as well as to the mathematics developed in the treatise. The first part deals with the various medieval versions of, and witnesses to, Menelaus' treatise; the second part provides mathematical commentaries, including texts and translations of remarks by medieval scholars; the third part gives a critical edition of a fragment (breaking off in prop. 36) of an early Arabic translation (**A**) [408–483] and the al-Māhānī/al-Harawī version (**M/H**) [500–777].[5] There is also a postface on spherical geometry and its history. The mathematical commentaries in the second part are useful for understanding the text and the critical editions, and the many partial editions and translations of medieval sources are an extremely valuable contribution to our state of knowledge about this text.

The **M/H** version of the *Spherics*, edited and translated, along with **A**, in 'Part III: Text and translation', is historically quite interesting, but al-Harawī's many interventions, along with his failure to grasp some of the mathematical details, introduce nearly as many problems as they resolve.[6] Al-Harawī has added two historical and philosophical prefaces to the text [500–505, 684–685]; inserted a number of lemmas [686–695], one of which is mathematically incorrect [692–995]; rewritten some propositions, sometimes incorrectly; and introduced some terminological innovations, which cause as much confusion as help and are not used in the other major medieval versions of the text [688–691]. Hence, this version of the treatise cannot be taken as a reader's text, and Naṣr Manṣūr ibn ʿIrāq's version, **N**, edited by Krause [1936], and the revision by Naṣīr al-Dīn al-Ṭūsī, available in Hyderabad series 1940–1941 and reprinted by Sezgin [1998], **T**, must still be consulted in order to understand the mathematics involved.

Another welcome contribution of Rashed and Papadopoulos' book is 'Part II: Mathematical commentary', which explains the mathematical details of

[5] I use bold letters, **X**, for versions of the text for which we possess one or more witness(es), and italics, *X*, for versions which have been lost, or are a matter of conjecture.

[6] For an overview of al-Harawī's version of the text, see Sidoli and Kusuba 2014.

the text and explains each proposition, including the relevant scholarship of both Ibn ʿIrāq and al-Ṭūsī. Hence, this section of the book provides a fairly clear picture of the mathematical issues involved, along with the interpretations of this text by two of its most important medieval readers.

In part 1, Rashed and Papadopoulos give an introduction to Menelaus and his work, and then discuss the text history of the *Spherics* in the medieval period. Our understanding of the relationships between the various medieval versions of the the treatise is still largely due to the scholarship of M. Krause [1936]. Rashed and Papadopoulos give a reevaluation of this material but the positions for which they provide clear evidence were already established in Krause 1936 and Hogendijk 1996. Their new suggestions remain conjectural and are, in my opinion, not convincing.

Since the situation with the medieval version of this treatise is rather involved, it may help to summarize this before describing Rashed and Papadopoulos' contribution. There are currently three known, complete, relatively early Arabic versions of Menelaus' *Spherics*:

- the version by al-Māhānī/al-Harawī, **M/H** [500–777];
- that by Ibn ʿIrāq, which has been edited in Krause 1936, **N**; and
- a revision, **T**, by al-Ṭūsī from **M/H** and **N**.

Furthermore, there is also

- a newly discovered fragment **A** [408–483], as well as
- a Latin version by Gerard of Cremona, **G** and a Hebrew version by Jacob ben Machir, **J**,
  - both of which Krause argued were produced from the same, now lost, Arabic version, whose existence he conjectured, *D*.

Krause showed that if *D* had indeed existed, it must have been made from a source that contained the al-Māhānī version before al-Harawī corrected it, *M*, for the first part, and the source of **N** for the second part.[7] The existence of *D* was then further confirmed when Hogendijk [1996] showed that Ibn Hūd, in composing his *Perfection* (*al-Istikmāl*), had worked from a version of the *Spherics* that had these same characteristics—namely, the first part from *M* and the second part from the source of **N**. As for the translators, Krause noted that the Hebrew manuscripts credit Isḥāq ibn Ḥunayn with the translation and claimed that the source translation for **N** must have

[7] At the time, Krause believed that the Sector Theorem, prop. 3.1 (prop. 66 in **M/H**) in *D* was not from the source of **N**; but it was later shown that this view is not tenable: see Lorch 2001, 332–334 and Sidoli 2006, 50.

been this translation, *bH*,[8] while the source translation for **M/H**, say, *U*,[9] was taken to be anonymous, and the translation mentioned in some of the marginal commentaries by Abū ʻUthmān al-Dimashqī with the corrections by Yūḥannā, *D/Y*, he considered to be completely lost.[10]

Much of the first part of Rashed and Papadopoulos' book confirms this overall picture, with two proposed changes. For example, they reconfirm that the source-translation for **M/H** and **N** differ, that **G** is based on *M* and the source of **N**, and that the source used by Ibn Hūd has the same characteristics as that for **G**. On the other hand, they believe that:

(a) the newly discovered fragment **A** is a translation unrelated to anything we previously knew about—that is, that **A** is not Krause's *U*—which, as I will argue below, is unconvincing, and that

(b) the source for **N** is not Krause's *bH* but rather *D/Y*. This is a possibility, but because they have not shown (a), it remains fairly unlikely and is contradicted by the direct testimony of the Hebrew sources.

Moreover, instead of directly addressing the issue of Krause's proposed *D*, they, strangely, raise the possibility of such a source as though it is a question arising from their own work and not already a concrete proposal argued for by both Krause and Hogendijk.

As for (a)—namely, the proposal that the new fragment that Rashed and Papadopoulos have discovered, **A**, is not the base translation for **M/H**—I do not find it convincing. In fact, I find nothing in the comparison of this fragment with **M/H** that rules against the likelihood that **A** is Krause's *U* and that it was indeed the source-translation for the production of **M/H**. There are, of course, many differences. The diagrams have been redrawn and relabeled and the letter-names are changed such that they are introduced in *abjad* order. The diagrams in **A** are, in fact, those included at the back of one

[8] Rashed and Papadopoulos note eight Hebrew manuscripts that mention the name of the translator [19 nn50, 51]. The claim in two of these that the translator was Ḥunayn ibn Isḥāq is a natural slip of replacing the less famous son with the more famous father.

[9] Krause calls this $\ddot{\text{U}}_1$.

[10] Krause 1936, 35 notes one mention of this version in the margin of a copy of **T**. Rashed and Papadopoulos have since found two other mentions of this version—one in a margin of a copy of **T** and the other in a margin of a copy of **M/H** [19–20]. In fact, these new citations of *D/Y* are both the same gloss and may simply have been transmitted as marginal scholia rather than drawn from independent inspections of *D/Y*.

of the **M/H** manuscripts, *BL Or.* 13127 f. 52a, and said to be ‘according to the first composition’ («على الوضع الأول»).[11] There are also extensive differences of terminology [402–403] and three minor differences in the way that the argument is developed [props. 4, 11, and 14]. But all of these changes could be included within the scope of al-Harawī’s claim that the text has been corrected in ‘expression’ («لفظ»), ‘sense’ («معنى»), and ‘proof’ («برهان») [503].

The three substantive differences between **A** and **M/H** in the mathematical arguments can all be explained as the interventions of the editors of **M/H**. For prop. 4 in **A**, only half of the proposition is set out in the exposition and proved, although the text of the proof is corrupted. Hence, the changes in **M/H**, also found in **G**, involve setting out the full exposition and completing the proof [29–30, 417 nn8, 10]. That is, the differences between **A** and **M/H** are easily explained by supposing that al-Māhānī corrected a garbled source—which description **A** here appears to fit. For prop. 11 in **M/H**, one of a pair of converses is shown, whereas the other converse is asserted, while in **A** only the other converse is stated and shown [33–35, 426 n23]. Since the first application of prop. 11 in the following theorem uses the converse not shown in **A** [248: see Krause 1936, 130], it is clear why a mathematically inclined editor would change the text to the version found in **M/H**, so that both converses are clearly stated. Finally, for prop. 14, **M/H** introduces a condition to the theorem, also found in **G** and **J** but not in **A** or **N**, which, however, is not necessary. Rashed and Papadopoulos believe that this is an indication of a different source [37, 430 n27, 532 n11]. But it can just as well be read as an intervention on the part of al-Māhānī—because, as pointed out by Rashed and Papadopoulos, the more restricted statement is enough for the application of this theorem in the following theorem [538: see Krause 1936, 134]. Hence, all three of these differences are explicable in terms of mathematical interventions on the part of al-Māhānī. On the contrary, the overall development of the propositions is the same in **A** and in **M/H**, including the peculiar props. 8 and 9. Moreover, the references to ‘the ancient translation’ («النقل القديم») or ‘the first composition’ («الوضع الأول») in the scholia to **M/H** point to material that we find in **A** [565, 595] and the diagrams at the end of *BL Or.* 13127. The wording of these references

[11] The situation with the diagrams in *BL Or.* 13127 is fully described in Sidoli and Kusuba 2014, 158–159. The description [492–493] by Rashed and Papadopoulos is somewhat misleading. There are slight differences between the diagrams in **A** and *BL Or.* 13127 in props. 12 and 35; and in the labeling for props. 18, 22, and 23. But otherwise they are identical.

is sometimes a little different, but the content is the same; and al-Harawī indicates that there was more than one correction of the source-translation in circulation in his time [503]. Consequently, we should not expect perfect verbal agreement. In fact, the places where **A** agrees with **N** against **M/H** can all be just as well, if not better, explained by the interventions of al-Māhānī than by the supposition of a different source. All in all, I see no compelling reason why we should not believe that **A** is a fragment from the tradition of the source translation that served, in some way, as the basis for **M/H**—in a word, that **A** is in fact a manuscript from the tradition of Krause's *U*.

As for (b), the thesis that the source translation for **N** was al-Dimashqī's *D* as opposed to Isḥāq's *bH* is possible, but not proven. Isḥāq is credited with some six other translations of Greco-Roman mathematical texts and al-Dimashqī is credited with one other full translation—although a rather advanced one—and perhaps some books of the *Elements*, so that either man is a possible candidate for the translator of the source of **N**. The argument in support of following Krause is that Isḥāq is directly associated with this version in the Hebrew tradition [see 17 n8], whereas the *D/Y* version is only mentioned in three glosses (to manuscripts of **T** and **M/H**), two of which are, in fact, the same, although in different versions of the text—and it is unclear from these glosses that there was a full, independent translation by al-Dimashqī, *D*, in circulation. The advantage of following Rashed and Papadopoulos is that we would not have to accept that the *D/Y* version has been completely lost. On the other hand, we would either need to suppose that *bH* is the basis of **M/H**—which is Rashed and Papadopoulos' position—or that *bH*, which was produced by one of the most famous translators of mathematical texts, has been lost completely, neither of which seems to me to be likely. The reason for my holding that it is unlikely that *bH* is the source of **M/H** is that al-Harawī says that the source translation was poor—which is also clear from the text itself, as was argued directly by myself and Kusuba [2104, 193–194]—whereas, based on what we know from other sources, Isḥāq's translations of mathematical works were generally fairly good. Indeed, **A** is much sloppier than any of the translations that are securely attributed to Isḥāq. Moreover, as was argued above, it is likely that **A** is a copy of the source translation for **M/H**, which would mean that in order to accept Rashed and Papadopoulos' claim we would also have to accept that a translation by one of the most famous translators of Greek mathematical works has disappeared without a trace.

These criticisms do not in any way diminish the value of Rashed and Papadopoulos' work, though their lack of acknowledgment of the work of previous scholars is disappointing. Sometimes they simply neglect to mention significant work, such as the recent collection of scholia to the *Almagest* citing the Greek text of Menelaus' *Spherics* made in Acerbi 2015. In other cases, they make no mention of the fact that some of their positions have already been put forward, and argued for, by others. For example, they argue at length that the first part of **G** is based on the same source as **M/H** (that is, *M*), whereas the second part is based on the same source as **N** [26–71]. But this was established by Krause [1936]. Likewise, in the section on Ibn Hūd, Rashed and Papadopoulos claim that the question of his source has 'not been correctly addressed until now' [74], though they use the same methodology as Hogendijk and come to the same conclusion—namely, that the first part of Ibn Hūd's source is from **M** and the second part from the same source translation as **N** [73–121]. That is, in both cases Rashed and Papadopoulos' actual contribution is to give further evidence, including edited texts, which serves to confirm previously established positions.

We are grateful to Rashed and Papadopoulos for their work in producing two new editions of the Menelaus' *Spherics* (**A** and **M/H**), in providing the original sources for much of the medieval scholarship on this important work, and in commenting on the overall mathematical development of the treatise. As the discussion above has shown, however, we should not read **M/H** by itself as Menelaus' text because it is a highly edited version of the treatise. In our current state of knowledge, it remains that we must read **M/H** along with both **N** and **T** in order to assess Menelaus' work fully, and we still await critical editions of the Latin and Hebrew versions before we can hope to understand the medieval transmission of the text.

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