Ptolemy's Philosophy: Mathematics as a Way of Life by Jacqueline Feke

Princeton, NJ/Oxford, UK: Princeton University Press, 2018. Pp. xiv + 235.
ISBN 978–0–691–17958–2. Cloth US$39.50

Reviewed by
Cristian Tolsa Domènech
Universität Osnabrück
ctolsa@gmail.com

Jacqueline Feke's book presents the results of her efforts to elucidate Claudius Ptolemy's philosophical system. That Ptolemy conceived his mathematical studies as forming part of a broader investigation is especially clear in the elaborate preface of the *Almagest*, in which he defends the thesis that mathematics is the only non-conjectural part of philosophy. For Ptolemy, only with mathematics can we advance in physics and theology, the two other branches of theoretical philosophy in the classification of knowledge that he proposes; and, furthermore, only mathematics helps us approach the divine. Thus, even if at the beginning of this preface he posits that ethics is independent of theoretical philosophy, Ptolemy ends up making it dependent upon it. As Feke notes [78], this is typical of several philosophical passages in Ptolemy's works, in which his writing takes the form of a serious, ongoing philosophical investigation, as in his discussion of the constitution of body and soul in *On the Criterion and the Commanding Faculty*. We would perhaps expect that short philosophical excursions by a mathematician adopt a more expositional, handbook tone; but they turn out to be original and valuable records that are worth studying for his thinking process, the vivid style of which (I dare propose) demonstrates Ptolemy's admiration for Plato's dialogues and of Aristotle's treatises.[1]

Ptolemy's philosophy is mainly to be connected with Middle Platonist trends that appropriated key concepts from Aristotle's esoteric works, which were made available sometime during the first century BC, the time of Androni-

[1] The two authorities seem to be alluded to in the preface of the *Almagest*: this is obvious for Aristotle, who is cited for the classification of the theoretical parts of philosophy, but a typical form of Socratic criticism of other philosophers in Plato's dialogues is probably also in Ptolemy's mind at the very beginning, when he uses the expression 'the true philosophers' (« οἱ γνησίως φιλοσοφήσαντες »). Cf., e.g., *Resp.* 473d οἱ βασιλῆς τε νῦν λεγόμενοι καὶ δυνάσται φιλοσοφήσωσι γνησίως τε καὶ ἱκανῶς.

ISSN 1549–4497 (online) ISSN 1549–4470 (print)

cus. Like other technical authors of his time, Ptolemy found a philosophical justification and setting for his lifelong mathematical pursuits; but his case is especially remarkable for being extremely sophisticated and without clear precedents in the history of mathematics. Unlike the case of medicine—the obvious comparable figure is Galen—there does not seem to have existed a tradition of mathematical works ingrained to a similar degree in any philosophical system, except perhaps in the case of harmonics, which, probably not coincidentally, Ptolemy studied at the beginning of his career. It should, then, not be surprising that the *Harmonics* is, among Ptolemy's mathematical works, the one in which we find a deeper engagement with philosophical issues, and that his only entirely philosophical text (in the traditional sense, that is, without mathematics), the above-mentioned essay *On the Criterion*, deals with epistemological topics that play an important role in the *Harmonics* (for which reason it is also ascribed to this first stage of Ptolemy's career).

Ptolemy thus subverting at once the propaedeutic role assigned to mathematics in Platonic thinking and the lesser importance attached to it in the Aristotelian hierarchy of the parts of philosophy, presents himself as a true philosopher, the creator and principal adherent of his own mathematical-philosophical system. That the concept of this global philosophical-mathematical system was ever influential is doubtful, since we do not hear of any contemporary followers, and since Ptolemy's influence in later authors seems restricted to the individual disciplines and to the more technical aspects of his works, be it the astronomical models of the *Almagest* and the *Handy Tables*, versions of which began to circulate in third-century papyri and ultimately replaced Babylonian-style astronomical tables in astrological practice, or the astrological doctrines themselves that are masterfully synthesized in the *Tetrabiblos*.[2] Again, the only strictly philosophical section (in the traditional sense) of Ptolemy's works which seems to have received attention in antiquity is the first part of the *Harmonics*, which was studied at length by Porphyry in his commentary on that work. But, as mentioned above, there was a long tradition going back to Archytas that discusses the criteria in harmonics. It is precisely the relation between Ptolemy's tenets and those of his predecessors in harmonics that Porphyry explores in his commentary.

After all, mathematicians were very few in antiquity, and perhaps Ptolemy's all-encompassing philosophical project was meaningful only in his own time. Thus, unlike Galen, who often portrays himself as surrounded by fol-

[2] *Pace* Feke, who in the conclusion [205–207] is more optimistic about the possible influence of Ptolemy's philosophy in antiquity and even in modern times.

lowers and peers who attend his demonstrations, who has in mind a myriad of dedicatees and students with different interests when writing his works, and who writes treatises on logical reasoning for his medical students, we hear only of one single reader of Ptolemy's works, the obscure Syrus, about whom we can deduce only that he was interested solely in astronomical (including astrological) matters, given that he appears as the dedicatee of Ptolemy's works devoted to these topics.[3] Such mathematical loneliness was of course not exclusive of Ptolemy. Consider the example of Archimedes, who bitterly regrets the death of his peer Conon in the preface of his *On the Sphere and the Cylinder*, probably because Conon was one of the few people in his time who was interested in, and could understand, his work.[4]

Feke's study derives from her PhD thesis [2009]; in fact, two and a half of the eight chapters in her book (the second part of chapter 3, and chapters 4 and 7) are but slight modifications (with due acknowledgement) from three published articles deriving from the same dissertation, which was supervised by the historian of ancient Greek astronomy Alexander Jones.

Astronomy has always been the major gateway to Ptolemy. Following the profound and encyclopedic work of Otto Neugebauer,[5] Jones has been a major contributor to this field [cf., e.g., Jones 1999]. Perhaps next in interest in the last decades has been Ptolemy's *Geography*, which has also been studied by Berggren and Jones [2000], but which has received more attention in the Berlin-based study-group responsible for the most recent edition of this work (which has probably reached a smaller audience due to the use of German) [see Stückelberger and Graßhoff 2017]. In parallel with Jones' work on astronomy, study of ancient Greek music has been revived by a handful of specialists, most importantly by Andrew Barker, whose clear and didactic annotated translation of virtually all ancient Greek texts on music theory has done an invaluable service in attracting people to the field.[6] Barker [2000] is also particularly relevant here because he has contributed

[3] For Galen's diverse readership, see Johnson 2010, 85–87.

[4] Archimedes, *Sph. et cyl.* 1 pref. ὤφειλε μὲν οὖν Κόνωνος ἔτι ζῶντος ἐκδίδοσθαι ταῦτα. For a survey of the number of mathematicians in the ancient Greek and Roman world, see Netz 1999, ch. 7.

[5] I am referring to the greatly influential Neugebauer 1975 and Neugebauer and van Hoesen 1987.

[6] Most of the ancient texts are included in Barker 1989. In addition, Porphyry's commentary has been recently reedited (without a new inspection of the manuscripts), translated, and annotated in Barker 2015. Cf. my review in Tolsa 2016a.

a monograph to examining Ptolemy's scientific method in the *Harmonics* [cf. Creese 2010].

As Feke recognizes [2], there have been very few scholars who have been interested in Ptolemy's philosophy for its own sake, that is, without being interested primarily in one of the particular mathematical fields in which Ptolemy was active—indeed, it is difficult to say where his philosophy ends and where his mathematics begins, for, according to him, mathematics is *the* main part of philosophy. A complete study of Ptolemy's philosophy should, therefore, be a study of the whole Ptolemaic corpus, an obviously titanic task out of the reach of a single individual in a relatively short interval. Thus, the title of the precursor to Feke's book, Liba Taub's *The Natural Philosophical and Ethical Foundations of Ptolemy's Astronomy* [1993] is, strictly speaking, a more apt description of the content of this kind of study, if perhaps a less catchy one. To find another longer piece on Ptolemy's philosophy *per se* we have to go back to Franz Boll [1894]. Nevertheless, Boll's study was to a significant degree concerned with defending the authenticity of the ascription to Ptolemy of the philosophical essay *On the Criterion*, and, especially, of the astrological treatise, the *Tetrabiblos*, and accordingly devoted more effort to underlining the coincidences between the philosophical tenets in the various works than in explaining them. Neither Taub nor Boll reviewed Ptolemy's *Harmonics*—I suspect Barker's work has been responsible for making this step possible—which is what leads to Feke's claim that 'this monograph is the first ever reconstruction and intellectual history of Ptolemy's general philosophical system' [2].

From a more general perspective, there are two scholarly fields which have seen a significant development in the last times, and which are relevant to research on Ptolemy's philosophy. One of them, signaled by Feke in her introduction [3–4], is the revitalized research on the philosophical milieu of early Roman times, especially the authors labelled as Middle Platonists. In particular, Feke points out clear affinities between Ptolemy and Alcinous' handbook (on the divisions of theoretical philosophy [30]), and with Albinus' introduction to Plato (on becoming similar to god [69]). Also noteworthy is the new interest in the work of the first Aristotelian commentators, Adrastus and Aspasius (second century AD), from which only parts of the latter's commentary on the *Nicomachean Ethics* are extant, and in which Feke finds an interesting parallel to Ptolemy's discussion of the possible dependency of the practical part of philosophy on the theoretical [54–55].

Finally, Ptolemy's output can be fruitfully compared with that of his scientific peers, though Feke does not go into this. Recently, there have been great

efforts in the scholarly community to understanding how knowledge was created, shaped, and presented in the early Roman empire. Again, Galen is the evident parallel for Ptolemy because of the deep philosophical entrenchment of his medical project.[7] But analysis of the work of other scientific authors such as Vitruvius, Hero of Alexandria, Theon of Smyrna, Nicomachus of Gerasa, or Plutarch (to name just a few) can also contribute to the appropriate contextualization of Ptolemy's endeavors.[8] Of course, this is an immense topic, and the individual researcher needs to choose where to set limits. Feke has decided to study all of Ptolemy's 'strictly' philosophical passages, but it would also be possible, and even desirable, to explore a particular facet of Ptolemy's project in relation with similar practices among his peers.

In my view, whereas Feke's study is highly valuable, the need for further contextualization should be emphasized, since we otherwise run the risk of isolating Ptolemy from his contemporaries. Feke does a great job of making sense of Ptolemy's system internally, surveying everything in Ptolemy that can be related to 'straight' philosophical texts in and around his time—including the preface of the *Almagest* [ch. 2–4], harmonic theory [ch. 5–6], psychology [ch. 7], astrology and cosmology [ch. 8)—but we are lacking a context explaining why Ptolemy presented his mathematics in this highly harmonized philosophical system. In this sense, not only comparison with other scientists is needed, but also with other intellectuals who used philosophical doctrines as a ready toolbox to present their special knowledge. I am thinking, for example, of Philo of Alexandria, who, a little more than a century before Ptolemy, explained the Bible using a mainly Platonic framework. My own research on Ptolemy, roughly parallel with Feke's, has shown that some elements of Ptolemy's system go back to such Alexandrian philosophers of the first century BC who made an impact on Philo as Eudorus and Aristo.[9]

This adds a geographical dimension that is relevant to Ptolemy, and which is totally absent in Feke's book. Philosophical ideas from the Hellenistic schools seem to have been transferred to the Alexandrian milieu only after Hellenistic times, where they were newly combined without the influence of the philosophical schools. It is perhaps not by coincidence that Alexandria

[7] See, e.g., the essays collected in Gill, Whitmarsh, and Wilkins 2009.

[8] See, e.g., the collected papers in König and Whitmarsh 2007, Taub and Doody 2009, and König and Woolf 2017.

[9] See the parallels with Plutarch, probably deriving from Eudorus, in Tolsa 2014. For the comparison of the criterion with a law court and the parallel with Aristo of Alexandria (famous for leaving the Academy for the Peripatos], see Tolsa 2016b.

was the seat of Potamo's eclectic sect of philosophy.[10] Admittedly, this is a very difficult topic, mainly due to the almost complete obliteration of the relevant sources. But from what survives, it has been established that, in the metaphysics of Alexandrian Middle Platonists such as Eudorus, mathematical objects played a new, mediating role between the physical world and the forms [see Bonazzi 2011]. This may have constituted the basis of Ptolemy's subversive claim that mathematics surpasses, and at the same time contributes to, the other branches of philosophy, thanks both to its non-conjectural epistemological status and to its mediating position. Such views had obviously to do with the revival of the *Timaeus* as the major Platonic text explaining the new globalized world, beginning with Stoics like Posidonius, interested in underlining the great interconnections and *mirabilia* of this new *oikoumene*.[11]

This leads to another interesting topic of early Roman intellectual history, namely, the packaging of knowledge in disciplines. Ptolemy synthesizes all mathematical astronomy in his *Almagest*—which he accordingly simply calls the *Astronomical Composition* (μαθηματικὴ σύνταξις)—all astrological knowledge in the *Tetrabiblos*, and the geographical coordinates of the whole inhabited world, as known by the Romans, in his geographical treatise. Such an encyclopedic, mathematical project, the result of combining Ptolemy's genius with that of all his predecessors in one book per discipline, is a typical Roman development. Critical collection of knowledge inherited from the past, combined with new insights and great synthesizing ability, are the essential ingredients of many Roman intellectual projects that prefigure the medieval curriculum of disciplines. The mathematical sciences of Plato's *Republic* 7, which finally came to form the quadrivium, were already the way in which Theon of Smyrna and Nicomachus of Gerasa organized their work; and we also hear of Varro's classification of the knowledges in nine disciplines [Vitruvius, *De arch.* 7.pr.14]. Ptolemy's discussion of the relation between astronomy and harmonics in *Harm.* 3.3 is illustrative of how conscious he is about such classifications; significantly enough, he adapts a traditional analogy—he calls astronomy and harmonics cousinly rather than sister sciences—that goes back to Archytas, and to which Nicomachus also recurs when presenting his Platonic division of knowledge [*De arith.* 3].

[10] Cf. the useful review of the main philosophical trends in early Roman Alexandria in Hatzimichali 2011, ch. 2.

[11] Regarding this Posidonian connection, it is noteworthy that both Eudorus and Aristo wrote a book on the source of the Nile flood, according to Strabo, *Geog.* 17.1.5.

It seems to me, however, that Feke's analysis lacks awareness of such passages that create a frame-story, since she wonders at length [131–140] about the fact that, whereas, for Ptolemy, both harmonics and astronomy share *a priori* the same status as incontrovertible knowledge, he ends up admitting the reality that complete agreement of the astronomical models with the observations is impossible [e.g. *Alm*. 3.1]. Feke asks herself why it is that, if Ptolemy makes astronomy dependent on geometry, he bothers to quantify his geometrical models, which makes the disagreement perceptible. Both astronomy and harmonics had long been independent sciences, and, although their similarities made them comparable for Archytas, Plato, and Ptolemy, they had different epistemologies. For astronomers of Ptolemy's time, it was a mandate to provide astrologers with the means of assessing the solar, lunar, and planetary longitudes; and Ptolemy was, as we know, greatly successful at this, even if, of course, the resulting numbers did not completely agree with the observations. Unlike in the case of astronomy, music is human-created, and small changes can be forced into the musical scales in order to make them fit a predefined pattern. This is why in *Harm*. 2.1, where Ptolemy 'proves' that real music as performed by a real citharode fits the mathematical ratios that he has established for his tetrachords in the previous book, he can stipulate that all intervals heard must be made of epimoric ratios, i.e., ratios of the form (n+1):n, which makes the job much easier.[12]

Nevertheless, Feke clarifies for us Ptolemy's quite idiosyncratic ordering of knowledge, showing that his Middle Platonic/Aristotelian division of philosophy in the preface of the *Almagest* can serve to illustrate many features of his work. Feke is particularly good at unpacking Ptolemy's dense language in his exposition of these divisions, and at explaining how sections or major parts of his work indeed consist in his announced application of mathematics to physics, be it that of harmonics to psychology (Ptolemy conceives the soul as material) and cosmology (the heavenly substance is aether), or that of astronomy to astrology and cosmology. However, Feke contends, this nice picture does not work for *On the Criterion* because it contains no mathematics [6, 145]. Here I remain unconvinced by her arguments that the essay does not treat mathematics because it was written before Ptolemy had conceived his global project. It may well be the case, and it seems indeed probable, that this text was written before the *Harmonics* and all other pieces; but I doubt that this is the reason why it had no mathematics. The criterion of

[12] Cf. on this important chapter and the curious transformations it suffers in Porphyry's commentary, Tolsa 2017a.

truth was for Ptolemy an epistemological prerequisite for his investigation in mathematical harmonics, and not the other way around. I would be at a loss to image what kind of mathematics could have been included in the text, if not examples, but the *Harmonics* already provides the best possible example of application of the criterion. The manual of Alcinous is again illustrative, since it presents the topic of the criterion in a separate chapter [4], even if philosophy has been previously divided into the practical and the theoretical [3] in almost the same way as Ptolemy, without including the criterion of truth. To compare with Galen: the criterion was important to the study of harmonics in a similar way as logic was for the doctor who wanted to make deductions correctly, and that is why Galen wrote treatises on logic (without, of course, applying medicine to logic).[13] Therefore, I would suggest that the criterion functions in Ptolemy's system as proto-mathematics. This would explain that the latter part of the essay deals with psychology (i.e., here Ptolemy would also apply mathematics to physics).

As for the application of mathematics to the other branch of theoretical philosophy, theology according to the preface of the *Almagest*, it is not clear whether we can find any tangible example in Ptolemy's works. Of course, Ptolemy implies that the very study of astronomy makes us followers of divine beauty, habituating or disposing us to the same state of soul, and this by itself could presuppose a contribution to theology. I would, however, add that Ptolemy was self-consciously designing his works to be beautiful, well-proportioned objects, and that they could in this way be conceived as offerings to the gods (and, therefore, as a form of theology). It is well known that the concepts of beauty and proportion are highly related in Platonism, as well as in Ptolemy's own thought [cf. *Harm.* 1.3.4]. The Canobic inscription, which was dedicated to a 'savior god' who preserves what is written, is divided into two sections of a relative length similar to that of the two main sections of the *Harmonics*. In both cases, the last section is devoted to applying harmonic ratios to the physical world: in the case of the *Harmonics* to the human soul and the heavens, and in the case of the inscription only to the heavens. *On the Criterion* also presents a similar division. Through textual analysis (especially of the *Harmonics*), it is possible to relate this rhetorical structure to the two-part discourse of *Timaeus* on the creation and the nature of the cosmos in Plato's dialogue, the second part beginning with the introduction of the receptacle (48b προσήκουσαν ἑτέραν ἀρχὴν)

[13] There are some hints at the Aristotelian categories in the text *On the Criterion*: cf. again Tolsa 2016b.

[cf. Tolsa 2014]. It is distinctive of Timaeus' speech that he calls upon the gods at the two beginnings, and also in opening the follow-up dialogue, the *Critias*, appealing as Ptolemy does, to the divine savior capacity granting success to the intellectual project.[14] My guess is that such proportionate division, common to these three works, is meant to make the text beautiful and thus, in a certain sense, agreeable to the gods, something which would have been at least desirable in the case of the Canobic inscription, itself an object dedicated to a deity.

Another kind of elaborate presentation can be seen in the *Almagest* and in the *Geography*, in which long catalogues (occupying the most part of the work in the latter case) appear in both cases carefully positioned toward the middle of the whole text, as if in a circular structure, perhaps mimicking the circular nature of the object of study. Again, Ptolemy was not alone here, since several authors adopted analogous textual strategies to underline the divine character of their works: to mention just a couple of examples, Vitruvius divided his architectural treatise into 10 books, which is surely not just a coincidence with the fact that this number was considered perfect in antiquity, as mentioned by himself [*De arch*. 3.1.8]; and Galen compares his master work, *De usu partium*, to an epode at the very end of the text,[15] explaining that this part of lyric poems was addressed to the gods (ὑμνοῦντες τοὺς θεούς).

BIBLIOGRAPHY

Barker, A. 1989. *Greek Musical Writings: vol. 2. Harmonic and Acoustic Theory*. Cambridge.

_____ 2000. *Scientific Method in Ptolemy's 'Harmonics'*. Cambridge, UK.

_____ 2015. *Porphyry's Commentary on Ptolemy's* Harmonics*: A Greek Text and Annotated Translation*. Cambridge, UK.

Berggren, J. L. and A. Jones. 2000. *Ptolemy's Geography: An Annotated Translation of the Theoretical Chapters*. Princeton, NJ.

[14] Plato, *Tim.* 27c, 48d; *Critias* 106a–b. On the significance of invoking the deity in the *Timaeus* and the *Almagest*, see Tolsa 2017b.

[15] Kuhn 4.365 ὁ λόγος οὗτος ὥσπερ ἀγαθός τις ἐπῳδὸς ἐξηγεῖται.

Boll, F. 1894. 'Studien über Claudius Ptolemäus: Ein Beitrag zur Geschichte der griechischen Philosophie und Astrologie'. *Jahrbücher für classische Philologie*. suppl. 2149–2244.

Bonazzi, M. 2011. 'Towards Transcendence: Philo and the Renewal of Platonism in the Early Imperial Age'. Pp. 233–252 in in F. Alesse ed. *Philo of Alexandria and Post-Aristotelian Philosophy*. Boston/Leiden.

Creese, D. 2010. *The Monochord in Ancient Greek Harmonic Science*. Cambridge, UK.

Feke, J. 2009. *Ptolemy in Philosophical Context: A Study of the Relationships between Physics, Mathematics, and Theology*. PhD dissertation, University of Toronto. Toronto, ON.

Gill, C.; T. Whitmarsh; and J. Wilkins. 2009. edd. *Galen and the World of Knowledge*. Cambridge, UK.

Hatzimichali, M. 2011. *Potamo of Alexandria and the Emergence of Eclecticism* Cambridge, UK.

Johnson, W. A. 2010. *Readers and Reading Culture in the High Roman Empire: A Study of Elite Communities*. Oxford, UK/New York.

Jones, A. 1999. *Astronomical Papyri from Oxyrhynchus*. 2 vols. Philadelphia.

König, J. and T. Whitmarsh. 2007. edd. *Ordering Knowledge in the Roman Empire*. Cambridge, UK.

König, J. and G. Woolf. 2017. edd. *Authority and Expertise in Ancient Scientific Culture*. Cambridge.UK.

Netz, R.. 1999. *The Shaping of Deduction in Greek Mathematics*. Cambridge, UK.

Neugebauer, O. 1975. *A History of Ancient Mathematical Astronomy*, 3 vols. Berlin.

Neugebauer, O. and H. B. van Hoesen. 1987. *Greek Horoscopes*. Philadelphia.

Stückelberger, A. and G. Graßhoff 2017. edd. *Ptolemaios. Handbuch der Geographie*. 2 vols. Berlin.

Taub, L. 1993. *The Natural Philosophical and Ethical Foundations of Ptolemy's Astronomy*. Chicago.

Taub, L. and A. Doody. 2009. edd. *Authorial Voices in Greco-Roman Technical Writing*. Trier.

Tolsa, C. 2014. 'Ptolemy and Plutarch's *On the Generation of the Soul* in the *Timaeus*: Three Parallels' *Greek, Roman, and Byzantine Studies* 54:444–461.

_____ 2016. Review of Barker 2015. *The Classical Review* 66:392–394.

Tolsa, C. 2016b. 'Ptolemy's Law Court Analogy and Alexandrian Philosophy'. *Ancient Philosophy* 36:465–477.

_____ 2017a. 'Porphyry's Distortion of Ptolemy *Harmonics* II.1'. *Phoenix* 71:44–65.

_____ 2017b. 'Ptolemy's Savior God, "Saving the Phenomena" and Plato's *Timaeus*'. *Museum Helveticum* 74:144–157.