

*An Ancient Greek Treatise on Magic Squares* by Jacques Sesiano

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The two earliest Arabic treatises explaining the construction of magic squares date from the 10th century AD. One is found in the *Commentary on the Arithmetical [Introduction]* (Kitāb tafsīr al-Arithmāṭiqī) by ‘Alī ibn Aḥmad al-Anṭākī (d. 376 H/AD 987). Only book 3 of the original three books is extant, and that in a single manuscript. This book is divided into three chapters: the first is a collection of arithmetical definitions, statements of propositions, and identities assembled from Greek and Arabic sources; the second is on magic squares; and the third deals with “hidden numbers”, in which a person thinks of a number and another discovers it after operations are performed on it. Curiously, none of these chapters have anything to do with Nicomachus’ *Arithmetical Introduction*, on which the book is supposed to be a commentary.

The other text is by Abū l-Wafā’ al-Būzjānī (d. 388 H/AD 998), the author of several well-known treatises on arithmetic, geometry, and astronomy. His *Book on the Arrangement of the Harmonious Number in Squares* (Kitāb fī tartīb al-‘adad al-wafq fī l-murabba‘āt) gives an elementary introduction to the construction of magic squares. (The phrase “harmonious number” (‘adad al-wafq) is the Arabic designation for “magic square”.)

In 2017, Sesiano published the Arabic text of the al-Anṭākī manuscript and selections from al-Būzjānī’s book, together with English translations. He noted that al-Būzjānī was a competent mathematician who wrote his book at an elementary level, while, on the other hand, al-Anṭākī was mathematically much less proficient (as we can tell from his first chapter) yet his treatment of magic squares is far more advanced [2017, 10]. Sesiano proposed, reasonably,

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that al-Anṭākī compiled his book from material lifted from other sources, and, further, that this manuscript

is unique in that it attests to the surprising results already obtained in the science of magic squares by the tenth century, thereby suggesting a much earlier, possibly Greek time for the first discoveries. [Sesiano 2017, 9]

There is unfortunately no extant ancient Greek text that even mentions magic squares. It is quite plausible, however, as Sesiano and others have suggested, that Greek authors, particularly those leaning in the direction of Neopythagorean number theory, might have worked in this area. Several people have in fact attempted to identify magic squares in different Greek writings. In 1927, George Sarton briefly suggested Theon of Smyrna; in 1953, H. E. Stapleton wrote that Theodorus, a student of Porphyry, knew the  $3 \times 3$  magic square; and more recently Nicolas Vinel [2005] has interpreted a passage in Iamblichus as alluding to magic squares [Hallum 2021, 62]. Sarton has been shown to be mistaken; Stapleton's argument remains unconvincing; and although Vinel's evidence is intriguing, it too has failed to convince others. The earliest Greek text describing magic squares remains that of the Byzantine scholar Manuel Moschopoulos, from ca 1300 AD. This work derives from Arabic or Persian sources, so it is not part of any Greek tradition [Sesiano 2017, 6].

Recently the Qatar National Library has digitized a number of manuscripts in high resolution color that belong to the British Library and put them online with no copyright restriction. In 2017, they made available *Delhi Arabic 110*, which contains two unrelated treatises, the second of which is titled *Collection of Harmonious Number* or, less literally, *Magic Square Collection* (Dīwān al-'adad al-wafq). This text was written in 517 H/AD 1123–1124, and the manuscript was copied in the late 9th H/15th or early 10th H/16th century [Hallum 2021, 118, 120]. It is a compilation from a variety of sources, with the borrowings from different authors interspersed throughout its eight chapters. Sesiano has discovered that one author in particular served as the source for al-Anṭākī's chapter on magic squares: al-Mufaḍḍal ibn Thābit al-Ḥarrānī, who “is doubtless related to the known translator Thābit ibn Qurra (836–901), perhaps his son” [12].

In the book under review, Sesiano has collated al-Mufaḍḍal's portions of *Delhi Arabic 110* and the second chapter of al-Anṭākī's book 3 to produce an edition with facing English translation [15–161]. Both manuscripts contain parts absent in the other, so the new edition is welcome. This constitutes the core of the book, which is preceded by two short chapters, the first a general introduction to magic squares and the second a description of

the manuscripts with a brief survey of the treatise [1–14]. After the edition and translation Sesiano gives a general commentary [163–219] covering the different sections of the treatise in order. The Arabic glossary that follows [221–229] gives references in the text to Arabic words, but without English equivalents, and similarly for the short Greek glossary in the second column of page 229. The book ends with a two-page bibliography and a four-page index. The title, “An Ancient Greek Treatise on Magic Squares”, that Sesiano has given his book refers to the text of al-Mufaḍḍal/al-Anṭākī, which he argues is a translation that al-Mufaḍḍal made of a now lost Greek original.

The sections of the Arabic treatise deal in order with:

- (a) odd-order bordered squares,
- (b) odd-order bordered squares with separation by parity,
- (c) even-order bordered squares, and
- (d) even-order composite squares.

Sesiano very clearly explains the methods of construction and the mathematics behind them in his commentary. Naturally, there is some overlap between this commentary and the one in his book of 2017, but he has not imported blocks of text from one into the other. The commentary here is new. Quite a bit of the first chapter, on “General Notions of Magic Squares”, is identical or nearly so to the corresponding part of the earlier book, but it would not have been worth the trouble to rewrite this elementary introduction.

Sesiano is not the only one who has examined [Delhi Arabic 110](#). Just this year Bink Hallum of the British Library published the article “New Light on Early Arabic *awfāq* Literature”, in which, he says, “The second half of this article will focus on a newly identified manuscript of a previously unknown *awfāq* treatise from the sixth/twelfth century” [[Hallum 2021](#), 60]. This manuscript is the same Delhi Arabic 110. Sesiano of course could not have known of this article, and the most recent of Sesiano’s writings in Hallum’s bibliography is [Sesiano 2017](#).

Hallum has researched the author singled out by Sesiano, and identifies him as Abū l-Khaṭṭāb al-Mufaḍḍal ibn Thābit al-Ḥarrānī (d. before 368 H/AD 978–979), a poet and mathematician who worked in Baghdad under the patronage of al-Muhallabī (d. 352 H/AD 963), *wazīr* to the Buyid *amīr Mu‘izz al-Dawla* (r. 334–356 H/AD 945–967). He was not Thābit ibn Qurra’s son, but descended from another branch of the same Harranian Sabian family [[Hallum 2021](#), 132].

Sesiano explains in his introduction how it is that he came to identify al-Mufaḍḍal's work as a translation from Greek (he labels the manuscript in the British Library as MS *D*):

In the edition of Anṭākī's text, the present writer, considering the weakness of some of Anṭākī's work and the high level of the part on magic squares, observed that this latter part suggested a *much earlier, possible Greek time for the first discoveries*. This is fully confirmed by MS *D*, which reproduces al-Mufaḍḍal ibn Thābit's introduction, omitted in Anṭākī's copy, which attests that we have here the translation of an ancient treatise. [12]

Let us look at the relevant part of al-Mufaḍḍal's introduction. Sesiano translates it as:

Then I found in the Library, among the books of the caliphs' collection, two books, for the greater part damaged by termites, so that one could understand just a little of them; the summary note about them was in al-Māhānī's handwriting and the first page for the most part in Ḥusayn ibn Mūsā al-Nawbakhtī's handwriting. Then I examined them, found elucidating them very arduous, (but) it occurred to me that it might be possible to make sense of those parts which had been damaged in one by what had been preserved in the other and to restore the proper meaning by replacing a word by another until the account was correct. [17]

Later, in his general commentary, Sesiano writes of this passage:

The two exemplars used by our translator for his work were found, he says, "in the Library, among the books of the caliphs' collection". Now this library must be that of the *Bayt al-ḥikma*, the "House of wisdom", indeed known to have preserved works collected by the early caliphs. [163–164]

Al-Mufaḍḍal says nothing about the language in which those two books were written, nor does he say that he translated them. These are Sesiano's inferences, which should be critically examined. To start, Sesiano's assumption that these insect-damaged books were written in Greek is at odds with what Dimitri Gutas has uncovered on reviewing the scanty primary evidence we have regarding the *bayt al-ḥikma* [Gutas 1998, 53–60].

The fact is that we have *exceedingly little historical* information about the *bayt al-ḥikma*. This in itself would indicate that it was not something grandiose or significant, and hence a minimalist interpretation would fit the historical record better. [Gutas 1998, 54]

Among the references we have, there is no mention of any Greek books being kept there. Further, the library

was certainly not a center for the translation of Greek works into Arabic; the Graeco-Arabic translation movement was completely unrelated to any of the

activities of the *bayt al-ḥikma*. Among the dozens of reports about the translation of Greek works into Arabic that we have, there is not even a *single* one that mentions the *bayt al-ḥikma*. [Gutas 1998, 59]

Instead, “[i]ts primary function was to house both the activity and the results of translations from Persian into Arabic of Sassanian history and culture” [Gutas 1998, 58]. Gutas and van Bladel later wrote:

The idea, developed in twentieth-century scholarship, that this *bayt al-ḥikma* was a bureau for the large-scale translation of Greek books into Arabic, operating along the lines of a modern research institute, or even a college, is entirely incorrect. [Gutas and van Bladel 2009]

They trace the myth of the House of Wisdom as a repository of Greek books and of Greek translation activity back to DeLacy O’Leary’s *Arabic Thought and Its Place in History* [1922]. Later authors gave even more unwarranted prominence to the library. Gutas writes:

The maximalist position is offered by Y. Eche, *Les bibliothèques arabes publiques et semi-publiques en Mésopotamie, Syrie, Egypte au Moyen Age*, Damascus, Institut Français de Damas, 1967, 9–57, i.e., 49 pages of imaginary reconstruction on the basis of barely a dozen one-line references in the sources. [Gutas 1998, 54 n44]

Indeed, in his commentary Sesiano gives as his source for the Caliphal library “Eche, *Bibliothèques*, 26–27, 36, 56–57” [164 n196].

Hallum has also translated al-Mufaḍḍal’s introduction. He renders the passage in question, with added reference numbers, as:

[5] Then, in a storehouse of the books of our elders, I found two books so destroyed by termites that only a little of each of them could be made out. [6] The shorter (*mukhtaṣar* [lit., “abridgment, summary”] of the two was [copied] by the hand of al-Māhānī, and the first folio of the longer was [copied] by the hand of al-Ḥasan b. Mūsā al-Nawbakhtī. [7] When I read them, I found comprehending them extremely fatiguing (*fa-wajadtu istikhrājahumā mu’tiban jiddan*), [8] but it occurred to me that it was possible to derive some of what was damaged in each one of them from what had survived in the other and from the fact that, because they deal with the same subject, the same terminology is used in the same ways, so that the collation can be confirmed. [Hallum 2021, 138–139]

Here, instead of “in the Library, among the books of the caliph’s collection”, we have “in a storehouse of the books of our elders”. In fact, the word «shaykh» (pl. «shuyūkh» in the manuscript) does not mean “caliph”, but more generically “old man” or “elder”, or, more specifically in this case, “a man whose age gives him a claim to reverence or respect” [Lane and

Lane-Poole 1955, 1629]. The specific title of caliph was, instead, «amīr al-muʾminīn», literally, “Commander of the Faithful”.

Hallum analyzes the passage:

Sentence 5 tells us that an ancestral library at Baghdad contained at least two manuscripts on *awfāq*, but to whom did al-Mufaḍḍal refer as *our elders* (shuyūkhunā)? With whom was he identifying when he wrote “our”? It is tempting to think that he meant the elders of the Sabian community at Baghdad, but there are other plausible explanations. For example, he may have been referring to the elders of some other group of which he felt a part, such as the *majlis* around the vizier al-Muhallabī. [Hallum 2021, 141]

There were many libraries in Baghdad in the 10th century and, from what we read in this passage, al-Mufaḍḍal was browsing not the famous *bayt al-ḥikma*, but some other “storehouse of books”. So, even if somehow Eche were to be vindicated and the glory of the *bayt al-ḥikma* as a repository of Greek books were to be restored, al-Mufaḍḍal did not find his two exemplars there. There is in fact nothing in al-Mufaḍḍal’s introduction that allows us to infer that the two books that he found were written in Greek.

It was the incongruity between al-Anṭākī’s chapter and al-Būzjānī’s book that initially gave Sesiano reason to propose a Greek origin for the former. Recalling this later, he wrote that

it seemed as if the second [al-Būzjānī] was providing the basic elements of a nascent science, whereas the first [al-Anṭākī] treated specialized topics certainly not accessible to general readers. [Sesiano 2019, v]

This anomaly was resolved for him by the discovery of the text of al-Mufaḍḍal in *Delhi Arabic 110*, which he first announced as a translation from Greek not in the book under review, but in his history and construction of magic squares [Sesiano 2019, v–vi, 8ff]. Since al-Mufaḍḍal

does not explain the basic elements of the science of magic squares, Būzjānī’s treatise is an attempt to do it. The origin of Arabic studies on magic squares is thus Thābit’s translation.<sup>1</sup> [Sesiano 2019, v–vi]

What made this an incongruity in the first place is that Sesiano found it implausible that a competent mathematician like al-Būzjānī would write such an elementary book if the theory of magic squares, starting from its foundations, had already been covered by Arabic writers. Thus, it made sense that al-Mufaḍḍal had translated an advanced book in Greek that assumes familiarity with elementary methods, that it was this translation that introduced magic squares to the Arabic-speaking world, and that al-Būzjānī

<sup>1</sup> At that time Sesiano had mistakenly identified the translator as Thābit ibn Qurra.

was one of the first to step in to provide the foundational principles lacking in it.

Now, this interpretation presupposes that competent mathematicians do not write elementary textbooks. Said another way, al-Būzjānī's book, like the books of other Arabic authors on mathematics, should be expected to extend to the limits of the author's knowledge. But al-Būzjānī himself wrote the textbook on practical geometry titled *Book on What Is Needed by the Artisan for Geometric Construction* (Kitāb fīmā yaḥtāju ilayhi al-ṣānī' min al-ʿmāl al-handasiyya). It is no contradiction that al-Būzjānī knew his Euclid well, and yet wrote a book that provides instructions useful for people who needed basic geometric procedures for practical work.

There are many other examples too of elementary textbooks penned by accomplished Arabic mathematicians, such as Ibn al-Haytham's *Arithmetic of Transactions* (Ḥisāb al-muʿamalāt), Ibn al-Bannā's *Condensed [Book] on the Workings of Calculation* (Talkhīṣ ʿmāl al-ḥisāb), and Naṣīr al-Dīn al-ṭūsī's *Collection of Calculation by Board and Dust* (Jāmiʿ al-ḥisāb bi'l-takht wa'l-turāb). In the case of Ibn al-Bannā, it is undeniable that his knowledge of *ḥisāb* extended well beyond what he wrote in his short book, given what we read in his own commentary on that book.

So, why cannot al-Būzjānī's *Book on the Arrangement of the Harmonious Number in Squares* also have been composed to introduce the topic to beginners? Sesiano himself writes that in al-Būzjānī's book "everything is thoroughly discussed and in a highly pedagogical way" and that "it represents an excellent introduction to the study of magic squares" [Sesiano 2017, 16]. Such is the description of a textbook.

There is in fact no incongruity between the texts of al-Anṭākī and al-Būzjānī, so there is no reason to propose that al-Anṭākī's book is ultimately a translation from Greek. It is perfectly plausible, contrary to Sesiano's account, that the books from which al-Mufaḍḍal copied were earlier, now lost Arabic books, and that other lost books of that time covered the basics that al-Būzjānī later re-presented. We are thus right back where we were before Sesiano wrote his study in 2017. We can say no more than that it is plausible, perhaps even likely, that Arabic authors learned of techniques of constructing magic squares from Greek sources.

Although the title of Sesiano's book, *An Ancient Greek Treatise on Magic Squares*, is unwarranted, the edition of the text is rigorous, the translation is overall clear and accurate, and the commentary presents the mathematics in a well-organized manner.

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